Chapter 1

Intentionality and Foundations of Logic: a New Approach to Neurocomputation

Gianfranco Basti

Pontifical Lateran University

Abstract

In this work we start from the idea that intentionality is the chief characteristic of intelligent behavior, both cognitive and deliberative. Investigating the "originality of intelligent life" from this standpoint means investigating "intentional behavior" in living organisms. In this work, we ask epistemological questions involved in making the intentional behavior the object of physical and mathematical inquiry. We show that the subjective component of intentionality can never become object of scientific inquiry, as related to self–consciousness. On the other hand, the inquiry on objective physical and logical components of intentional acts is central to scientific inquiry. Such inquiry concerns logical and semantic questions, like reference and truth of logical symbols constituted as such, as well as their relationship to the "complexity" of brain networking. These suggestions concern cognitive neuroscience and computability theory, so to constitute one of the most intriguing intellectual challenges of our age. Such metalogical inquiry suggests indeed some hypotheses about the amazing "parallelism", "plasticity" and "storing capacity" that mammalian and ever human brains might exhibit. Such properties, despite neurons are over five orders of magnitude slower than microchips, make biological neural nets much more efficient than artificial ones even in execution of simple cognitive and behavioral tasks.

Keywords: intentionality, cognitive science, artificial intelligence, connectionism, neural networks, foundations of logic, diagonalization.

1.1 Introduction

In this work, we limit ourselves to “the originality of intelligent life”. We begin with the hypothesis that such originality depends in logic and psychology on *intentionality*. We work from *cognitive neurosciences*, because this approach allows us to deal with intentionality from a more rigorous theoretical perspective than from classical ones, such cognitive psychology or phenomenological analysis. This methodology allows us to deal with our problem in *an objective* and, at the same time, *non–redutionistic way*. In the study of mental processes, it links the *neurophysiological* component with the *logical* (semantic) and thus the *psychological* component — from the objective standpoint of the information processing, not from the subjective one of the introspection on consciousness states. The theoretical character of this analysis allows us to attain the *ontological* level of the analysis. I.e., allows us to discuss the *metaphysical* question of the originality of the intelligent life (traditionally defined as the problem of the immaterial character of intelligence) by using the "picklock" of *metalogic*. In other terms, it becomes possible to deal with the metaphysical question of the originality of the intelligent life, starting from the foundations of semantic "objects" such as "truth", "reference", "meaningfulness" of statements in a given language. Particularly, we start from the hypothesis that the process of logical constitution of these semantic relations and operations requires to be "implemented" in physical structures provided with given properties.

Our work is divided into two main sections. In the *First Section*, we deal with the study of intentionality, as characteristic of intelligent life, in the framework of *Artificial Intelligence* (AI) and of *connectionism* (Neural Networks, NN) research programs. We show the logical and meta–logical limitations of these two approaches to the problem of intentionality. In the *Second Section*, we discuss the relevance of a particular approach to the problem of logical foundations after the *Gödel incompleteness theorems* and its relevance for the problem of intentionality. This approach constitutes the *logical* counterpart of the well–known *epistemological* theory of true knowledge as *self–conforming* (*adaequatio*) of the mind to reality. This foundational theory consists in a particular application to the constitution of the logical objects of Thomas Aquinas's general ontology. This ontology is founded on the real distinction between *being as essence* and *being as existence*, considered as two metaphysical and/or metalogical constituents of each thing (either physical or logical). We emphasize particularly the relevance of this approach for dealing with characteristic problems related to the Gödel incompleteness theorems for formal systems. The only way to avoid such limitation theorems is to allow a change of axioms in the formal system concerned, so to make it "dynamic" or "recursive" in a deeply new sense. We suggest the relevance of such an approach for a logically consistent theory of intentionality, as well as for the solution of cognitive neuroscience problems related to neural dynamics and neural computations relationships — e.g., the true question of "parallelism" in brain computations, the "plasticity" and the "memorization capabilities" of brain computations, with respect to their artificial simulations. They are all questions for which neither AI, nor NN approaches to cognitive neuroscience, have satisfying solutions.

1.2 Intentionality and Cognitive Neuroscience

1.2.1 The Functionalist Approach in Cognitive Neuroscience

1.2.1.1 The Origins of the Functionalist Approach

When the AI research program was born in late 50's, it was generally held that a new age in psychological and neurophysiological studies was starting: the age of *cognitive sciences* [1]. Effectively, this approach seemed to constitute an escape from the old dichotomy in scientific psychology between:

1. the *subjectivism* of the introspective method of phenomenological psychology, typical of the *cognitivism* of *Gestalttheorie*; and
2. the *objectivism* of the mechanistic method of associative psychology, typical of *behaviorism.*

By way of difference, the *functionalist theory of mind* introduced by Hilary Putnam [2], argued that the objective correlate of a subjective state of consciousness is double. It is constituted by the *information flow of the logical operations* in the brain, considered as a logical (computational) machinery, and not by the simple *energy flow of its physical operations.* Philosophically, the problem of the mind – body relationship could be reduced to the problem of the relationship between the *software* and the *hardware* of the computational architecture of the brain.

The functionalist approach in the study of mind is the final chapter of a long history in the modern theory of mind that has the following main steps:

1. The first step was the development of a *rationalist theory of mind* by modern philosophers such as Descartes, Leibniz and Kant. This theory

identifies the thought processes with *formal inferences,* with logical procedures manipulation of symbols according to formal rules. For Kant these rules and procedures are determined *a priori* in human minds and largely *unconsciously.* They can become aware only after a long study, so that only when they are thought *in abstracto* they become objects of a particular science such as the *formal logic* [3]. Particularly, the core of the perception is for Kant an act of *productive fantasy.* It consists in the development of a particular *schemes* or "rules for the fantasy synthesis" for each abstract concept. By this scheme, a given sensible intuition can be organized according to a given formal concept for producing a determinate perception. In short, in our mind we do not have the image of a dog. We have a rule for the constitution of different images of the singular dogs that our sensibility presents to us in different contexts. By this deductive scheme constitution, for each abstract concept there exists a formal scheme for its application on a domain of sensible objects. This "deductive schematism" is thus defined by Kant as "an art concealed in the depth of the human soul, whose real modes of activity is hardly likely ever to allow us to discover" [4].

2. The second step toward the functionalist theory of mind is the development during the last century and the first half of our century of *symbolic* or *mathematical logic*. The aim of this research program, started since the seventeenth century with Leibniz's *characteristica universalis,* was the rigorous construction of the formal logic as a *logical calculus*. This construction reached its apogee at the end of the last century with G. Frege's work. Both the notion of *propositional function,* as a formal scheme with free variables for proposition construction, and the notions of *logical quantifiers,* for the construction of the class logic in the form of a predicate calculus were essential. This improvement made possible the rigorous systematization of the logical calculus into its main three branches of the *predicate* (*class*) *calculus,* of the *propositional calculus,* and of the *relation calculus.*
3. The third step toward the functionalist theory of mind was the demonstration of a *fundamental theorem of computability theory* by the English mathematician A.M. Turing [5]. According to this theorem, each *computable function* of the mathematical and/or of logical calculus can be *recursively* calculated through a *finite procedure* by an appropriate elementary computational architecture called *Turing Machine* (TM). Of course, the behavior of each TM can be simulated by another TM, on condition that, onto the

"ribbon" (memory) of the second one, all the instructions to execute the calculations of the first one are explicitly written in the language of the second TM. Fundamental consequence of this theorem is that the *universality* in computations can be granted *iff* each singular TM in turn can be simulated by a *Universal Turing Machine* (UTM) with an infinite "ribbon". In other words, it is supposed that the universality of the *codes* or "alphabets" used by each single TM for executing its recursive computations can be founded only through the isomorphism (biunivocal correspondence) between these alphabets and the *universal fixed alphabet* of the UTM.

A formal consequence of this theory and of the notion of "λ–calculus" developed by A. Church is the so called *Church's-Turing's thesis* according to which the class of *all the computable functions* is equivalent to the class of the recursively computable functions and this class, in turn, is equivalent to the class of functions computable by a TM. This thesis, because of Gödel incompleteness theorems [6], cannot be formally demonstrated so to remain only a hypothesis. Immediately related with such a limitation theorem is the other one according to which it cannot be formally demonstrated that a UTM can calculate through an ending procedure. This is the famous *halting problem* demonstrated by Turing himself.

However, the anthropological consequence of this theory is that, if we accept the rationalist theory of mind, that is, if we reduce the human thought to a logical calculus, each individual human mind has to be considered as logically equivalent to a TM. Hence each singular mind has to be considered as a function of some "universal mind", defined in the rigorous terms of a UTM. Such a consequence, that constitutes the metaphysically *monist* core of any functionalist theory of mind (see, for instance, [7]), became effective when the final step toward this theory was available in modern scientific psychology.

4. The fourth and ultimate step toward the cognitive sciences was the main hypothesis underlying the so – called "genetic" approach to the study of intelligence. This approach was developed by the Swiss psychologist J. Piaget within the classical approach of cognitive psychology [8]. According to this hypothesis, the development of abstract intelligence in human individuals corresponds to the acquisition of the *operative schemes* of four fundamental logical operations (identical, inverse, reciprocal and correlative). These operations constitute the so called "group of the four transfor-

> mations” granting the relations of *reflexivity, transitivity* and *symmetry* (and hence of *equivalence* and *(extensional) identity*) of the logical reasoning. These schemes are owned by the subject at the unconscious level, so to recover to modern cognitive psychology the notion of the *cognitive unconscious* (see above p. 4) of the Kantian theory of mind schematism [9].
>
> An essential difference with the Kantian schematism has, however, to be soon emphasized. It is essential indeed for our aims of a theoretical treatment of the perception problem within the framework of the cognitive sciences. While the Kantian schematism is essentially *deductive*, Piaget’s schematism would be *inductive*. What is essential for Piaget’s theory of perception is in fact that the perceptual schemes of the operative intelligence are submitted to a procedure of continuous redefinition with respect to changing reality. It becomes possible by supposing a mechanism of *assimilation – accommodation* of the schemes. That is, the new sensible knowledge, as far as it cannot be assimilated to the old *a priori* schemes, determines an accommodation of these schemes to the new occurrences. In this way it grants the development in time of the intelligent capabilities of the subject. This “evolutionary” idea of the scheme constitution recovers thus to modern cognitive psychology the core of the Scholastic theory of *an inductive schematism* typical of its theory of perceptual intentionality [10].

Until now, Piaget’s idea has not found a proper operational correlate in the modern theory of computability. It relies on reasons we illustrate in the next paragraph (See pp. 7ff.), ultimately depending on the same foundations of modern logic and mathematics (See pp. 27ff.). Our systematic effort is thus related to a re-consideration of the foundations of logic and mathematics to overcome these essential limitations. They involve not only the psychology of perception and the cognitive science, but also the modern theory of computability in its many applications in all the fields of modern science.

Finally, and more deeply, these limitations involve the same destiny of realism in modern epistemology (See pp. 16ff. and pp. 27ff.).

However, as far as we do not consider this essential point of the *inductive* versus a *deductive* procedure of scheme constitution, and we uncritically accept an a priori constitution of the schemes in the *cognitive unconscious* of human mind, the following conclusion is not hazardous. The functionalist approach to cognitive sciences is a sort of operational translation of the Kantian transcendental philosophy of mind [11]. Namely, just as the very same *software* can be implemented into different *hardware's,* so, in the framework of the functionalist approach, it could be possible to intend a computer simulating a formal operational scheme like a transcendental counterpart of what individual minds do at the empirical level.

More precisely, this fundamental statement of the functionalist approach to the study of mind can be synthesized in D. R. Hofstadter's terms by the principle of the "AI dogma". Every time the computer simulated successfully a human intelligent behavior, the software of this computation *must necessarily* imply some essential isomorphism with the "software" running in the human brain [7].

This "dogma" exemplifies in one only statement the core of the famous *Turing test* [12], because it is a direct consequence of the computability theory for TM's. Let us suppose that we have to test whether is it a human individual or a computer the mysterious individual "who" is giving us the "intelligent" responses to our questions we are setting "him". "His" mystery is that we cannot see "him" because "he" is in another room and we can communicate with "him" only through a teletypewriter. If a computer effectively gives these intelligent responses, but they are indistinguishable from those normally given by a human individual, the intelligent human behavior has been perfectly simulated by the computer. Hence, according to "AI dogma", some fundamental isomorphism *must* exist between the software running in the machine and the software running in the human mind. The possibility that each TM can be perfectly simulated by another TM "instructed", "programmed" in a suitable way, implies this consequence for the cognitive sciences.

1.2.1.2 Formal semantics and the problem of schematism

This possibility exemplifies also the response (see, for instance [11], [13-14]) that the *functionalist approach* tried to give to the problem of *conscious intentionality* in terms of A. Tarski's [15-16] and R. Carnap's [17] *formal semantics.* Indeed, what the Scholastic philosophy enhanced since Middle Age and modern phenomenological and cognitive psychology rediscovered since the pioneering work of F. Brentano [18], is the *intentional character of any psychical act as such.* In other words, what characterizes any psychical act, as far as it is distinguished from a physical act, is its intrinsic *reference to a content,* or "aboutness". This content has to be considered both in its *extensional* sense (that is, as a given object either physical or ideal) and in its *intensional* sense (with "s", i.e., the intended meaning we associate to that object). In short, *intending* an intensional content and *referring to* some extensional content is what constitutes a psychical act as *intentional*. Hence, considering the act of thought in a purely formalistic way without any reference to a content, in the sense of Descartes' *cogito* or of Kant's *Ich denke überhaupt,* is rightly considered by the phenomenology as a misleading abstraction from the real situation of human psychology. When I think, I desire, I will, I feel, I perceive, etc., I think, desire, will, feel or perceive always *something*! In this way, the problem of logical *truth* of a given proposition has, from the psychological standpoint, an intentional character and from the formal logic standpoint a *semantic* character. Semantics is indeed the logical discipline which "deals with certain relations between expressions of a language and the objects (or "state of affairs") 'referred to' by those expressions" [16]. A. Tarski indeed, for the first time in the history of modern logic, defined in a rigorous way for formal languages this semantic relationship to a content (*reference*) and the semantic relationship of *truth* within a purely *extensional* and *formalistic* approach to this problem.

Tarski solves the problem of a formal definition of semantic concepts like truth by affirming the necessary *semantically open* character of any formal language whose truthfulness has to be rigorously defined and hence (recursively) proved. That is, in discussing the problem of the formal, *consistent* (i.e., that does not imply contradictions) definition of semantic concepts, we have *always* to distinguish between two different languages. The first, the *object – language,* is the language to be checked. The definition of truth we are seeking applies to propositions of this language. The second, the *meta – language,* is the language in which we "talk about" the first one and in terms of which we can construct a consistent definition of truth for the first language propositions. Of course, the two notions are *relative* and not absolute. Indeed, if we want to check the truth of the proposition of the meta–language, we have to consider it the object – language of another meta–language, and so on. The conclusion that no formal language can be the meta–language of itself is directly related with Gödel's demonstration of incompleteness of formal arithmetic (Peano's axiomatic arithmetic), against original Hilbert's formalistic program [16]. This is because formal semantics must use a *recursive* procedure of *satisfaction* for defining formally the notions of truth and reference[1]. Now, for our aims, three reflections about Tarski's semantic theory of truth are to be made:

1. *From the computational standpoint*, Tarski emphasizes that the recursive procedure of satisfaction of a given propositional function with n free variables is a relation with $n + 1$ terms (e.g., for unary functions is a binary relation, for binary functions is a ternary relation, and so on). So, in defining the notion of satisfaction for formal languages with propositional functions of an arbitrary number of free variables, we are not faced with only one notion of satisfaction, but with infinitely many notions that must be introduced simultaneously because they cannot be defined independently. In this way, the core of a recursive procedure of satisfaction, is to define a recursive procedure of substitution of a many–termed relation between propositional functions and an indefinite number of objects, with a binary relation between functions and finite *sequences* of objects with an arbitrary

[1] The satisfaction is a particular semantic relation between arbitrary objects and propositional functions. Generally an object (e.g., *snow*) satisfies a propositional function (e.g., *x is white*) if the latter becomes a true proposition when the name of the object is used to replace the free variables in it (e.g., *snow is white*). Of course, in our case we cannot use this definition of satisfaction for defining truth, since it supposes the definition of truth. Tarski must use thus a recursive procedure for the definition of satisfaction. Starting from objects satisfying the simplest propositional functions (e.g., for natural numbers, all the numbers x and y satisfying the functions "*x is greater than y*", or "*x is equal to y*"), we can define the conditions under which compound functions are satisfied too (e.g., the logical disjunction "*x is greater than y*, or *x is equal to y*" is satisfied for all x and y satisfying at least one of the above simplest functions). In this way, we can construct the formal definition of truth in terms of satisfaction: *A proposition is true if it is satisfied by all the objects and it is false otherwise* [16]. Where, of course, the totality of the objects of which we are speaking about are to be interpreted as the totality of the objects to which the propositions of the object – language refer. With similar recursive procedures, it is possible for formal semantics to give rigorous definitions – in the same framework of the distinction between two different formal languages: the object - language and the meta - language – also of other semantic terms, such as the notion of *reference and/or designation* (e.g., "Columbus designates (denotes) the discoverer of America") as well as the notion of *definition* (e.g., "$x \cdot 2 = 1$ defines (uniquely determines) the number ½").

number of terms [16]. The relationship of this theory of truth with Gödel theorems is thus immediate. Indeed, such a recursive substitution implies to have only one unary function for enumerating recursively a collection of objects, so to have only one ordered sequence of them. In this way, a recursive procedure of substitution is identical with that recursive procedure of *coding* called Gödel *numbering.* The essential result of Gödel theorems is indeed that such a coding function cannot be written in the same formal language (arithmetic) in which the objects and/or the functions to be enumerated are written. That is, it is not possible to conceive such a substitution procedure as a *diagonalization procedure*. A diagonalization procedure can be defined as the iterative procedure of substitution of an *n*–ary function with an unary function. For instance, given a binary function of the type $h(x, z)$ or $f_z(x)$, the diagonalization would consist in its iterative substitution with the unary functions $h(x, x)$ or $f_x(x)$. This last way of writing a unary function, $f_x(x)$, is notable because in it the same x plays the double role of argument and of index of the same function. This suggests that the diagonalization procedure is effectively a procedure of *class closure by diagonalization,* that is, the computational counterpart of the logical notion of *complete induction* [19]. This suggestion is much more than a suspect in the case of the substitution procedure relative to the notion of satisfaction in Tarski's theory of truth. Is not Tarski's definition of truth identified with the satisfaction of a propositional function *simultaneously* for *all* the objects of a given linguistic domain (see note 1)? If Tarski poses the distinction between an object–language and a higher order meta–language as necessary and sufficient condition for his formal (recursive) definition of truth, is thus precisely because such a class closure by diagonalization cannot be performed without contradiction inside the same formal language. This demonstration is indeed the main result of Gödel theorems of incompleteness of the formal arithmetic. This result, precisely through the work of Tarski and Turing, can be thus extended to any formal language.

2. *From the logical standpoint,* another consequence derives from the precedent discussion. As Tarski himself and Gödel rightly emphasized, from such a semantic approach *no absolute* notion of truth becomes possible, even in a *local sense,* i.e., for a *finite* domain of objects. In this regard, it is important to avoid a possible misunderstanding. It is really true that Gödel theorems hold only for *general recursive functions.* That is, they

properly hold only for functions defined on all their *infinite* domain of application. On the contrary, it is impossible to exclude the convergence of the recursive procedure for *partial recursive functions* [20]. Namely, it is impossible to demonstrate Gödel results for recursive functions defined on only a finite subset of their infinite domain of application. In this case, indeed, the recursive procedure could converge within the domain of application even though out of the (sub–)domain of definition. In this sense, it is formally correct to invoke with Kleene a healthy *finitism* to avoid the more destructive effects of Gödel theorems in computability theory [20]. On the contrary, in the case of the semantic notion of truth such a finitism has no effect. Either finite or infinite a domain of objects is, to meet Tarski's criterion of satisfaction, it is necessary to migrate outside a formal language for judging from a higher logical order the truthfulness of its propositions. For this unavoidable necessity of a higher level meta–language, such a formal definition of truth implies that truth notion cannot be absolute at all, but always *relative*. Commenting on this evidence, Gödel in his philosophical reflections rightly quotes Plato's theory of truth. Especially, this result is consistent with the truth theory expressed in Plato's famous *Letter VII*. According to this text, any true knowledge necessarily exceeds any procedure of demonstration as well as any "fixed form" of language. That is to say, truth exceeds any "formal language" that pretends to assert *forever* its primitives and its rules. In short, for Plato as well as for Gödel, the logical universals, either exist ultimately by themselves, or no consistent procedure of construction (i.e., of formal definition and/or of formal demonstration) could ever pretend to constitute them.

3. Finally, *from the epistemological standpoint*, another consequence must be drawn from the previous discussion that is essential for our aims. Owing to its pretension of meeting the Aristotelian notion and hence the common sense notion of truth, namely the notion of truth as "correspondence to reality", it seems that the semantic theory of truth implies by itself an epistemological position of *realism*. Effectively, K. R. Popper tried to interpret it as a theory of truth as *correspondence to facts* [21], as if Tarski's theory of truth was able to give to Popper's biology–inspired epistemology its rigorous formal, and hence scientific foundation. This interpretation of Tarski's results is absolutely inconsistent and the possibility of interpreting the semantic theory of truth as supporting a position of epistemological real-

ism was always explicitly rejected by Tarski [16]. The semantic theory of truth has nothing to say about the conditions under which a given simple ("atomic" in L. Wittengstein's terms) proposition (and overall an empirical proposition) like *snow is white* can be asserted. As he correctly affirms, his theory implies only that whenever we assert or reject this proposition, we must be ready to accept or reject the correlated meta–proposition: *the sentence "snow is white" is true.* In other terms, the semantic theory of truth has nothing to do with the problem of *the formal constitution of true propositions* but only with the problem of *the formal justification of true propositions.* In other words, it is completely immersed within the *axiomatic method* identifying logic with the "logic of justification" of proposition already constituted, and not within the *analytic method,* identifying logic with the "logic of discovery" (See pp. 27ff.). For this reason, in the approaches of the formal semantics and of the functionalist theory to the problem of *reference* there is no room for the treatment of the problem of the *real reference* (See pp. 33ff.). This problem is methodologically excluded in them. So, J. A. Fodor, quoting R. Carnap, rightly emphasized that the treatment of the intentionality problem within the functionalist theory of mind has to be conjugated with a rigorous principle of *methodological solipsism* [13]. In fact, the functionalist theory has nothing to do with the problem of the *reference to reality* of some mental state. Better yet, if we accept the use of Tarski's and Carnap's formal semantics within the functionalist theory of mind as the only possible *scientific* counterpart of the *naive* notion of intentionality in the "folk psychology", the epistemological realism can be only negated in the name of the above remembered methodological solipsism [11.13-14]. Any mind–state that we might characterize as a "propositional attitude" (= the psychological counterpart of a propositional function in the functionalist theory of mind) can refers only to another mind–state or "mental representation", like to the object capable of satisfying it, for constructing valid propositions. And this is unavoidable in a functionalist theory of mind, precisely for the same reason for which in formal logic and in formal semantics, "the fundamental conventions regarding the use of any language require that in any utterance we make about an object, it is the name of the object which must be employed and not the object itself" ([16], p. 55). This is the core of the mentalist *representationalism* and of the logic *nominalism* intrinsic to the functionalist approach [11.13-14]. It justifies completely Fodor's pretension that the functionalist approach is an operational counterpart of the Kantian theory of mind and of his epistemological representationalism, against the epistemological realism.

If the formal semantics constitutes the operational counterpart of the intentionality in the functionalist theory of mind, there is no room in the functionalist approach for Piaget's *inductive schematism.* A process of scheme accommodation poses itself at the level of the formal constitution of the scheme itself. But it is precisely about this procedure of formal constitution of the logical symbols that formal semantics has *in principle* nothing to say.

1.2.1.3 Intentionality and the metaphor of the three "rooms"

It is hard to defend the functionalist pretension of using Tarski's semantics for dealing with psychological intentionality, overall in the study of perception. Indeed, what we mean by "intentionality" is not only the act of *reflexive thought* of formal manipulation of logical symbols and relations already otherwise constituted in our mind. In this sense intentionality could be in agreement with the methodological solipsism of functionalist theory, as well as with nominalism of Tarski's formal semantics. On the contrary, intentionality essentially means the act of *productive thinking* of new logical symbols and hence of new logical relations. In short, intentionality is essentially related to the act of *constitution of symbols,* and in the case of constitution of *true* symbols, intentionality is essentially related to the problem of constitution of symbols *adequate* to the singular context of their use. So, any scientific theory of intentionality must deal with the problem of intentionality at the *pre–symbolic level*.

In summary, the formalist method requires that functionalism posits intentionality only at the symbolic level of mental information processing rather than at the more fundamental pre–symbolic level of the constitution of symbols.

This criticism against the functionalist approach to intentionality has been developed in the last twenty years. In this regard, two other counterexamples of the famous "Turing room" metaphor have been proposed: the "Searle room" and, more recently, the "Putnam room". These two metaphors exemplify indeed two main criticisms that can be posed to the symbolic treatment of the intentionality problems in the functionalist approach. These criticisms are, respectively:

1. from the standpoint of the *intensional* (with *s*) *logic* approach to the theory

of intentionality;

2. from the standpoint of the *theory of coding* in the logical *foundations of computability theory.*

Let us begin with J. Searle's criticism.

1.2.1.4 Searle's "room" and the intensional approach to intentionality

In order to exemplify in which sense the Turing test fails in proposing a valid proof of the equivalence between a mind and a computer, J. Searle proposed the counterexample of his "Chinese room" [22-23]. Let us imagine that a person, who does not know at all Chinese, has to translate a English text into Chinese. Let us suppose to give him a dictionary as well as the complete set of rules sufficient for the exact translation of the text concerned. Even though this person produced a text resulting in an absolutely correct translation for Chinese people, nevertheless this person, *just like a machine,* would have not understood anything of what he produced. In other words, even though a Turing test satisfies the criteria of an extensional approach to the problem of meaning, nevertheless it is impossible to affirm that this approach can be considered as a satisfying operational translation of what we designate as an intentional act of knowing [22-23]. The "relation to a content" as characteristic of any intentional act implies not only the *extensional* reference to names of objects, but also the *intension* of a conscious significance by which we associate names and objects in different contexts. *Intending* a meaning and by it *referring* to an object are not the same thing, even though they are effectively always together in any conscious intentional act.

This reciprocal irreducible character of the intensional and of the extensional components of any intentional act is evident also in the logic of their linguistic expression. In the intensional logic indeed the *extensionality* axiom and the related *substitution* axiom do not hold [24]. For instance, from the extensional standpoint, the notion of "water" and the notion of "H_2O" are to be considered as synonyms, since they apply to the same collection of objects. From the intensional standpoint, however, they do not have the same meaning; just substitute the term "water" with the scientific term "H_2O" in some poetic or religious discourse. The result is meaningless. Owing to the exclusively extensional character of the treatment of the semantic content in the functionalist approach, this approach is absolutely not sufficient for cognitive psychology.

Unfortunately, the constructive part of Searle's theory of intentionality is void of any theoretic and scientific significance. Nevertheless, what Searle's criticism rightly emphasizes is that the functionalist approach to the study of mind cannot be at all adequate owing to its exclusively extensional approach to semantic problems.

For these very same reasons W. V. O. Quine stated that the mind–body problem is essentially a *linguistic* and not ontological problem [25]. So, because of the extensional character of any scientific language, for him intentionality cannot be at all object of scientific inquiry [26]. This reductionism, typical of the logical empiricism of Quine's philosophy is typical also of P. Churchland's interpretation of the connectionist approach to cognitive neuroscience [27].

On the other hand, E. Husserl's early attempt of an intensional approach to foundations of formal logic cannot in principle lead to any constructive approach to the semantic problems of *truth* and of *reference.* Indeed, also the intensional logic solution to the problem of truth supposes a sort of axiom of completeness in formal logic. It supposes completeness at least in the fundamental sense of an equivalence principle between the *non–contradiction principle* and *excluded middle principle*. Only by this equivalence can truth be intensionally founded on the conscious *evidence* [28]. The necessity of this equivalence for any intensional theory of truth as evidence is the deep formal reason for which Husserl abandoned his early attempts of an intensional foundation of formal logic after the publication of Gödel results two years later the publication of his main work on formal logic, *Formal Logic and Transcendental Logic* [29]. Indeed, the incompleteness of any formal language implies the unavoidable presence in it of undecidable statements. That is, in any formal language there is the unavoidable presence of true statements for which it is not possible to demonstrate them or their negation, so to violate the excluded middle principle. Also for late Husserl works, just like for Tarski and Gödel, truth can be thus only a sort of regulative idea in Kantian sense: something that is "beyond" any formal language and demonstration procedure as Husserl's late idea of *universal teleology* exemplifies.

1.2.1.5 Putnam's "room": intentionality and the problem of coding

One of the most exciting events in the brief history of cognitive sciences is the abandonment of the functionalist approach by its supporter who introduced it into the scientific and philosophical debate: the mathematician and philosopher Hilary Putnam. This is related to the unsolvable problems of *reference and truth* characterizing any intentional act, when approached from the standpoint of the computability theory [30-31].

As we can expect from a cultivated logician and mathematician as Putnam is, his complete theoretical conversion form the early functionalism posed the intentionality question at the right place, both from the computational and from the logic points of view. As we saw before (See pp. 7), any formal theory of reference and truth is faced with the Gödelian limits making impossible a recursive procedure of satisfaction in a semantically closed formal language (see also note 1). What we emphasized as the core of the problem is that such a recursive procedure for being complete would imply the solution of the *coding* problem through a diagonalization procedure; that is, the solution of the so–called "Gödel numbering" problem. In computational terms, the impossibility of solving the coding problem through a diagonalization procedure means that no TM can constitute by itself the "basic symbols", the primitives, of it own computations. For this reason Tarski rightly stated that, at the level of the propositional calculus, the semantic theory of truth has nothing to say about the conditions under which a given simple ("atomic" in L. Wittengstein's terms) proposition can be asserted. And for this very same reason, in a fundamental paper about *The meaning of "meaning"* [30], Putnam stated that no ultimate solution exists either in extensional or in intensional logic both of the problem of *reference* and, at the level of linguistic analysis, of the problem of *naming*.

In this sense, Putnam stated, we would have to consider ultimately names as *rigid designators* "one - to - one" of objects in S. Kripke's sense [32]. But no room exists both in intensional and in extensional logic for defining this natural language notion of *rigid designation* in terms of a logical relation, since any logical relation only holds among terms and not between terms and objects, as Tarski reminded us. Hence a formal language has always to suppose the existence of names as rigid designators and cannot give them a foundation.

To explain by an example the destructive consequences of this point for a functionalist theory of mind, Putnam suggested a sort of third version of the famous "room – metaphor", after the original "Turing test" version of this metaphor and J. Searle's "Chinese – room" version of it. Effectively, Putnam proposed by his metaphor a further test that a TM cannot solve and that, for the reasons just explained, has much deeper implications than the counterexample to the Turing test proposed by Searle. For instance, Putnam said, if we ask "how many objects are in this room?", the answer supposes a previous decision about which are to be considered the "real" objects to be enumerated — i.e., rigidly designated by numerical units. So, one could answer that the objects in that room are only three (a desk, a chair and a lamp over the desk). However, by changing the enumeration axiom, another one could answer that the objects are many billions, because we have to consider also the molecules of which the former objects are constituted.

Out of metaphor, any computational procedure of a TM (and any computational procedure at all, if we accept Church's thesis) supposes the determination of the basic symbols on which the computations have to be carried on. Hence, from the semantic standpoint, any computational procedure supposes that such numbers are *encoding* (i.e., unambiguously naming as rigid designators) as many "real objects" of the computation domain (See [31], p. 116). In short, owing to the coding problem, the determination of the *basic symbols* (numbers) on which the computation is carried on, *cannot have any computational solution* at the actual state of development of the formal computability theory.

To sum up, for Putnam's analysis, the functionalist approach to cognitive intentionality has to do essentially with an *inductive schematism* of concepts and therefore with the act of *productive thinking* for the constitution of logic symbols (See p. 13). On the contrary, the functionalist approach can at last give some limited operational version of the *deductive schematism* and hence of the intentional act intended as an act of *reflexive thought* on symbols already constituted. In other words, neither the problem of real reference nor of inductive schematism, essential for a scientific theory of human and animal perception, have in principle any solution from the functionalist approach to cognitive science.

1.2.2 The connectionist approach in cognitive neuroscience

1.2.2.1 What is the connectionist approach?

Generally a Neural Network (NN) is conceived as a computational architecture simulating brain dynamics and in a *pre–symbolic* form cognitive behaviors. Where "pre–symbolic" has to be intended in the "weak" sense that this computational architecture is conceived for reducing the relevance of the programming operation, not in the "strong" sense of "constitution of logic symbols".

From the engineering standpoint, NN's are useful for their capability of *automatic extraction* of statistical relations in the input data of a *higher order* than simple averages, so to perform operations generally very difficult for classical symbolic AI models, such as *pattern recognition* and temporal series previsions in complex systems.

From the architectural standpoint, artificial NN's are networks interconnected in *parallel ways* composed of simple *adaptive* elements (neurons) and by their hierarchical organization, designed for interacting with real world in a way similar to *biological* NN's. A "neuron" of an artificial NN is effectively a *threshold logic unit* of the logical circuit implemented in a classical digital computer. The different architectures of NN's depend on different modalities of determination of the *threshold* and of the *interconnections* among neurons. For instance, in the "formal neuron" (see Figure 1(a)) of the first model of artificial NN, i.e., McCulloch's and Pitt's "formal" NN, the output frequency η of each unit as a function of an input ξ_j, $j = 1, 2, ..., n$ is given by the following function:

$$\eta = \text{cost}\, \mathbf{1}\left(\sum_{j=1}^{n} \mu_j \xi_j - \theta \right)$$

where $\mathbf{1}(\bullet)$ is a classical Heaviside step function, μ_j are the statistical weights among the connections and θ is the threshold. Because of the threshold θ the output is discretized, so that, if the input is discretized too, it is possible to demonstrate that, for suitable values of μ and θ, a net composed of these neurons can compute whichever Boolean function.

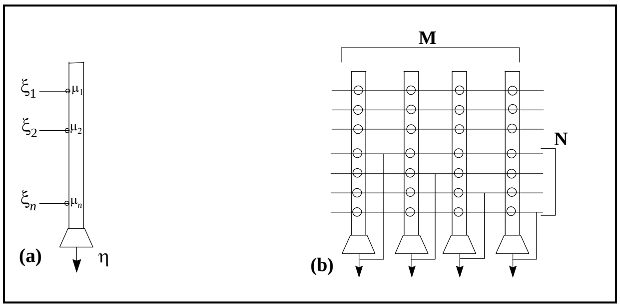

Figure 1. (a): Scheme of the "formal neuron" of McCulloch and Pitts. (b): Scheme of a self–organizing neuron module according to the general Eqs. (1) - (3)

The *adaptive* procedure in artificial NN's essentially consists in making the weights of the connections among the units *variable in time* as a function of the statistics of the neuron output. The fundamental rule by which this modification is performed is the so-called Hebbian rule [33]. This is a frequentistic rule according to which the weights μ_j change as a function of the product between input and output among the elements, so to reinforce inputs that produced stable outputs. In this way, the spontaneous formation of modules of reciprocally exciting neurons becomes possible, formally corresponding to the presence of statistic correlations intrinsic to different components of the input data. Mathematically, the Hebbian rule implies:

$$dw_j / dt = \alpha y x_j - \beta(y) \cdot w_j$$

where w_j are variable weights, x is the input, y is the output and $\beta\,(y)$ is a positive function of y.

Hence, given the matrix notation $\mathbf{x} = [x_1, x_2, ..., x_n]^T$ and $\mathbf{w} = [w_1, w_2, ..., w_n]^T$, where T is the transposed, if $\mathbf{C}_{xx}$ is the correlation matrix of $\mathbf{x}$, if x_j are stochastic variables with statistical stationary properties, then the w_j converge asymptotically to values such that $\mathbf{w}$ represents the eigenvector of the maximum eigenvalue of $\mathbf{C}_{xx}$. In this way, through the presence of lateral feedbacks among neuron arrays, it becomes possible to speak formally of *self - organization* of computation modules in NN's (see Figure 1(b)), according to the following equations [34]:

$$d\mathbf{y}/dt = f(\mathbf{x},\mathbf{y},\mathbf{M},\mathbf{N}) \qquad (1)$$

$$d\mathbf{M}/dt = g(\mathbf{x},\mathbf{y},\mathbf{M}) \qquad (2)$$

$$d\mathbf{N}/dt = h(\mathbf{y},\mathbf{N}) \qquad (3)$$

where **x** is the vector of all the inputs at the neural module concerned, **y** is the vector of all the outputs and **M** e **N** are two adaptive connection matrixes. *Biologically,* Eq.(1) is a relaxation equation of the electrical activities of neuron modules for short *t*; Eqs. (2) and (3) are adaptive equations evolving on longer time scales and concerning *structural modifications* of the net. In particular, Eq. (3) represents the function of an *associative memory.* To understand this essential notion, it is necessary to introduce the distinction among two different dynamics concerning the effective functioning of an artificial NN:

1. *A learning phase* concerning the dynamics on the weights by which the net self - organizes its internal computational modules;
2. *A test phase* by which the net, after the learning, performs its own task (e.g., pattern recognition). In this phase, if we consider a NN as a dynamic system characterized by a given set of differential equations. The dynamics concerns the activation of different neurons and/or of different neuron modules according to equations that assume generally the following form for a single layer NN:

$$z_j = f\left(\sum_{i=1}^{n} w_{ji} x_i - h_i\right), \quad j = 1,\ldots,k$$

where z_j is the output of the j-th neuron, w_{ij} the connection weight between two neurons x_i is the input of the i -th neuron, h_i is a threshold and f is a non-linear function. It is thus evident that such a dynamic system effectively operates a non - linear mapping T_W between the input set **X** and the output set **Y** exemplifying the notion of *associative memory*:

$$T_W\colon \mathbf{X} \rightarrow \mathbf{Y}$$

From these very simple notions it is easy to understand the core of a connectionist architecture of calculation with respect to classical sequential architectures of AI. While in a sequential architecture there is a strong distinction between the logic unit of calculation (the CPU of a normal computer) and the unity(ies) of information storage (the hard disk(s) and RAM devices of a normal computer), in a connectionist architecture no distinction exists between these two components. The same units (neuron modules) devoted to process information are those devoted to the information storage too. The information stored is distributed along the weight connections where it is processed. For this reason, in the connectionist realm, we speak of *parallel distributed processing* of information in such architectures [35].

1.2.2.2 Theoretical limitations of connectionism

From the logic and computational standpoint, a NN after the learning is equivalent to a TM, reproducing in itself all the theoretical limitations we discussed above, with respect to *reference* and *truth*. Of course, the novelty with respect to classical symbolic methods of AI is the pre–symbolic task of the learning phase by which a NN seems to constitute by itself the logical symbols of its predicate calculus. The theoretical problem is the following: is a connectionist NN in learning a computational architecture able to constitute formally its own basic symbols intended as *rigid designators* of changing objects of the real world? The answer is evidently negative. A NN could be effectively able to constitute its own basic symbols *iff*, during the learning phase, was able to modify, *depending on input*, not only the statistical weights of its fixed topology of connections, but the same geometrical topology of the connections. On the other hand, only in this case a NN will assume the typical *dynamic* and *computational* characteristics of biological networking. That is:

1. From the *dynamic standpoint*, it will assume the characteristics of an *unstable* and even *non–stationary* dynamics. Indeed, in the connectionist NN's, despite the non–linear character of such dynamic systems, the information (e.g., a pattern) is stored in each stable final state (fixed point attractor) of its dynamics. That is, it is stored in some absolute minimum of the "energy" landscape (i.e., of some complex function measuring the distance between the actual state and some target state) of the dynamics. On the contrary, what is typical of real brain networking is the *unstable* character of the signal transmission and processing among neurons. For instance, in real brains, the firing rate of neuron spikes is continuously changing. In this way, it becomes despairing any attempt to interpret in a frequentistic way the learning rule for weight connections as, on the contrary, the Hebbian rule pretends to do . Moreover, there is evidence in real brains of more

complex oscillatory and even chaotic (i.e., unstable in itself, even though pseudo–stable or pseudo–periodic with respect to a properly chosen interval ε): see Figure 2) global dynamic behaviors [37-40].

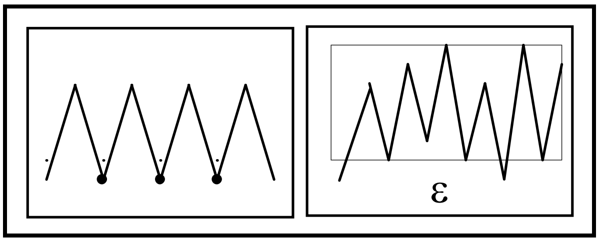

Figure 2. The difference between a stable (periodic) [left] and an unstable (aperiodic) [right] time series. An unstable time series can be intuitively defined as pseudo–periodic or *chaotic* if it can be characterized by recurrences that are periodic within a given interval ε.

The informational advantages of chaotic behavior in neural dynamics, become evident as soon as we consider the information richness hidden in the pseudo–cycles of a chaotic dynamics. Roughly speaking, in the energy landscape of a classical non-linear neural net, such as a Hopfield net, it is possible to memorize less than one pattern for each of the n minima [41]. In a chaotic memory it would be possible to profit *in real time*[2], on a deterministic basis, of all the cyclic combinations of these minima, with an exponential increment of the memory capability (theoretically it is possible to improve the memory capacity till 2^n patterns. See Figure 3). In our view, in this dynamic use of the brain dynamic instability is hidden the secret of straightforward memorization capacity of the biological and specifically the human brain. Computationally, the main difficulty is that till now there were no effective computational techniques of pseudo–cycle extraction of any length, because of the complexity of chaotic behavior. This complexity in-

---

[2] Because a chaotic net does not memorize patterns “statically” into fixed points of the dynamics but into unstable cycles that can be recovered on a deterministic basis, it is not necessary to reset the net after a recognition for the next one, as with static nets. It is sufficient to change a parameter value for switching from a cycle into another.

deed makes inapplicable to deterministic chaos classical statistical methods of signal analysis. In the last four years, however, one of us, developed a new effective technique of pseudo–cycle extraction of any length, with a computation time growing only linearly with the cycle length [42-45]. We have the definitive experimental evidence that this method, based on the new foundational ideas discussed in the next Section, can extract practically *all* the pseudo–cycles of a chaotic dynamics.

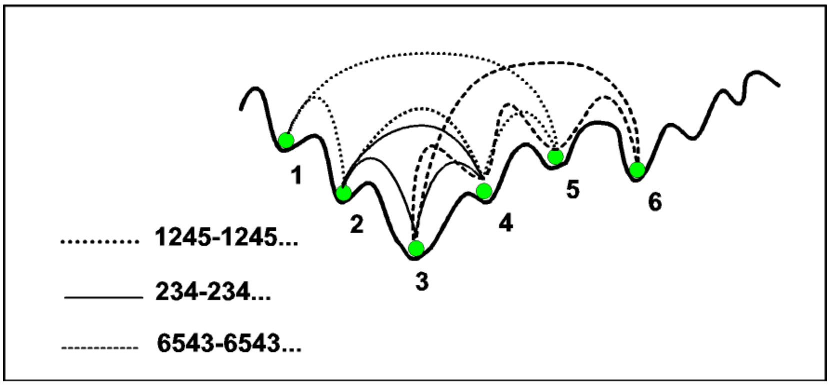

Figure 3. Intuitive representation of the storing capacity of a chaotic dynamics into the 2^n pseudo–cycles among the *n* minima of its energy landscape. For instance, if we imagine that each minimum corresponds to a memorized feature of a visual object, it is easy to understand that each class of object corresponds to a cycle, i.e., a given combination of features. Moreover, by a simple phase change (e.g., a change in the ordering of minima within a give cycle) the net could easily recognize the sameness of the object also under three-dimensional rotation in the space. Finally, because we are faced here with pseudo–cycles and not with cycles, it becomes easy to explain also the physical basis of the phenomenon of similarity recognition (analogy) through such a dynamic structure of recognition.

Finally, there is an amazing *evidence* of the *non – stationary character* of real brain networking. For instance, Positron Emission Tomography (PET) techniques of inquiry give a sort of biological evidence of what logicians intend with the notion of names as *rigid designators* of objects. Namely, in cognition tasks, such as attention focusing or moving object tracking, completely different neuron networks are excited to designate the very same object [46]. It is as if the real brain is continuously modifying the geometrical connection topology of its computation network, to match the object modifications. On the other hand, this sort of accommodation of the basic symbol space for matching varying objects is precisely what is needed from a NN for being able of performing *really parallel computations.* Let us illustrate briefly this essential point.

2. *From the computational standpoint,* a connectionist NN cannot be considered as a really parallel computational architecture because the inner units are *fully connected* with the input units x_k (see Figure 4 (a)). A really parallel computation implies that the inner units compute functions $p_i\,(X)$ defined only on some subset of the input units [47]. For considering such functions as rigid designators of varying external objects it is thus necessary that the supports $S_{p_i}(X)$ of these functions are varying with the objects (See Figure 4(b)). The non–stationary character of brain networking displays all its intrinsic computational value, if interpreted in this sense [48-52]. In the next Section we hint briefly to such a neural net, $\Psi^D\,(X)$, called *dynamic perceptron* (See Figure 4(b_1-b_2)). It is characterized by an automatic pre–processing devoted to modify the net connection geometry, depending on the correlations of each singular input — practically it is in continuous learning, not on the weights, but on the connection topology. This architecture was developed by one of us [42.45.48], as a partial implementation of some ideas of Thomas Aquinas's theory of intentionality. In any case, there are straightforward neurophysiological evidences of the so–called "dynamic receptive field" of neurons belonging to different sensory systems of mammalians that could find by the notion of "dynamic perceptron" their computational model, showing the informational relevance of such a strange behavior[3]. The dynamic receptive field has been ob-

[3] We thank prof. I. Tsuda of the Dept. Of Mathematics of Hokkaido University in Sapporo (Japan) for this personal communication about the relationship between the pre–processing of our "dynamic perceptron" and the neurophysiological evidence of the "dynamic receptive field" in sensory cortex. We are preparing with prof. Tsuda a specific paper on this topics.

served in mammalian retina [53], auditory cortex [54-55]; primary visual cortex [56-57]. It was found that there exist subfields, some of which are activated only during 20-50 msec for a continual presentation of stimuli, and the combination of activated subfields varies even for a static presentation of stimuli. In primary visual cortex, it is well known that there exist neurons with orientation specificity. Another type of neurons, whose orientation specificity — i.e., a tuning — is dynamically changeable, was found in relation to the dynamic receptive field [56]. In this context, a classical receptive field can be reformulated as a spatio-temporal summation of dynamic receptive fields. The spatial summation is taken over an entire receptive field, and the temporal summation over a few hundreds miliseconds. Since the time scale 20-50 msec is almost equal to a "unit" of psychological time, the dynamic receptive field may be considered as a neural correlate of internal dynamics for the reorganization of mental space. Namely, the presence of dynamic receptive field suggests the presence of the process of dynamic re–modelling due to dynamic interactions between higher and lower levels of information processing [56-57].

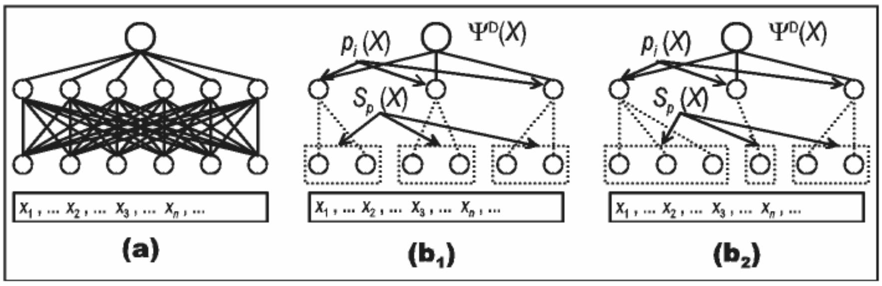

Figure 4.

1.2.2.3 A first conclusion

In cognitive neuroscience it is generally held that a given neural circuitry is a *code* of some given perceptual *belief.* However, in the light of all the precedent discussion, we feel that any honest computational approach to the study of mind cannot limit itself to state simply that a given brain circuitry is a “code” of a “belief”, i.e., of a mental representation of a given thing. At the actual state of development of the computability theory, *there is not* and *cannot be* any formal demonstration of this *threefold correspondence* among the *referential thing,* the *neural code* and the *belief*.

This sort of correspondence can be only a *matter of convention*, depending on the meta–language we choose to define this correspondence. Namely, this correspondence is only an *interpretation* in the technical sense of the model theory, just as to say that a given activated circuitry in a computer or a sequence of signals in a telegraph corresponds to the letter "A".

However, a distinction is necessary.

1. The problem of formally defining the *coreference* (i.e., to have the same reference) between a belief statement, expressed in intentional language (*I-talk*, e.g.: "I (believe to) see a red colour") according to intensional logic, and a related observation statement (*O-talk*), of some neurological (e.g.: "a modification in the variable *y* is measured at time *t* in the brain location *z* as a response to a given input *x*"), computational, psychological etc. theory, according to extensional logic, *is not a solvable problem* (See [25], pp. 132-134; [31], p. 116).
2. On the contrary, to solve the problem of the *real reference,* that is the problem of the correspondence between *a neural code* – not necessarily constituted according to a Hebbian law – and an *external thing,* it is sufficient to demonstrate that a biological brain is able to compute functions not computable for a TM, as opposed to Church's thesis. In other words, to solve the real reference problem for a scientific theory of perception it is sufficient to demonstrate that what characterizes a biological brain (and more generally any biological organism) is its *capability of redefining the basic symbols*, the codes, of its own computations, in dependence of singular different occurrences of their own objects.

To understand this point, we need a completely different approach to the real reference problem in the light of the pre–modern logic and particularly in the light of the classical Aristotelian–Thomistic theory of intentionality.

1.3 Intentionality and foundations of logic after Gödel

1.3.1 Analytic versus axiomatic method in logic after Gödel

The preceding discussion about the core of the human intentionality is expressed in the language of cognitive science as the capability of human mind of re–defining the basic symbols of its computations (See pp. 21ff.). The double opposition between *inductive* versus *deductive* schematism (See p. 6) and *productive* versus *reflexive* thought (See p. 13), has a logical counterpart in the opposition between *analytic* and *axiomatic* method in logic. Namely, in the opposition between a logic defining its own role as *logic of discovery* of new hypotheses, and a logic reducing its role to the simple *logic of justification,* the logic of proving statements by deductive procedures, starting from fixed premises or *axioms.* Effectively, after Gödel — and, more recently, in the heterogeneous universe of the computer sciences — the necessity of studying logical procedures allowing change in axioms during calculations is an argument of ever growing importance. In fact, for contemporary logic, computer science and cognitive sciences there is the shared necessity of avoiding the multifarious limitation theorems which have their formal origins in Gödel's [58-60].
The interest for recovering to modern logic and modern sciences *the analytic method*[4] *of classical, pre–modern logic* depends on the fact that it is in principle impossible to allow axiom changes within formal systems. Following Cellucci's reconstruction, the historical origin of the analytic method is in Plato's logic and it consists in affirming that the premises of any deductive procedure consist in pure *hypotheses,* since it is impossible to attain the truth of any mathematical entity. Each hypothesis consists thus in a "step" toward the further, more general one in a never ended bottom-up process. The aim of logic would consist in the continuous progress toward ever more general principles, without the possibility of stopping such a process[5].

[4] As we explain after (See note 7), "analytic" has here to be intended in a radically different way as to its modern sense, the sense used by Pappo, Descartes, Newton and Leibniz.

[5] See, for instance Plato, *Parmenides,* 136c,1-7, *Letters,* VII, 342a-343c. The necessity of an infinite character of this process is however negated in *Republic*, VI, 511b6-8 where it is said that knowledge is a sort of ascension–descent through a sort of universal deduction tree. That is, knowledge is intended in *Republic,* before as a bottom-up

The historical origin of the *axiomatic method* is in Greek geometry and namely in the prototype of any axiomatic system: Euclid's *Elements.* It is based on the supposition that we can attain self–evident principles, without developing research toward more fundamental hypotheses. Further, Aristotle's logic transformed the axiomatic method into the proper object of logic, and proposed the axiomatic method in mathematics as a model for any other science. On the other hand, he refused the idea of a mathematical science of nature, typical of Pythagorean and Platonic traditions. Nevertheless, for Aristotle, the analysis still plays an important role as method of discovery of the so–called "middle–term" in any syllogistic procedure, that is the term connecting the "major premise" of the syllogism to its "conclusion"[6]. This use of the analytic method, related with inductive strategies, is functional to the axiomatic one. For Aristotle, analysis is a terminating procedure, or a "reduction" procedure, whose end is some new axiomatic definition — characterized by an immediate relation subject–predicate, i.e., axiomatic definitions are essence definitions — and/or some statement easily reducible to some axiomatic truth (See [61-62]. See also [60], pp. 291ff.). The construction of a deduction system following the axiomatic method in its syllogistic version within each scientific discipline constitutes the deductive "synthetic" moment, after the "analytic" devoted to the principle discovery. In this sense, the synthetic component has functions to make scientist's discoveries *rigorously expressible and profitable for all*. So, for Aristotle cannot exist one only axiomatic system for expressing all the mathematical truths or the true contents of any science. The analytic method for discovering new principles and finding new truths plays thus an essential, though subordinate, role in Aristotle's logical and epistemological theory.

In the modern age, the axiomatic method was established with important differences from Aristotelian teaching. The most important one was the rejection of axioms as "real definitions" or essence definitions, because of Galileian science self–limitation to *quantitative properties* of the physical things. This rejection was confirmed by Newtonian physics, vindicating *the absolute phenomenal*

process by the *resolution* (finitely *analytic*) method toward a final not–hypotetical principle, for re–descending thereafter to all the consequences through a top–down process by the *synthetic* (deductive) method. This program, at least for geometry, was effectively fulfilled by Euclid's *Elements.*

[6] See Aristotle, *Post. An.*, I, 22, 83b,39-84a,2.

character of the new physics, and the *purely formal character* of the three laws of dynamics, as conditions for justifying the calculus and geometrical predictability of quantitative phenomena. The *Logique* of Port–Royal, re–proposing former reflections of B. Pascal, asserted the necessity for mathematics of using only “nominal definitions”, by a separation between “definitions” and “existence assertions”.

In modern mathematical logic and in Hilbert’s formalism, the nominal character of definitions implied the rejection of Frege’s logicism, by renouncing the necessity of supposing “truth” and “meaningfulness” of formal system axioms for maintaining only their *coherence*. “Truth”, “meaningfulness”, as well as “coherence” are metalogical properties of formal systems and must be metalogically checked by algorithmic procedures. A set of axioms is not *coherent* because it is *true* and *existent the objects* to which these axioms refer. On the contrary, because the set of axioms is coherent and its coherence can be proved by a finite recursive (algorithmic) procedure, they are also *true* and *existent* their objects. On this basis, Hilbert pursued the possibility of constructing one only formal system for all mathematics. He stated also the possibility of using the axiomatic method for the “logic of discovery”, by supposing the possibility of an algorithm able to determine, for each statement expressible in a given formal language, whether is it demonstrable or not within this language. Church–Rosser theorem denies such an algorithm can exist in formal systems. Moreover, Gödel’s incompleteness theorems for arithmetic and their extension to all formal systems in the work of Turing and Tarski, ruled out the idea that the notion of mathematical truth can be exhausted by any formal system.

In this sense it has been asserted that logic and mathematical systems must be *open* systems in which the analytic method must recover its ancient role as logical method of new axiom discovery [58-63][7]. In other words, the incompleteness destiny for logical systems is unavoidable only *iff* we want to maintain

---

[7] This is true, though some distinctions have to be made with respect to the difference between: 1) Plato’s and Aristotle’s definition of the analytic method as bottom–up process for the definition of new hypotheses and/or as process for the definition of new axioms for making possible a demonstration; and 2) the modern definition of analysis, all depending on Pappo’s definition of it, as top–down process of decomposition of a compound in its parts. Of course (2) cannot be reduced to (1). See [60], pp. 292-299 and pp. 349-351.

fixed principles of demonstration, affirming that the formal systems are the *only* logical systems and the axiomatic method is the *only* method of logic. The *logic of discovery*, the logical method for new principle detection for the continuous construction of scientific (demonstrative) procedures, is the most important part of logic, *since only this type of non–determinism can avoid undecidability spectres.*

According to Cellucci, the reasons for which the discovery of limitation theorems for formal systems can be interpreted as a necessity for recovering the analytic method in its early Platonic version (see note 7) against the monism of the axiomatic method are very deep. They are essentially three:

1. *For avoiding the incompleteness*, it is not sufficient to construct a series of formal systems, each obtained by adding as new axiom the undecidable proposition of the precedent one. Indeed the main question is whether there are complete formal systems successions obtained in such a way. The answer is very limited and substantially negative. Formulas of the type $\forall x\, A(x)$, where $A(x)$ is *a decidable* property, are demonstrable, even though there cannot be an algorithmic (finite) procedure for deciding the truth of the formula $\forall x\, A(x)$. On the contrary, formulas of the type $\forall x\, \exists y\, A(x, y)$, where $A(x, y)$ is a decidable relation, are not demonstrable, though they are true in the system [63]. More generally, the solution to Gödel's incompleness theorems for formal systems cannot consist in a series of systems chosen through an effective procedure. Some sort of non–determinism is necessary in the construction of the systems and hence in the construction of the axioms.
2. *The only non–determinism sufficient for avoiding Gödel's incompleteness* in formal systems consists *in the introduction of new axioms* and not in the simple possibility of non–deterministic multiple choices [64]. "The non–determinism required by Gödel result is the non–determinism related to the possibility of introducing at each step new axioms in a non algorithmic way" (See [60], p. 326). This denies that the system might be considered "formal" in classical sense and that the method used might be axiomatic — or "analytic" in modern sense (Gentzen's natural deduction methods included. See note 7).
3. *Turning to "mathematical intuition"* for justifying the discovery of new axioms, as Gödel himself did, implies a double unpleasant consequence. Before all, it means that there must exist ultimately in logic and mathemat-

ics (and hence in any science, metaphysics included) an irrational, subjective component [65-66]. "Here we are not in the realm of science, but of poetry" [67]. On the other hand, also if we intend the "intuition" in the strong sense of Gödel's "ideal intuition" of abstract concepts — that would be relative to infinite mathematical objects and that would be the result of a difficult training of the mathematician — we are faced with unavoidable limitations. The certainty thus obtained is unusable for granting that *certainty* in concrete mathematical choices we are searching for. For instance, let us suppose that in the system *S* there exists a true formula *A*, undecidable (i.e., both *A* and non-*A* cannot be demonstrated in this system) and a given abstract concept of set, $\mathfrak{I}$, known for intuition and for which the axioms of *S* are equally true. For a corollary of the first incompleteness theorem of Gödel, there must exist also another formula *B* and another intuitive notion of set, $\mathfrak{I}'$, for which the axioms of *S* are equally true, but such that *B* is *true* for $\mathfrak{I}'$ and false for $\mathfrak{I}$ (it is sufficient that we pose as *B* the statement not–*A*). In this case the ideal intuition cannot be used for deciding whether $\mathfrak{I}$ or $\mathfrak{I}'$ is the correct set notion. An examplification of such a case is whether we pose *S* as the Zermelo-Franklin set theory (ZF), $\mathfrak{I}$ as ZF notion of set and $\mathfrak{I}'$ as Cohen notion of set, after his demonstration of the independence of continuum hypothesis from the axiom of choice (Cohen 1966). It is thus evident that ideal intuition, in spite of its implicit reference to infinitary method of demonstration[8], cannot grant that absolute mathematical certainty which formal systems are searching for (See [60], p. 254).

However, the reference to *infinitary methods* in mathematics, which can abstractly grant coherence and truth but with weak effectiveness, can offer another contribution to a better understanding of Plato's analytic method limitations. Plato's impossibility of reaching mathematical truth and his preference for hypotheses and not for self–evident axioms are both based on an *ontological assumption*. This assumption is in many senses equivalent to the core of Tarski's demonstration of impossibility of defining semantic notions such as truth, cohe-

---

[8] It is to be remembered that G. Gentzen demonstrated the coherence of number theory by extending the mathematical induction till ε_0 [68]. In this way he explained why Hilbert's finitary arithmetic cannot give a similar demonstration of number theory coherence, so to satisfy Gödel's second incompleteness theorem.

rence, reference, etc. by a purely formal recursive procedure of satisfaction, without attaining formal languages of ever higher logical types (see above § 1.2.1.2 and note 1). The common ontological assumption here concerned is summarized in the following quotation from Plato's Dialogue, *Parmenides*:

> (If you pose the essence and the existence of a given object), you have to examine *simultaneously all* the consequences of such a hypothesis, both with respect to the object itself, and with respect to each other object individually considered, as well as with respect to whichever collection of these objects and with respect to the other objects considered all together. (…) This task is never ending (*Parmenides* 136c, 1-6).

It is evident that this is the same problem identified by Tarski for demonstrating the undecidabilty of semantic notions in formal systems because of the necessity of posing "infinitely many notions of satisfaction that must be introduced simultaneously because they cannot be defined independently" (see § 1.2.1.2 and note 1).

The solution suggested by Cellucci for avoiding this limitation would recover in a post–modern (post–Gödel) way the core of Plato's analytic method. Effectively it results very close to M. Minsky's "society of mind" [69]. It consists in supposing many systems connected together in a variable way, without, of course, because of Gödel's, Church's and Turing's limitation theorems, any possible formal rule and/or algorithmic procedure governing this variation and/or the choice among the systems. Logically, it means to attribute only *a hypothetical existence and essence* to the different logical objects expressed in the different axiom collections, available in this way. The advantage of such an approach is that, using different discovery inferences — overall "induction" and "analogy" —, this method chooses in the dichotomy characterizing the so–called "inference paradox" (either *certainty* or *knowledge amplification*) the second alternative.

In other words, Cellucci concludes, we are faced today with a constraining alternative.

1. From one side, there is logic following *the axiomatic method* that not only gives no knowledge amplification, but it is no longer able, after Gödel, to grant absolute certainty. So, to avoid contradictions, this approach weakens the strength of logical implications (See, for instance, P.J. Cohen's "generic

set" theory [70]), demonstrating only generic and therefore *useless* propositions.

2. On the other side, we have *the analytic method* that, without granting certainty, is able at least to amplify knowledge by providing, new hypotheses for demonstrating useful propositions. It may be obvious that post–modern logic has to choose the second alternative: if it is possible to have logical systems only with a local and limited demonstrative power, at least they showed be able to demonstrate useful propositions! This is the reason for Cellucci's preference for the analytic method.

Nevertheless, it is hard to understand how different this solution is from the one criticized by Cellucci himself and appealing to subjectivity of mathematical intuition. In the next Section we want to suggest another strategy, which recovers the Aristotelian–Thomistic integration between the analytic and the axiomatic method.

It is useful to conclude this subsection by recalling the result of this overview about the opposition of the two logical methods (the analytic one and the axiomatic one), in the light of Gödel's theorems. This result is that the true problem is only one: how to grant an "open character" to logical systems, that is, *a procedure making a logical system able to change its axioms* to avoid undecidable situations in formal systems. In this light it is easy to understand that this is the same problem we faced in cognitive science, discussing H. Putnam's approach to intentionality problem (See § 1.2.1.5). Not casually he is the more trained in foundational questions among cognitive scientists. Because the notion of intentionality was introduced and discussed for the first time in western thought by Scholastic philosophy in the Middle Ages, it gives us useful suggestions for a solution to the related problems of *intentionality* in cognitive systems and *openness* in logical systems.

1.3.2 An after Gödel reconsideration of Thomas Aquinas' theory of logic

To comply with this double unique problem, we deepen Thomas Aquinas's[9] logic, in its more original suggestions with respect to Plato's and Aristotle's

[9] Thomas Aquinas (1225-1274) was an Italian philosopher and Theologian, who lived and worked between Paris' (France) and Naples' (Italy) universities, during the first half of XIII Century.

logical theories.

In addition, according to Cellucci's reconstruction, it is difficult to find in modern logic what we need. Gödel's theorems constrain modern logicians "to see beyond modern age", both forward and backward. If we want to build a "post–modern" age that is not the irrational realm of "weak thought", we have to solve the problem of *the logic of discovery,* without falling into a purely subjective approach to the problem of axiom change in deductive systems. Given the prevalence of the axiomatic method in modern science since Descartes, Galilei and Newton, for a post–modern approach to the logic of discovery, we are obliged to search for it within the "pre–modern" age, without any Enlightenment preclusion for the Middle Age.

In this research of a suitable logic of discovery, we stopped with Cellucci by the classic analytic method. We agree with him in emphasizing the strong distinction between the use of analytic method in Plato and in Aristotle. We agree also in affirming that all the differences ultimately consist in their opposite approach to the notion of "essence" knowledge. For Plato, this knowledge is ultimately unreachable; for Aristotle it is something available by a process of abstraction–intuition. His axiomatic method in formal logic depends precisely on this, so that each principle of a categorical demonstration by his syllogistic method consists in an "essence definition". Formally this definition consists in *a non–tautological identity*.

To modern people affected by the Galileian–Newtonian refusal of the "essences" it is sufficient to recall that by "essence" both Plato and Aristotle intended *the infinite totality of relations* making each thing[10] identical with itself and different with respect to other things, or collections of "things" (see note 10), in the universe. It is evident that, when we are faced with the problem of dealing simultaneously with infinitely many satisfaction relations, as in Tarski–Gödel formal theory of semantics (See note 1) — apart from different words and cultural contexts —, we are effectively faced with the very same problem of truth as "*essence* knowledge" of our Ancestors. In both cases, however, what

---

[10] From now on, for sake of simplicity and clarity, we name as "thing" each existent being (whether it is a "substance" or an "event", or a "relation" or a "quality" or a "quantity" or a "collection" or whichever else). This denotation is aimed to avoid confusions with the term "being", from now on intended exclusively as nominal form of the verb "to be".

is effectively being discussed is a valid justification of universality and necessity in logic. This is the problem of *certainty* in scientific knowledge.

Generally, in the history of philosophy of the Middle Ages, Thomas Aquinas's philosophy is considered as an original synthesis of Platonic and Aristotelian traditions. The core of Aquinas's originality is the doctrine of *real distinction between essence and existence* in the notion of "being" (See note 10). According to Thomas Aquinas, the error of both Aristotle and Plato in dealing with the essence problem — and hence with the justification of universality in logic — consists in not distinguishing adequately between *essence* and *existence* of a given thing.

For Plato essence and existence are not really distinguished: the essences exist as immaterial individuals and the "being" of each material "thing" is only a limited participation to this ultimate way of existing. In fact, he — with the greatest majority of Western logical and mathematical thinkers — established the existence of a given individual by the satisfaction of the formal relation of self–identity. It is intended as negation of any negation of identity, i.e., as negation of any qualitative difference[11] with everything else, and hence by *an actual infinity of relations* (See [71] 185a; [72] 139b-e; 146a-147b). In this way, only the immaterial essences fully exist as individuals. Through this opposition between what is relative (the quality) and what is absolute (the essence), the ultimate being and truth of each thing become for Plato unknowable. The knowledge of essence would require *the simultaneous exhaustion of all the infinite qualitative differences* from which only the absoluteness of self–identity and individuality of a thing can emerge.

Aristotle tried to solve Plato's problem in two steps:

1. by distinguishing between *substance*, intended as individual existent thing, and its *essence* that could be common to many individuals, so to deny that essences are existent individuals on their turn, belonging to some immaterial world,
2. by putting in the *material* constituent of any essence the root of the diffe-

[11] The qualitative difference, is that allows to say that a given individual is something and is *not* anything else. To say it in extensional logic (class theory) terms, any class must to be close to other classes, that is it must contain as null–class the class of all the elements not belonging to it. This necessity of negative definitions for consistent logical constructions is the ultimate formal root of all the logical antinomies.

rentiation process, both of the different essences and of different individuals, sharing the same essence.

For Aristotle, the knowledge of essence becomes possible for humans in this way. Knowing an essence does not mean, as for Plato, dealing *actually* with an infinity of relations, but only each time with a finite totality of relations, since all the other ones exist only *in potency*, hidden in the common indefinite material substratum of all the things. It is evident that Aristotelian ontology is perfectly coherent with his treatment of the analytic–synthetic method in logic as heuristic component of an overall axiomatic method[12]. If we applied Aristotelian ontology to our post–Gödelian problems in foundations, we would obtain at most very weak results. It would be possible to preserve a formal system despite its inner incoherent statements — whose presence is granted by Gödel theorems — as long as it is possible to maintain these statements "implicit" or "in potency". In other words, Aristotelian ontology is supposed in any modern attempt to solve logical antinomies by weakening the strength of the logical implication (See § 1.3.1).

So, what we need is that, from one side, instead of being hidden or existent in potency in some universal collection, the relations not concerned in some effective calculation and/or demonstration *do not exist* at all. Nevertheless, *universality* could be granted if the essence of a given object, instead of being conceived as the simultaneous existence of an infinity of relations, was conceived as another primitive besides relations irreducible to them. Universality could be thus granted if we were able to attribute to essences the *capability* (of course passive, i.e., relative to an active power as it is in any causal relationship) of generating relations, each time it was *necessary* for converging in calculations and/or for making a demonstration effective. The astonishing plasticity of human brain and of human cortex in redefining continually its finite connection topology, with rapid responses to impinging inputs, without loosing time in combinatorial searches, would be a limited but efficacious neurophysiological "icon" of such a metalogical and metaphysical idea.

---

[12] Namely, this ontology explains why Aristotle's application of the analytic method for the discovery of the lacking "middle–term" for constructing a syllogistic demonstration (see § 1.3.2), is only a reduction procedure. That is a procedure always terminating in some universal statement directly derivable from some main axiom, as far as this statement was "implicit", "hidden" or "potentially existing" in it.

On the other hand, how could we wish, without falling into subjectivism, to make "open" the logic systems, and simultaneously pretend not to insert as primitives of these "open" systems logical objects with the power of *generating* other logical objects? What we need is an ontology able to make the existence of infinite logical relations *virtual*. The relations and the terms they connect have to be conceived neither as existent "in potency" nor as existent "in act". They have to be conceived as *virtually existent*, i.e., *relatively to some "principle" with the power of making existent* a different subset of the infinite totality concerned, for each different concrete context, for satisfying universal logical laws. Only at this price it is possible to give back to a post–modern logic of discovery all the rigor of *logical* method. That is, to make this method a set of logical rules deriving in a strong deductive sense by universal logical laws, without any concession to irrationalism.

Thomas Aquinas ontology is useful at this point. For different historical motivations from ours (more theological and metaphysical than metalogical), both his metaphysics and his logic are based on the definition of causal principles for the existence of each thing (both physical and logical or linguistic) belonging to the universe. From one side Thomas' ontology accepts Plato's instance that universality depends on essence and on its capability of embracing an infinity of relations. On the other hand, it accepts Aristotle's instance that, for each concrete, individual application, only a finite subset of the infinity is effective, even though this subset is always changing. Nevertheless, if on one side Thomas criticized the unattainable character of truth in Plato's philosophy, from the other side he negated Aristotle's solution of distinguishing different senses of existence — "in potency" and "in act", with a continuum of intermediate states — was logically and ontologically consistent. Particularly, he criticized Aristotle's justification of existence contingency (i.e., the "being-in-potency" of a thing) as a supposed "indifference to being and not–being", because violating non–contradiction principle (See [73] n. 184; [61] pp. 50-69). In fact, Aristotelian ontology can grant at most a non–determinism in choosing among a set of alternatives already fixed — i.e., existent in potency in some universal substratum. But we have demonstrated that this is insufficient in principle for avoiding the limitations related with Gödel theorems.

Aquinas's solution is more radical than Aristotle's. From one side, he distinguishes another sense in the notion of "being" absolutely different from those identified by Aristotle. The different senses of being identified by Aristotle, making the notion of being an “analogical” (“multivocal”) and not “univocal” notion, are only *different modalities of existing* (necessarily, contingently, potentially, actually, etc.). When we speak about essences we need another sense of “being” distinct from all senses of “being” as “existing” in the different modalities detected by Aristotle. This sense is related to the use of the copula “is” in the construction of elementary definition statements (See [61] I, v, 71ff.; [74] II, 23). For instance, when we say that “the phoenix is the bird reborn from its ashes” we are saying nothing about its existence. Similarly, to define “the runner” as “who (or what) runs” says nothing about the existence of somebody (or of something) effectively running. In the construction of definition statements we are dealing with the “beingness” (*entitas*) of the object concerned, with “*what* it is” not with its “existence” (*existentia*), with the “*it is*” of some “what”. When in a realistic epistemology we speak about “real reference” of a given elementary (subject–predicate) statement, the “being” the statement is referring to, is properly the “beingness” of the object concerned, not its “existence”. It is thus evident that definition statements do not catch all the essence of the object but only its “whatness” (*quidditas*), a finite subset of relations to distinguish the object within the finite semantic context of a given linguistic occurrence. E.g., Aquinas said (See [75] II,vii,472-475), defining humans as “rational animals” is sufficient to distinguish them both from immaterial things and from non–living bodies, as well as, among organisms, from plants and irrational animals. But this definition is not able to fulfil all the human essence. In some case, we could be constrained to adequate this definition, by coming back to human “beingness” to “pick up” some other quality from the human essence. For instance — to give a modern example of this ancient idea — if some extraterrestrial individual arrived on the earth and fulfilled the definition of human “whatness” as “rational animal”, it would be necessary to change such a definition to avoid an undecidable situation. That is, we would be constrained to come back to human “beingness” to extract from human essence other properties to improve the discriminating power of our human “whatness” definition. So we could define humans as “*terrestrial* rational animals”, a definition absolutely redundant in the actual context where no E. T.s are officially discovered! It is evident that we are faced with the realistic epistemological counterpart of what in the precedent subsection we defined the discovery of new axioms for solving undecidable situations.

Aquinas explains (See [76], VII, 2, ad 1) that we can properly use in logic the meta–predicate “to exist” — as coextensive to the meta–predicate “to be true” (See [77], I, 1c) — only after having properly constructed the “whatness” statement that is argument of these meta–predicates (See [78], VIII, II, 1d, ad 1). By this rule, we can state, for instance, that “it is false that ‘the phoenix is the bird reborn from its ashes’ and therefore (this sort of) phoenix does not exist”. On the contrary, “it is true that ‘the phoenix is the *mythological* bird reborn from its ashes’ and therefore (this sort of) phoenix exists”. It is evident that, besides the extensional sense of being as *existence* with all its modalities — whose linguistic counterparts are object of several modal logic theories —, there is another purely intensional sense of being. Aquinas defines it as “beingness” (*entitas*), or “essence being”, being–of–the–essence, for distinguishing it from “existence” (*existentia*), or “existence being”, being–of–the–existence. The former is involved in all the answers to the question “what is it?” (*quid est*), the latter in all the answers to the question “is there?” (*an est*). But for Aquinas it is possible to answer the second question *only* after having answered the first one. In this way, it is easy to solve all linguistic paradoxes related to the use of negative terms, such as “liar paradox”, in as many confused uses of “being” notion as “beingness” or as “existence”. A typical case concerns the theological paradox of the “existence of evil”, as far as the beingness of evil consists in a “privation of being”, i.e. in a privation of some qualities characterizing the beingness of a given thing. Though “evil is a not–being”, nevertheless “it exists” in given contexts (e.g., physiologically as sickness, morally as sin, physically as natural disaster, etc. See Thomas Aquinas, [79], III, 7).

This solution of the formal and semantic logical antinomies does not involve any “type theory”, either in Russell’s “ramified version” — attaining languages of higher logical order — or in its “simple version” — avoiding higher order logic, by supposing in the meta–language an appropriate choice of primitive terms [80]. By contrast, Aquinas’s metalogical distinction between existential and essential (definitory) statements is based on a constructive logical theory of the essential statements directly depending on his metaphysics.

The main consequence of this logic is however the following. Because of his strong distinction between beingness and existence, Aquinas can change the logical notion of *identity* in a fundamental point. Two existent things are identical not because they are “the same thing” (two individuals cannot be at all the same one) but because they have *one only essence*. In Aquinas’s logic, the symbol “=” interposed between two equiform tokens (in formulae of the type

"$a = a$") or not equiform tokens (in formulae of the type "$a = b$"), cannot be metalinguistically interpreted as a sign that the two symbols it connects "are denoting the same thing" (e.g., two equivalent classes, if they are class symbols). The equality symbol has to be interpreted as a sign that the two symbols it connects are "referring to *one only essence*, even though denoting two distinct things", also when the two symbols are equiform (See [81], V, xvii, 1021). Where "reference" in Aquinas's semantic theory is a *constructive operation* and not a binary relation. That is "f refers to e" means that "e constitutes f as true", where e is (an essence determining the beingness of) the denoted object and f is a formula of a given language.

What in the classical axiomatic approach is a contradictory formula could become here only an equivocal formula. It could be possible indeed to find appropriate conditions under which to attain the essence for generating new symbols to remove the ambiguity. This is the main property of an open constructive theory such as Aquinas's[13]. E.g., if I say "Andrea is a man" and "Andrea is not a man", the contradiction is a simple ambiguity if by "man" I am always referring to Andrea's same humanity (his beingness). But in the first formula I am denoting Andrea as a living body and in the second Andrea as a corpse. We have ultimately and inevitably contradictions *iff* we fix the axioms (and/or the definitions), that is if we pretend that identity expresses sameness with respect to existent things.

Allow us to put this in terms of modern computability theory. Against axiomatic method, to perform effective, concrete calculations, it is inconceivable to

[13] "It is not sufficient name identity with the difference of the thing that it denotes: this brings *to equivocating* (not to contradicting)" (See [73], I, ix, 116). This has for Thomas an immediate consequence for mathematics, as to numbers applied to concrete measurements and/or calculations: "A number, as far as existing in numbered things, is not the same for all, but different by different things" (See [82], I,10,1c). E.g., the "two" used for numbering "two horses" is not the *same* "two" used for numbering "two mosquitoes". The two "two's" share only the essence of "two–ness", as their common generating principle. This means that in mathematics, by denoting the "two's" with the very same digit "2", I am referring to the common essence of "two–ness", but for denoting in my applied calculations two different existential instantiations of the same essence. If I pretend to use in my applied calculations the same instantiation, they cannot converge to a solution.

suppose *one only axiomatic system of natural numbers.* In this connection, Gödel's theorems result a confirmation of this ancient foundational idea. The *equality* token in arithmetic and, more generally, in logic and mathematics, cannot mean "sameness" with respect to existing things!

From the axiomatic system theory standpoint, following A. L. Perrone [42-43], a theory of logical foundations incorporating Aquinas's distinction between "being as essence" and "being as existence" is characterized by two kinds of primitives, *essences* and *relations*, and not only one, the relations, as in modern mathematics [83]. The foundational theories proposed during last years by E. De Giorgi and his colleagues are a useful approximation to this idea. Also they recognize another kind of primitives besides relations: the "qualities" [84]-[86]. Qualities characterize each object belonging to the theory: e.g., there exists "the quality of being a set", "the quality of being a relation", "the quality of being a natural number" and so on. Also "'the quality of being a quality' is a quality", where this self–referential properties of qualities is granted in that axiomatic language by allowing the graph of relations among all the objects belonging to the universe of this theory to remain partially undetermined.

In other words, also for De Giorgi the way to avoid inconsistencies is to allow the demonstration of only *generic* propositions. This foundational approach is biased by the Platonic prejudice of considering the "qualities", like Plato's "essences", as existing objects in a purely *extensional* sense. That is, they are reciprocally distinguished by their property to determine different collections within the Universal Collection *V.* For these collections, *the extensionality axiom* holds, that is, two equivalent collections are the *same* collection[14]. This purely extensional definition of identity emphasizes that De Giorgi's "qualities" do not differentiate another realm of being besides the (relational) existence like Aquinas's.

Aquinas's essences are, on the contrary, absolutely *monadic*. The only relation they have, in Perrone's formalization, is each one with itself. This relation however is not *self–referential*, so to determine the *existence* of the relative collection in *V*. Their auto–relational property is only to emphasize that they constitute the ultimate anti–predicative level in any chain of predicative definitions. In other words, they emphasize the only proper level at which identities occur.

---

[14] In Aquinas's as in ours approaches, two equivalent collections are not he same, they refer to (i.e., they are generated by) the same essence.

In this way, in Aquinas's approach, self–referential expressions are strongly prohibited for essences (See [74], II, 23; [81], IV, viii, 649). As it is inadmissible to say that "the race (intended as 'running') runs", one cannot say that "the essence of being an essence is an essence". Hence it cannot exist as an individual or as a thing. "It is" as a generating co–principle entering into the ontological constitution of each existent thing[15]. In this logic collections (classes, sets, families, etc.), both finite and infinite, are thus to be conceived as *evolving* objects. They do not contain as existent all their elements, but they contain virtually all the things that can be made progressively existent (generated) according to given modalities (See [88], 113; [82], I, 18, 4 ad 3; [89], III, 11, 385).

1.3.3 An application to foundations of arithmetic

Here, we are faced with the "dynamic" character of the collections (sets, classes, etc.), because they are made able to enrich themselves of new objects, as far as the conditions making necessary the existence of new element(s) in them occur [42]-[43]. This implies the definition of a "dynamic" counterpart "H" of the "equality" relation "=" because two distinct things now can be posed as "equal" with respect to a given operation *r* on which the equality "$\overset{r}{\text{☟}}$" is defined (see Thomas' quotation in note 15).

The main axiom of this foundational theory concerns the existence binary operator $\overset{\text{☟}}{\exists}$ whose action consists in making existent a given object x within the universal collection V, $x \in V$, by applying itself to the essence of x, Ex, every time the conditions c making necessary the existence of x occur, i.e., $c = 1$ (See [43], 270):

> **Axiom 1**: $\overset{\text{☟}}{\exists} x := \overset{\text{☟}}{\exists}(E, x, c)$ is an existence binary operator. It applies to the essence Ex and gives the object $x \in V$ as existent ($\exists x$) or non–existent ($\sim\exists x$), depending on conditions necessitating the existence of x through the

[15] "The essence has not directly the existence: it passes to existence through some individual thing to which only existence pertains because the producing action terminates onto it" [87].

operation *r* on which the equality $\overset{r}{\cong}$ is defined. These conditions are summarized in the value of the constant *c*, i.e., *iff* $c = 1$ the passage from *Ex* to $\exists x$ occurs, otherwise $c = 0$.

In this foundational theory, the *non–contradictory* character of a given formula is insufficient for granting its *truth* and *existence* of the relative object. I.e., it is not true that: $\forall x \neg(\neg P(x)) \Rightarrow \exists x P(x)$. This differs from intuitionistic mathematics, however, because in the intuitionistic approach the existential operator acts only if there exists already an effective calculation for the single *x* value. In our approach, there exists in principle the possibility to construct an effective procedure for calculating what we need in each given condition.
This depends essentially on the possibility of defining a relation of dynamic equality between natural numbers defined as successors on different axiomatic arithmetic's[16] g_i, $g_j \in G$, where *G* is the collection containing virtually all the axiomatic arithmetic's. For obtaining this result it is sufficient to define the successor relation *S* as follows (Perrone 1996, 272):

Axiom 2: S_{g_i} is a binary relation. It is defined as follows: $\exists i, j : \forall x_{g_i}, y_{g_i} \in \mathbb{N}_{g_i}; \forall x_{g_j}, y_{g_j} \in \mathbb{N}_{g_j}$ such that the following holds:

$$S_{g_i}\left(x_{g_i} + y_{g_i}\right) \cong S_{g_j}\left(x_{g_j} + y_{g_j}\right)$$

All this means that the integers *i*, *j*, or better the correspondent axiomatic theories of natural numbers g_i, g_j belonging to the collection *G* there exist ($c = 1$), *iff* there is a relation $S_{g_i}\left(x_{g_i} + y_{g_i}\right) \cong S_{g_j}\left(x_{g_j} + y_{g_j}\right)$ to be fulfilled. In other words, the collection *G* *evolves* by specifying its own elements g_k depending on the *necessity* ($c = 1$) imposed by the relations to be fulfilled. In

[16] We remember that a corollary of Gödel's first incompleteness theorem is that does not exist and cannot exist one only axiomatic arithmetic in which it is possible to demonstrate all the true arithmetic propositions.

the case that the two axiomatic theories are the same g_i, the following holds:

Lemma 1: $S_{g_i}\left(x_{g_i} + y_{g_i}\right) = x_{g_j} + S_{g_j}\left(y_{g_j}\right)$

Demonstration: it is sufficient to consider the Axiom 2 by positing $i = j$ allowing Peano's classical successor.

In other terms, Peano's axiomatic arithmetic is a subset of this "open" arithmetic, given the successor operator defined on one only "closed" axiomatic system. Following Perrone's demonstration it is possible to see how "open" arithmetic, by such an operational version of Aquinas's ontology here briefly discussed, can be interpreted as a collection of axiomatic systems. Effectively, they are a collection of arithmetical systems "in progressive construction". The construction of each is governed by rules, satisfying a semantic interpretation of universal laws of logic, even though these rules are not "algorithmic" in classical Church–Turing sense [42]-[43].
Moreover, it can be demonstrated that the recursive functions constructed by such a "dynamic" approach are defined within different axiomatic theories of natural numbers. Such functions are endowed with a higher computational power than the partial recursive ones $\phi(x)$ that are defined not for all the values of x [20]. Partial recursive functions allow only recursive calculation schemes characterized by some *aleatory definition* of the codomain, as in the *non–deterministic Turing Machine*. On the contrary, in Perrone's approach, the relation defined in Axiom 2 grants that the choice of the number succession on which the function develops its computation at the next calculation step is not aleatory. It depends on what we have to calculate (the input), and within which conditions. On this basis, with further axiomatic constraints that we cannot discuss here (See [43], p. 276f.), it is possible to demonstrate that such recursive functions $\Psi(x)$ are *virtually* general recursive. I.e., they are defined on *all x* values (they are not partial), even though such a definition is not given *simultaneously*. That is, they cannot be general recursive in the classical axiomatic sense. Gödel theorems prevent a diagonalization procedure for a general recursive function defined by only one "closed" axiomatic system of natural numbers. However because they are ranging on "open" systems, they are general in the sense they have the power of being defined on all the domain, even though,

each time, *only the part of this domain necessary to conclude an effective computation* is given.
Perrone has developed several applications of these foundational ideas in different fields of computer science. They concern:

- *Automatic pattern recognition* in high energy physics experiments by the application of "dynamic perceptron" scheme (see § 1.2.2.2) [48].
- *Chaotic systems* characterization based on an effectively computable technique of the pseudo–cycles of any length [42],[43],[45].
- *Data compression techniques* based on the possibility of a "dynamic quantization" of the coefficients of the mathematical transform (wavelet, DCT, etc.) used [44], [90].

1.3.4 Thomas Aquinas' theory of intentionality

Quoting Putnam's work about *The meaning of "meaning"* [30], R. McIntyre [91] rightly emphasizes the core of any realistic theory of intentionality as to the problem of real reference. In any extensional and/or intensional approach to the problem of reference, *logical domain* of a given symbol determines the object. On the contrary, in a realistic approach *the real object must determine* the logical domain of a symbol. We gave different illustrations of such an idea in this paper, producing evidence both from the theoretical and from the experimental standpoints to sustain it. The preceding statement, however, recovers the core of the notion of intentional reference of Aquinas, which differs from the modern treatment of this notion.
In light of the previous discussion about Aquinas's approach to logic foundations, we have abandoned the idea that the logical notion of *reference* can be interpreted as a *logical twofold relation* between names and real objects. Generally, a logical relation has always its *reciprocal*. E.g., if $A = 2B$, then $B = ½ A$; if $A \geq B$, then $B < A$, if A causes B, then B is an effect of A, etc. On the contrary, it is well known that reference relation is without reciprocal: if A refers to B, B is not referring to A. It is related to B by some other relation. In Aquinas's foundational theory this picture is further complicated by the fact that the relation linking the object to the symbol referring to it is a *symbol constitution operation*, of which logical and ontological "machinery" we discussed in the previous subsection. Let us see the same idea from the epistemological standpoint.

As Aquinas emphasizes (See [77], I, 1; [78], b. I, XIX, 5, 2 ad 2um; [81], V, xvii, 1027), for granting real reference, we must consider the referential object *constituting* the symbol that *refers to* object "beingness" and hence that *names* the object. Particularly, (See [79], II, chs. 12-15) we must consider the referential object to be what *makes existing* in a logical sense the *true* proposition naming it. For instance, following Aquinas, the proposition "the sky is blue" is a true logical symbol of the object I am observing *iff* the blue sky I am *actually* observing is able to modify both the extension and the intension of the predicate "being blue", so to include in its domain the singularity of this object with its absolute novelty. But the reciprocal of such an act of constitution does not hold. In fact, if for any reason we pretended to designate the same object by the false proposition "the sky is yellow", the blue sky is not made yellow. Logically, we are thus constrained to say that the reference is neither *a logical nor a causal twofold relation,* but a *metalogical operation* of symbol constitution by the referential object.

We know from the discussion above that the referential object here concerned is not the "existent thing" but its "essence" in its being a constitutive metaphysical principle of the *beingness* such an existent thing. By this idea of reference as an operation of logical constitution, we can understand how for Aquinas the same object in different contexts – effectively, the same essence in different existential istantiation — will modify the universal symbol that designates it as a "one – to – one universal"[17], i.e., as a "rigid designator". Psychologically, such a logical operation corresponds to the famous theory of truth as *self – conforming* (*adaequatio* in Latin) of the intellect to the thing. The knowing act is the operation by which the senses and the intellect inner state *assimilate themselves* continuously to the changing referential thing. It is not the thing that must accommodate itself to the *a priori* of human minds, as in modern approach after Kant, but it is the *a priori* of the human mind modified continuously to make itself adequate to the referential thing. The domains of the predicates are not constituted *a priori*, but they are constructed step by step for including symbols designating new objects and/or states of affairs. The "logical machinery" of such an epistemological and psychological theory of intentionality could now be more intelligible in the light of the previous subsections.

[17] A "one–to–one universal" is a name that designates universally a singular object. Classical universals of such a type are the proper names.

Because of the formal justification of real reference as *a constitution operation* (generation of symbols) and not as a simple asymmetric relation, it is possible to define formal languages as "top – down" (to the referential object) and not "bottom – up" (to higher order meta–language) *semantically open*. In this way, one can imagine also new promising approaches both to the problem of inductive schematism in cognitive psychology of perception (see § 1.2.1.2) and to the two difficult formal problems with which both computer science and cognitive neuroscience are today faced (see § 1.2.2.2). I.e., the problem of an effective mathematical characterization of unstable and non - stationary dynamic systems and the problem of really parallel computation in natural and artificial NN's.

1.4 Conclusion

In this paper we deepened the relationship existing between the intentionality problem in cognitive science, and the problem of foundations in logic and computer science. The main result of this research is the necessity of overcoming the "axiomatic ideology" both in logic and computer science for allowing the construction of "open" logical and mathematical systems. In this way, also the problem of simulating intentional behavior can hope to find a solution in cognitive science. What is indeed characteristic of the intentional mind is its capability of "changing the basic symbols" of its logical computations for locking itself onto the changing reality. This idea recovers the essential of Aquinas's approach to foundations of logic as well as to intentionality problem.
In other words, Immanuel Kant's philosophical "Copernican Revolution" placed human intellect not the object at the center of modern science construction, just as Copernicus placed sun in the center of the solar system instead of earth. This philosophical and cultural revolution was justified by the wondrous victories of Newtonian calculus and of the axiomatic method after Descartes. Euclidean geometry became the paradigm of the new Galileian science. The evolution of numerical calculus; the necessity of overcoming the "fixity" of classical axiomatic method and hence the "stupidity" of actual computers, as well as the necessity of not abandoning logic and mathematics foundations to the "weakness" of subjectivism, all this imposes today a counterrevolution. This revolution however is not and cannot be the counterpart of a return to Pto-

lemy in cosmology. Einstein's cosmology discovered that universe has no center, because it is not static. What post–modern science needs for growing up, with an higher awareness of its limitations, but just for this with a more effective control on its ever increasing power, is a logic of "open" formal systems. From that, an epistemology of truth as unending process of self–conforming of intentional mind to an always-changing reality can suggest new more effective solutions to artificial simulations of cognitive behavior.

References

[1] H. Gardner, T*he new cognitive science. A History of Cognitive Revolution*, Cambridge, MA, MIT Press (1985).

[2] H. Putnam, «Minds and machines», in: *Dimensions of mind*, S. Hook, ed., New York, Cambridge University Press (1960).

[3] I. Kant, *Logic*, translated by Hartman R. S. and W. Schwarz, New York, Bobbs – Merrill (1800, 1974).

[4] I. Kant, *Kritik der reinen Vernunft*, in: *Kant's gesammelte Schriften*, hrsg. von Deutschen Akademie der Wissenschaften, Berlin, pp. A142/B181 (1781/7, 1900).

[5] A.M. Turing, «On computable numbers with an application to the Entscheidung problem», *Proceedings of the London Mathematical Society* 42:230-265 (1937).

[6] K. Gödel, «Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme», *Montash. für Math. u. Phys.* 38:173-198 (1931).

[7] D.R. Hofstadter, *Gödel, Hescher, Bach. An Eternal Golden Braid*, New York, Basic Books (1979).

[8] J. Piaget, *La Psychologie de l'Intelligence*, Paris, Colin (1952^5).

[9] U. Neisser, *Psicologia cognitivista*, tr.it., Firenze, Giunti-Martello (1976).

[10] C. Fabro, *Percezione e pensiero*, Milano, Vita e Pensiero (1941).

[11] J.A. Fodor, «Fodor's guide to mental representation: the intelligent auntie's vademecum», *Mind* 94:76-100 (1985).

[12] A.M. Turing, «Computing machinery and intelligence», *Mind* 59:433-460 (1950).

[13] J.A. Fodor, «Methodological solipsism considered as a research strategy in cognitive psychology», *The Behavioral and Brain Sciences* 3:63-109 (1980).

[14] J.A. Fodor, *Psychosemantics. The problem of meaning in the philosophy of mind*, Cambridge, MA, MIT Press (1987).

[15] A. Tarski, «Der Wahreitsbegriff in formalisierten Sprachen» (German trans-

lation of a book in Polish), *Studia philosophica* 1, 261-405 (1935).

[16] A. Tarski, «The semantic conception of truth and the foundations of semantics», in: *Readings in philosophical analysis*, H. Feigl, ed., New York, Cambridge University Press, pp. 52-84 (1944, 1949).

[17] R. Carnap, *Studies in semantics: introduction to semantics*, 3 vv., Cambridge, MA, MIT Press (1942).

[18] F. Brentano, *Psychologie vom empirischen Standpunkt*, Leipzig, Hahn (1874, 1924-28).

[19] J.C. Webb, *Mechanism, mentalism and metamathematics*, Dordrecht, Reidel (1980).

[20] S.C. Kleene, *Introduction to Metamathematics*, Amsterdam, North-Holland (1952).

[21] K.R. Popper, *Conjectures and Refutations*, London, Routledge and Keegan Paul (1969).

[22] J.R. Searle, «Mind, brains and programs. A debate on artificial intelligence», *The Behavioral and Brain Science* 3:128-135 (1980).

[23] J.R. Searle, *Intentionality. An essay in the philosophy of mind*, New York, Cambridge University Press (1983).

[24] E. Zalta, *Intensional logic and the metaphysics of intentionality*, Cambridge, MA, MIT Press (1988).

[25] W.V.O. Quine, *Quiddities. An intermittently philosophical dictionary,* Cambridge, MA, Harvard University Press (1987).

[26] W.V.O. Quine, *Word and object*, Cambridge, MA, MIT Press (1960).

[27] P.S. Churchland, *Neurophilosophy. Toward a unified science of the mind-brain*, Cambridge MA, MIT Press (1986).

[28] E. Husserl, *Recherches Logiques*, translated by H. Elie, Vol. I: «Prolégomènes à la logique pure», Paris, P.U.F. (1913, 1962), pp. 119-203.

[29] E. Husserl, *Formal and Transcendental Logic*, translated by D. Cairns, Le Hague, Nijhoff (1929, 1969).

[30] H. Putnam, «The mening of 'meaning'». In: *Philosophical papers: mind, language and reality,* New York, Cambridge University Press (1972), pp. 215-271.

[31] H. Putnam, *Representations and reality*, Cambridge, MA, MIT Press (1988).

[32] S. Kripke, *Naming and necessity,* Cambridge MA, Harvard University Press (1972, 1996).

[33] D.O. Hebb, *Organization of behavior. A neuropsychological theory*, New York, Wiley (1949).

[34] T. Kohonen, *Self - organization and associative memory. Second Edition*,

Berlin, Springer (1988).

[35] J.L. McClelland, D.E. Rumelhart and the PDP Group *Parallel distributed processing*, Cambridge, MA, MIT Press (1986).

[36] C. Von der Marlsburg, and E. Bienenstock «Statistical coding and short-term synaptic plasticity: a scheme for knowledge representation in the brain», in: *Disordered systems and biological organization*, NATO ASI Series F20, Berlin, Springer, pp. 312-317 (1986).

[37] C.M. Gray, P. Koenig, A.K. Engel and W. Singer «Oscillatory responses in cat visual cortex exhibit inter-columnar synchronization which reflects global stimulus properties», *Nature* **338**, pp. 334-337 (1989).

[38] R. Eckhorn, H.J. Reitboeck, M. Arndt and P. Dicke «Feature linking via synchronization among distributed assemblies: simulation of results from cat visual cortex», *Neural Computation* 2:293-307 (1990).

[39] A.K. Engel, P. Koenig, C.M. Gray and W. Singer «Synchronization of oscillatory responses: a mechanism for stimulus-dependent assembly formation in act visual cortex», in: *Parallel processing in neural systems and computers*, R. Eckmiller, ed., Amsterdam, Elsevier, pp. 212-217 (1990).

[40] C.A. Skarda, and W.J. Freeman «How brains make chaos in order to make sense of the world», *Behavioral and Brain Sciences* 10:161-195 (1987).

[41] G. Parisi, «Asymmetric Neural Nets and the Process of Learning», *Journ.of Phys. A: Math.Gen*, 19, L675-680 (1986).

[42] A.L. Perrone, «A formal scheme to avoid undecidabilities: an application to chaotic dynamics characterization and parallel computation» in: *Cognitive and dynamical systems. Lecture Notes in Computer Science,* S.I. Andersson, ed., 888:9-52 (1995).

[43] A. L. Perrone, «Verso una teoria dinamica della computazione», in: G. Basti and A.L. Perrone, *Le radici forti del pensiero debole: dalla metafisica, alla matematica, al calcolo*, Padova-Roma, Il Poligrafo e Pontificia Università Lateranense, pp. 255-332 (Percorsi della scienza. Storia testi e problemi, 7) (1996).

[44] A.L. Perrone, «Applications of chaos theory to lossy image compression», *Nuclear Instruments and Methods in Physics Research. Section A.,* 389, pp. 221-225 (1997).

[45] A. L. Perrone, «The cognitive role of chaos in neural information processing». In: *Proceedings of the International School of Biocybernetics: «Processes in the perception representation mechanisms»*, C. Taddei-Ferretti and C. Muzio eds., Singapore-London, World Scientific (1999) In press.

[46] M.I. Posner, and M.E. Raichle *Images of mind*, New York, Scientific Ameri-

can Library (1994).

[47] M. Minsky, and S. Papert *Perceptrons.* Second Edition, Cambridge Mass., MIT Press (1988).

[48] A.L. Perrone, Basti G., Messi R., Pasqualucci E., Paoluzi L. «Offline Analysis of HEP events by the ‘dynamic perceptron’ neural network», *Nuclear Instruments and Methods in Physics Research. Section A*., 389**,** 210-213 (1997).

[49] G. Basti, and A.L. Perrone «Per un ruolo costitutivo del caos deterministico nei sistemi neurali biologici ed artificiali», in: *Biosistemi e complessità*, E. Belardinelli and S. Ceutti, eds, Bologna, Pàtron, pp. 239-289 (1993).

[50] G. Basti, A.L. Perrone and P. Cocciolo «Using chaotic neural nets to compress, store and trasmit information», in: *Applications of Artificial Neural Networks, V*, SPIE–Proceedings Series, 2243, S.K. Rogers and D.W. Ruck, eds, Washington, D.C., SPIE Press, pp. 540-551 (1994).

[51] G. Basti, A.L. Perrone, E. Pasqualucci, R. Messi, P. Picozza, W. Pecorella, L. Paoluzi, «Principles of computational dynamics: applications to parallel and neural computations». In: *Applications and Science of Artificial Neural Networks, VII*, SPIE–Proceedings Series 2760, S.K. ROGERS AND D.W. RUCK (EDS.), Bellingham WA, SPIE Press, pp. 738-752 (1996).

[52] F.T. Arecchi, G. Basti, S. Boccaletti and A.L. Perrone «Adaptive recognition of chaos», *Europhysics Letters* 26, pp. 327-332 (1994).

[53] M. Tsukada, (1998). Private communication for his experimental findings in late 1970s on dynamic receptive field in cat retinal ganglion cells.

[54] J. J. Eggermont, A. M. Aertsen, H. J. Hermes, and P.I.M. Johannesma, «Spectro-Temporal Characterization of Auditory Neurons: Redundant or Necessary?», *Hearing Research* 5, pp. 109-121 (1981).

[55] M. P. Kilgard and M. M. Merzenich, «Cortical map reorganization enabled by nucleus basalis activity», *Science* 279, pp. 1714-1718 (1998).

[56] H. Dinse, «A Temporal Structure of Cortical Information Processing», *Concepts in Neuroscience* 1, pp. 199-238 (1990).

[57] H. Dinse, «A time-based approach towards cortical functions: neural mechanisms underlying dynamic aspects of information processing before and
after postontogenetic plastic processes», *Physica D* 75, pp. 129-150 (1994).

[58] I. Lakatos, *Mathematics, science and epistemology,* Cambridge, Cambridge University Press (1978).

[59] D. R. Hofstadter, *Fluid concepts and creative analogies,* New York, Harper Collins Publ. (1995).

[60] C. Cellucci, *Le ragioni della logica,* Roma–Bari, Laterza (1998).

[61] G. Basti, «Per una lettura tomista dei fondamenti della logica e della matematica», in: Basti G. and Perrone A.L., *Le radici forti del pensiero debole: dalla metafisica, alla matematica, al calcolo*, Padova-Roma, Il Poligrafo e Pontificia Università Lateranense, pp. 19-254 (1996).

[62] G. Basti «L'approccio aristotelico-tomista alle aporie dell'induzione». In: *Il fare della scienza. I fondamenti e le palafitte,* F. Barone, G. Basti, C. Testi eds., Padova, Il Poligrafo, pp. 41-95 (1997).

[63] S. Feferman, «Transfinite recursive progressions of axiomatic theories», *The Journal of symbolic logic* 27, pp. 259-316 (1962).

[64] T. G. McCarthy, «Self–reference and incompleteness in a non–monotonic setting», *The Journal of philosophical logic* 23, pp. 423-449 (1994).

[65] A.M. Turing, «Systems of logic based on ordinals», in: *The Undecidable*, P. Davies ed., New York, Raven Press, pp. 208f. (1967).

[66] K. R. Popper, *Logic of scientific discovery*, London, Routledge and Keegan P. (1959).

[67] J.-Y. Girard, «Le champ du signe ou la faillite du réductionisme», in: E. Nagel and J. R. Newmann, *Le théorème de Gödel*, Paris, Editions du Seuil, p. 161 (1989).

[68] G. Gentzen, «Die Wiederspruchsfreiheit der reinen Zahlenthorie», in *Mathematische Annalen*, 112, pp. 439-565 (1936).

[69] M. Minsky, *The society of mind*, New York, Simon and Schuster (1987).

[70] P.J. Cohen, *Set theory and the continuum hypothesis*, New York (1966).

[71] Platone, «Tehetetus», in: *Opere*, G. Giannantoni, ed., Roma–Bari, Laterza, 1971ff.

[72] Platone, «Parmenides», in: *Opere*, G. Giannantoni, ed., Roma–Bari, Laterza, 1971ff.

[73] Thomas Aquinas, *In Aristotelis Libros Perì Hermeneias et Posteriorum Analyticorum Expositio*, SPIAZZI R. (ED.). Torino, Marietti (1964).

[74] Thomas Aquinas, «In Librum Boetii De Hebdomadibus Expositio», M. Calcaterra ed., in: *Opuscula Theologica*, II v.: *De Re Spirituali*, R. Spiazzi ed.. Torino, Marietti (1954).

[75] Thomas Aquinas, «In Aristotelis Libros Posteriorum Analyticorum Expositio», in: *In Aristotelis Libros Perì Hermeneias et Posteriorum Analyticorum Expositio*, R.M. Spiazzi, ed., Torino, Marietti (1964).

[76] Thomas Aquinas, «Quaestiones de Potentia», in: *Quaestiones Disputatae*, II v.: P. Bazzi et Al. eds., Torino, Marietti, (1965).

[77] Thomas Aquinas, «De Veritate», in: *Quaestiones Disputatae*, vol. I, R.M. Spiazzi, ed., Torino, Marietti, (1953).

[78] Thomas Aquinas, «In Quatruor Libros Sententiarum Magistri Petri Lombardi Commentarium. Liber Quartus», in: *Opera Omnia*, tt.VI-VII, Parma, Editio Leonina, 1865-68.

[79] Thomas Aquinas, *Summa contra Gentiles*, D.P. Marc, ed., Torino, Marietti, (1967).

[80] F. P. Ramsey, «The foundations of mathematics», in: *The foundations of mathematics and other logical essays*, New York (1931, 1950).

[81] Thomas Aquinas, «In Duodecim Libros Metaphysicorum Aristotelis Expositio», R.M. Spiazzi, ed., Torino, Marietti, (1964).

[82] Thomas Aquinas (*S. Th.*) *Summa Theologiae*, P. Caramello Ed., Torino, Marietti, 1952-1956.

[83] E. Weyl, *Il Continuo. Indagine critiche sui fondamenti dell'analisi*, A. B. Veit Riccioli ed., Napoli, Bibliopolis (1932, 1977) .

[84] De Giorgi E., Forti M., Lenzi G. «Verità e giudizi in una nuova prospettiva assiomatica», in: *Il fare della scienza. I fondamenti e le palafitte*, F. Barone, G. Basti, C. Testi eds., Padova, Il Poligrafo, pp. 233-252 (1996).

[85] M. Forti, L. Galleni, «An axiomatization of biological concepts within the foundational theory of Ennio De Giorgi», *Biology Forum* **92**, pp. 77-104 (1999).

[86] M. Forti, «The foundational theories of Ennio De Giorgi», in: *Foundations in mathematics and biology: problems, prospects, interactions*, G. Basti and A. L. Perrone eds., Rome-Milan, Mursia and Pontifical Lateran University Press (1999). In press.

[87] Thomas Aquinas «De Natura Generis», in: *Opuscula Philosophica*, R.M. Spiazzi, ed., Torino, Marietti, (1954).

[88] Thomas Aquinas, *In librum Beati Dionysii de divinis nominibus expositio*, C. Pera, ed., Torino, Marietti (1950).

[89] Thomas Aquinas (*In Phys.*). *In Octo Libros Physicorum Aristotelis Expositio*, P. Maggiolo Ed., Torino, Marietti, (1965).

[90] A.L. Perrone, G. Basti, M. Ricciardi, «Lossy plus lossless residual encoding with dynamic pre–processing for Hubble space telescope fits images», In: *Applications and Science of Computational Intelligence, II*. K.L. Priddy, P.E. Keller, D.B. Fogel, J. C. Bezdek Eds., SPIE - Proceedings Series vol. 3077, SPIE- The Int. Soc. for Optical Engineer, Bellingham, WA, 1999, p. 532-541.

[91] R. McIntyre, «Intending and referring», in: *Husserl, intentionality and cognitive science*, H.L. Dreyfus and H. Hall, eds, Cambridge MA., MIT Press, pp. 219-235 (1982).