



Gianfranco Basti

Philosophy of Nature and of Science

Volume I: The Foundations

Translation by
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not revised by the
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[*Image on the cover*. Capture, occurred about 40 million years ago, of the spiral galaxy (IC2163) into the gravitational field of another one (NGC2207). They are destined, in few billion years, to become one only galaxy. (NASA, Hubble Space Telescope Institute©)].

0. General Introduction

Introductory remarks concerning the purpose and structure of this work as well as its contents

0.1 General Framework

This book intends to be a manual with the ambitious goal of defining a systematic link between two disciplines: *the philosophy of nature* and *the philosophy of science*, in terms of their often problematic relationship with *mathematical and natural sciences*.

The *Philosophy of Nature* is a special discipline within *General Metaphysics*, which has as its object the universe of *physical beings* with their specific *structures, properties and causal relationships*, examined on the level of their *fundamental ontology*. The Philosophy of Nature is therefore distinguished from the other *natural sciences* (physical, biological and cognitive), which, from a modern perspective, are limited to the study of *natural phenomena*, in as much as they are measurable, and of *laws*, usually formalized with the help of mathematics, which govern *the evolution in time* of these phenomena.

The *Philosophy of Science*, on the other hand, is recent a discipline within the philosophical panorama, dating from less than a century ago. It is characterized as a special discipline of the *Philosophy of Knowledge* or *Gnoseology*, and has as its object the *logical and epistemological foundations* of the natural sciences and mathematics. Often, due to the eclipse of the philosophy of nature in modern culture, it has come to occupy a vicarious role for the philosophy of nature, dealing with questions of an ontological nature, if not directly metaphysical, i.e., the origin and destiny of the universe in cosmology; the nature of matter, space and time in the physical sciences; the nature of life in the biological sciences; or the nature of knowledge or of the mind in cognitive sciences.

Given that the question concerning the foundations is the key to understand the complex relations between science and mathematics in modernity, and, today, also between philosophy of science and philosophy of nature, the development of these issues in the first two parts of the book will entail a good part of this work.

This work is therefore divided into volumes. The First Volume is exclusively dedicated to the First and Second part of the treatise, i.e., the definition of the historical and theoretical framework of the issue concerning the foundations. The Second Volume is dedicated to the other three parts that deal with the applications of logical and metaphysical principles, discussed in the First Volume, and with the objects of the physical, biological and cognitive sciences.

Volume I: Philosophy of Nature and of Science. The Foundations.

First Part: Historical Framing of the Issue. – 0. Introduction – 1. Science: from the origins until the Nineteenth Century – 2. The Twentieth Century and its scientific revolutions – 3. The search for foundations in the Twentieth Century.

Second Part: Theoretical Framing of the Issue. – 4. Philosophy of Nature and Philosophy of Science. – 5. Classical metaphysics and modern thought. – 6. Highlights concerning a metaphysics of physical being.

Volume II. Philosophy of Nature and of Science. Physical, Biological and Cognitive Sciences.

Third Part: Physical Sciences and their Objects – 1. The structure of matter and its complexity – 2. Scientific cosmology and the origin of the universe.

Fourth Part: Biological Sciences and their Objects. – 3. Different ontologies about living beings and the biological sciences. – 4. Scientific hypothesis concerning evolution and the origins of life.

Fifth Part: Cognitive Sciences and their Objects – 5. Cognitive functions and their neurophysiological bases – 6. Freedom and psycho-physical determinisms – 7. Psychophysical unity of the person and the origins of human life.

0.2 Contents of this volume

0.2.1 Advice on how to read this book

Three suggestions
for reading this
book

The vastness and complexity of this first volume of the complete work may appear to be in contradiction of the desire to construct a ‘manual’. In order to make reading easier, aside from the indexes and the glossary of scientific terms at the end of the book, chapters summaries have been introduced to synthesize the content of each one.

What we recommend to the reader and especially to students is to proceed step-by-step while reading this book.

1. *Above all, read this introduction carefully* which will clarify the aims of the whole treatise, placing it within the modern and contemporary debate concerning the issue, and summarizing the logical development of the entire work. For this reason, frequent allusions are made to different parts of the book which deal with the same arguments in a more complete way. An attentive reader will be able to locate those areas of the book which interest him the most.
2. *Secondly, read the chapter summaries.* From this second level of understanding, one can obtain a clear idea of the different parts of the treatise while at the same time retaining a clear vision of its unity.
3. *Thirdly, start reading the contents of each chapter,* understanding the subject of the entire work by way of concentric circles.

0.2.2 Truth and causality in Science

Three basic
questions of this
Volume

In this first volume, dedicated to the «foundations», we will examine some basic questions concerning the problem of the relation between philosophy of nature on the one hand, and between philosophy of nature and philosophy of science on the other hand. There are two questions of immediate metaphysical importance which will occupy our attention and which are particularly relevant for modern thinkers:

1. Truth and realism
of scientific
knowledge

- ◆ The problem concerning the *truth* and *realism* of scientific constructions within the natural sciences, with particular reference (from within the hypothetical-deductive method) to the question of the real foundation of the *hypotheses* that are at the base of different scientific theories; against the thesis (common to both logical neopositivism and Popperian falsificationism) of the non rational character of invention and of the construction of the same theories.

2. Logical necessity,
causal necessity
and their relation

- ◆ The problem of *causality* and its foundation in the natural sciences in relation to the problem of the logical *necessity* of demonstrations and of its *foundation*. Notice that classical thought and modern thought are at opposite ends of the debate concerning this issue. While for classical thought, especially Aristotelian-thomistic, it was the necessity of *real relations* (causal) – those that determine the existence (or common being) and the being (or being of the essence) of the individual natural beings – that through abstraction grounded the ne-

cessity of *logical relations*, for modern thought after Kant, the exact opposite is true. It is the necessity of the *self-evidence* of the logical *a priori*, of the formal relation between logical antecedent and consequent that grounded the necessity of the causal relation between predecessor and empirical successor, placed in temporal succession and made *evident* by experimental knowledge. In such a way, Kant intended to solve Hume's famous "problem of induction". Such a solution was derived from two tacit suppositions:

Critique of the
Humian-Kantian
model of causality
in the sciences:
a. Causal relation
grounded by time
and therefore by
consciousness

b. Apodictic
Foundation of
necessity based on
evidence and
therefore on
consciousness

- a.** One supposition that tied the causal relation exclusively to the *temporal* relation between predecessor and successor of the process, because the paradigm used was that of Newtonian *mechanics**¹ of dynamic integrated systems, where the final state of the process depends *univocally* on the initial conditions. In this way, for the first time causality was tied to the knowledge of the phenomenon, instead of to the being of the thing, since without the mind there can not be even time.
- b.** A second supposition dealt with a apodictic foundation of the logical *a priori*, a foundation based on the Cartesian principle of evidence, evidence not only of the first principles of formal logic (principle of non-contradiction, of identity, etc.) and of their application to «being in terms of being» of metaphysics, as it was for classical thought, but also of the postulates of the different sciences. Above all, the postulates of Euclidean *geometry** in its «algebraic» or «analytical» form as proposed by Descartes were considered self-evident and therefore apodictic. However, after Newton, the three fundamental postulates (laws) of mechanics were also considered in this manner. In this way, we see an abandoning of the classical distinction between self-evident *axioms* (or «first principles» of logics and metaphysics) and *postulates* (or «first principles» of the various scientific, physical and mathematical disciplines), anything but self evident for classical thought – one need think only of the difficulties that the famous fifth postulate of Euclid caused the ancient mathematicians -- , given that they did not deal with the common being (existence) of any being as with general

¹ The terms indicated with an asterisk (*) are defined in the «Glossary», added as an Appendix at the end of the book, because they are terms which are not defined and explained within the treatise itself. In order to understand the text, it is important to be able to suppose that the reader has grasped their meaning within a philosophical and scientific context. These terms are generally only indicated the first time they are used.

metaphysics, but rather with the proper being (being-ness) of the different species of physical and logico-mathematical beings. The first principles of the various sciences, far from being self-evident, for classical thought and above all for the authentic Aristotelian tradition were tied to the study of the proper objects of the various sciences, being essentially of an *inductive* nature.

Vice-versa, modern thought, intending to extend the notion of self-evidence proper of the meta-logical axioms of formal logic and general metaphysics to the logical postulates of the various sciences and in particular of mathematics (Descartes) and of physics (Newton), *bound science to the cart of the mind*. This time, the cart of self-consciousness was given in terms of it being «transcendental» or «pre-categorical» of the self-evidence of the axioms of the various mathematical and natural sciences, as well as of a «critical» metaphysics – Kantian – very different from a classical metaphysics. In fact, the three «ideas» of a rational Wolffian metaphysics were reduced to mere fideistic postulates, self-evident for practical reason and for a formalistic ethics of «duty for duty's sake».

Classical and
modern meaning of
Transcendental

These two problems, that of *truth* and that of *causality*, are intimately connected, and the connection consists of the transcendental foundation of truth: either *transcendental consciousness* and *the logical relations* of modern thought, or *being* and *its real relations* (causal) of classical thought.

0.2.3 Instrumentalizations of Philosophy and Science

Post-modernism
and overcoming the
classical-modern
opposition

The main question this books attempts to answer is the following: this opposition between classical and modern thought still matters, or has it been definitively «overcome», in a quasi-Hegelian sense of the term, by post-modern thought? A dialectical sense of «overcoming» which would be justified by the antithetical character of the aforementioned opposition between classical and modern thought.

Although I would never be considered a Hegelian, the thesis of this book is that one must answer in the positive to the question just mentioned. The post-modern context in which we live is not simply an artificial juxtaposition of pieces of classical and modern thought, a «surrounding of a Corinthian capital with neon lighting» as someone ill with a longing for 'old-fashioned modernism' would still maintain. In ontology, logics and epistemology – the three fields of investigation that are

developed in this treatise – post-modern thought (as we thinkers of the third millennium can construct it) is the historical occasion that is offered to contemporary thought to take up in a new and synthetic way the essential aspects of classical thought – its content, «being» – and the essential aspects of modern thought – its form, «critical rigor of investigation» – beyond both the ingenuity of classical thought and iconoclastic and self-defeating old-fashioned modern thought. Such a synthesis can finally put to rest those dead ends in both classical and modern thought – yet another Hegelian reference, this time to his famous «caput mortuum of the *Encyclopedia*» – that both shared and that have obscured the brilliance of both, becoming the profound motivation of their irreducible and ideological opposition during the entire modern period. And those ‘dead ends’ have a common identity, a very precise one for both parties involved. The pretension that metaphysics on one hand and science on the other each construct a type of *absolute and self-sufficient knowledge*, each capable of reducing the other to itself.

The origin of the opposition between classical and modern thought in the ‘Galileo’ question

On one hand, we have an ideological metaphysics, incarnated in the principle of the modern period by some «Aristotelian» philosophers from Florence and Pisa and from their ranks those who opposed Galileo. They intended to derive the principles of «natural philosophy» from the axioms of Aristotelian metaphysics, forgetting that for Aristotle himself and for a large portion of the Aristotelian tradition – including that of the scholastics, beginning with Thomas Aquinas – the postulates of the physical sciences were of an inductive origin, not deduced from metaphysical principles, and that the demonstrations which sprang from them, given that they dealt with *contingent* causal processes, had a *hypothetical* value. From the same laws, therefore, it was necessary to subject the truth of concrete cases to experimental control each time.

False opposition between two apodictic systems of knowledge: physical and metaphysical

On the other hand, we have a physico-mathematical science from the origins of modernity that, to reclaim a proper autonomy from metaphysics, intended to give a foundation to its demonstrations that would be *apodictic*, *absolute* like that of metaphysics, basing them on the supposed *self-evidence* of its postulates: whether they be postulates of Euclidean geometry, in its progressive algebraization begun by Descartes and applied by Leibniz and Newton to the nascent *mathematical analysis** and to its applications in mechanics; or the three laws of *dynamics**, according to its modern Newtonian systematization.

Different ideological instrumentalizations of this opposition

In this way, bases were placed for an ideological instrumentalization of modern science which can be called «scientism», in which against the reductionist pretension of a rationalist metaphysics with regards to the

physics proposed by the anti-Galilean Aristotelians, another reductionist pretension was opposed by those who, in the name of «modernity», wanted to reduce classical theology and metaphysics to a simple «superstition». Along this road of such an ideological opposition, it was quickly seen that in modern science and philosophy, a new paradigm of reason was ushered in, where «enlightenment brilliance» would replace «the darkness» of superstition and conservatism, viewed as contradicting the forces of «progress». It was this radicalization of the opposition that moved Hegel to elaborate the theory of the «two cultures» and to theorize the logical and historical necessity (from his historical-dialectical method) of the unavoidable «overcoming» of the first at the hands of the second.

Modern period as
the era of the
'visions of the world'

It is difficult to fault Heidegger who defined the modern period as the «era of the *visions of the world*»: an era which caused upheavals both in science and philosophy during the 19th and the first half of the 20th centuries, and which seemed definitively «overcome», such as to justify – in the absence of a better definition – the designation of «post-modern» for that period apparently «without absolute certainties» – neither those metaphysical-theological of the classical era, nor those scientific of modernity – which we are living today.

Anticipating synthetically what we will try to demonstrate in the rest of this volume, it is worth while to briefly consider some theoretical aspects of the «Galileo question», paradigmatic of the modern opposition between the two cultures, not only because of its symbolic value, but rather because the questions that were addressed there and the way of presenting them will condition the rest of the historical development of this opposition. Today, in fact, when the Church on one hand and methodological reflections on science on the other hand (actually before the Church) have each offered their respective *mea culpa*, renouncing once and for all the ideological error of their supposed and opposed reductionist absolutisms, it is possible to view the question with greater objectivity, identifying in past errors the indication of the road to follow for future developments: i.e., for the construction of a new synthesis between metaphysics and natural sciences that can give the term «post-modern», a «positive» *connotation** and not only a «negative» one in terms of a pure and simple negation of the ancient and modern ideological «isms» of the past.

0.2.4 The Galilean Question

The double defeat of Galileo and Bellarmine in modernity and their post-modern rematch

At first blush, one could say that the scientific and theological revolutions of the 19th and 20th centuries have re-instated the two «champions» of the opposing fronts: Cardinal Robert Bellarmine and Galileo Galilei who ended up united in battle and in defeat, in the name of consolidated principles in the same Aristotelian and Scholastic authentic tradition, against the integralist Aristotelianism of the natural philosophers and of their theological companions. Historically, it was Bellarmine who proposed in 1615 to a Carmelite priest from Naples – who had written a brief work trying to reconcile Copernican theories with the affirmations in the Bible – and to Galileo himself during his famous visit to Rome in 1615 in order to persuade the Church not to take official positions against Copernicanism, to treat Copernicanism only as *hypothetically* and not *apodictically* true (Drake 1990).

The neo-gnostic risk of conceiving mathematical physics as absolute knowledge, another way beyond revelation of knowing the thought of God

As expert theologian and refined epistemologist, Bellarmine well knew the theoretical danger of affirming an *apodictic* value of physico-mathematical demonstrations. A serious risk is run if like Galileo (albeit in good faith), may have affirmed – based on the premise, indubitable for the man of faith, that it is the same God who made nature and who spoke in the Bible – that physico-mathematical science is understood as another way, parallel to that of the Revelation of Scripture, to behold God's very thought, reading the «great book of Nature»: a book written by God, according to Galileo, in mathematical language. The risk is that of undervaluing Revelation as the only way available to the man of faith in order to arrive at God's mind, the only source of *absolute truth* for the believer. For that very reason, the risk becomes that of conceiving faith in a neo-gnostic manner, as a «refuge for the ignorant», until eventually one equates (as Spinoza did at the end of the 17th century) «God» with «geometrical determination of the laws of nature»: *Deus sive natura*.

The atheistic evolution of such a conception: Spinoza's *Deus sive natura*

In such a way, not only were the bases placed for modern theoretical atheism, but the road was paved for that anti-metaphysical and anti-religious program which characterizes the ideological recourse of modern science and which goes by the name of «scientism». Therefore, when (to exemplify to what extent Spinoza had banalized the issue by equating God and nature) the religious man saw in a certain natural event the sign of a providential causality, and the physicist saw in the same event the result of certain laws of nature, they would not be looking at the same complex object, according to two distinct and incommensurable levels of investigation, never in contradiction between themselves; rather, given the identity between God and the laws of nature, we would be confronting two equivalent propositions that affirm

the deterministic happening of an event: one with the pre-scientific language of ancient superstition, the other with the rational language of modern science.

'Scientistic
reductionism'

The authentic worry was precisely this scientistic and neo-gnostic reductionism, a completely legitimate worry as subsequent history would point out, that animated the keenest theologians – aside from the integralists' instrumentalizations – in discussing the complex Galileo question which is at the origin of the birth of modern thought.

Theological validità
of the Galilean
distinction between
truth of faith and
truth of science

Thus, equating God the Creator with the God of Revelation is perfectly valid when it is invoked by Galileo in his famous *Letter to Castelli* in 1613 as the principle to justify the metaphorical value of certain phrases in the Bible concerning natural phenomena, when they would seem to be in contrast with physico-mathematical evidence. In such a way, in purely physical questions, the Bible «should be placed last», after all the empirical evidence has been carefully considered. Galileo also reminded us, in his *Letter to the Grand Duchess Cristina* in 1615, St. Augustine's advice to not make any astronomical position an article of faith is always valid, otherwise some heretic better informed about science could take advantage of every error to cast doubt on properly theological doctrines. It was therefore formally incorrect that the philosophers and integralist theologians would invoke the authority of the Scriptures to justify the geocentric theory.

Official recognition
of the Church of the
validità of such a
distinction

The Catholic Church in the 20th century, above all with Vatican Council II in the Constitution *Dei Verbum* on divine revelation has solemnly made its own this theory of the different literary genres of which Galileo is a pioneer, placing it as a constitutive part of its own doctrine concerning the authentic interpretation of the meaning of Scripture. Finally, from 1978 on, until the solemn proclamation in 1992 of an *ad hoc* Committee, the Church itself, in the words of the Pope, John Paul II, has fully recognized that Galileo showed himself to be a better theologian than those of the Inquisition that condemned him in 1633, thus bringing to a close the centuries old debate.

0.2.5 Hypotheses and Truth

The true problem of
the Galilean
question: the
epistemic value of
scientific
hypotheses

In this light, one can say that (post-) modern contemporary thought has benefitted from the errors that from both sides has characterized the beginnings of modernity, recognizing the *hypothetical* character of every scientific theory, both physical and mathematical, and at the same time its *epistemological* autonomy from metaphysics and theology, in the now mature awareness of contemporary mentality that theories (whether scien-

tific, philosophical or metaphysical) can not pretend to be all-encompassing, or have exclusive or definitive rights over truth, as Pope John Paul II has solemnly stated in the Encyclical *Fides et Ratio* in 1998.

Philosophers are the first to understand the need for self-criticism, the correction of errors and extension of the too restricted terms in which their thinking has been framed. In particular, it is necessary to keep in mind the unity of truth, even if its formulations are shaped by history and produced by human reason wounded and weakened by sin. *This is why no historical form of philosophy can legitimately claim to embrace the totality of truth, nor to be the complete explanation of the human being, of the world and of the human being's relationship with God* (#51, my italics).

Hypothetical nature of formal systems and relative, universal and necessary truth, and their applied

However, while all that is true, things are not as simple as they would seem in this initial analysis: we still need to consider the relation between the hypothetical nature of scientific theories, in terms of formal systems, and the relative, limited yet authentic *truth* and therefore the *universality* and *necessity* of the derived models in their experimental and/or technological application to the study of their *real* objects of investigation.

Ideological nature of the belief of absolute formulations of truth in formal systems and the necessity of the partial truth of their applied models in terms of determinate objects

If it is perfectly correct to affirm that the (relative) *consistency** of a purely formal demonstration – just like a pure manipulation of linguistic signs without any semantic context, without symbolic value – is completely independent of the truth of the premises – in a hypothetical argument true consequences can be validly derived from both true and false premises – such is not the case for semantic models derived from that *formal system**. In the use of a formal system applied to a determinate object of investigation, or in other words in the use within different natural sciences and sciences applied to a formal theory, it is essential to be able to decide under which conditions and within which limits certain premises are true, i.e., adequate for the object of study, and where on the other hand they should be modified to correspond to their changing object. In sum, it is one thing to say *there does not exist, there has never existed, and there can never exist, because logically there can not exist an absolute and complete formulation of the truth in an demonstrative system* concerning any limited or absolute² object (thing) whatever; it is another thing to affirm that there

² To say that absolute demonstrative formulations of truth about some being do not exist is not the same as saying that there can not exist true affirmations *tout-court* concerning an eventual Absolute Being, as long as one postulates or demonstrates its existence. The Absolute and the Perfect One can be an object of true investigation and theory, just as much as that which is relative and limited (against nihilism). The important thing is that one never pretends that a certain theory of the Absolute should be absolutely true (against integralism). Absolute truth can never be entirely reached by human reason, but

do not exist nor can there exist *limited formulations, but precisely for this reason authentic and authenticatable of the truth in a deductive theory* concerning certain objects. The first affirmation not only does not exclude, but in a certain way it implies the second. The truth of the first affirmation, which denies the existence of absolute and complete formulations of the truth in demonstrative systems, can be validly understood as true in only a limited way, never absolutely, otherwise it would be self contradictory. And that should be remembered especially today, when so many fashionable nihilistic logical and epistemological currents of thought circulate.

0.1.1 Two Errors

Aside from the theoretical difficulty of the question about which contemporary logical and epistemological reflection is very much aware, a series of *theoretical and historical errors*, both at the beginning and at the end of modernity, has made this question even more intricate and almost unsolvable.

0.1.1.1 Absolutizing Metaphysics

Ideological use of Bellarmine's solution by metaphysical anti-scientific rationalism: hypotheses as mental functions without real foundation

The first and fundamental error concerns the substantially incorrect ideological use made by ancient Aristotelian philosophers, and taken up by certain philosophers and theologians in Galileo's time, of Bellarmine's wise and profound suggestion about the hypothetical nature of physico-mathematical theories. In order to understand what this theoretical incorrectness is about, it is necessary to historically frame Bellarmine's proposed solution.

This solution was similar to another more ancient idea that Gemino proposed in the 2nd century B.C. to reconcile the astronomical observations and measurements made by Aristarchus of Samos, with the theory of the concentric heavenly spheres which had the Earth as the center. A theory that Aristotle in his *Metaphysics* borrowed from Eudossus and which in the 2nd century A.D., will be taken up by Ptolemy, adjusted with his famous theory of the «epicycles». According to the account of Simplicius, commentator of the Aristotle's *Physics* of the 6th century A.D. and presented by Stillman Drake in his book that reconstructs the Galilei question (Drake 1990, 59-60):

only partially and in an indefinite progressive way. And even when revealed by the Divine Mind, absolute truth can only be understood by the human mind, due to its limitations, in an always perfectible way and never definitively («the Spirit will guide you to the fullness of truth» Jesus said in the Gospel of John). In this sense, one must understand the traditional affirmation of theology and metaphysics concerning the existence of a sole Absolute Truth, which can be reached by human reasoning only in a partial way.

The classical origins of such an instrumentalization in Greek thought of the 2nd century B.C.

Gemino's commentary, inspired by Aristotle's ideas, is the following (...). Astronomy explains only those things that it can establish by means of arithmetic and geometry. In many instances the astronomer on one hand and the physicist (i.e., the natural philosopher, in the Aristotelian sense of the word, *Nat.*) on the other will attempt to prove the same point, for example that the Sun is very large and the Earth is round; but they will not proceed along the same path. The physicist will demonstrate each fact with considerations about essence and substance, about forces, about how good things are as they are, or about generation and change.

The astronomer will demonstrate things based on the properties of the figures or of the sizes or by way of the quantity of movement and time, appropriate to it. In many cases, a physicist can even reach the cause, observing the creative force; but the astronomer, when he demonstrates facts from external conditions, is not qualified to judge about the cause, as when for example he affirms that the Earth or the stars are round. And perhaps he does not even want to ascertain the cause, as when he considers an eclipse, and other times he invents, *by way of hypothesis* and affirms certain expedients through which *the phenomena will be saved* (my italics).

Hypothetical and inductive nature of physical laws according to Aristotle

The accurate reference of Drake to Aristotle consists in the fact that for the Stagirite the premises of demonstrations in physics, dealing with contingent facts, were hypothetical and not categorical, i.e., not always true, like those of metaphysics. What is more, on the basis of this distinction, the famous difference between *axioms* and *postulates* of the various sciences was formed. The distinction, that is, between principles of first philosophy, the only ones self-evident and therefore properly speaking «axioms» (literally, worthy of being believed absolutely), and the principles of the various natural sciences, all in one way or another of an inductive origin, i.e., «postulates» of a *hypothetical* nature (not necessarily and always *true*, and therefore about conditions which are sufficient but not necessary), given the *contingent* character (not necessarily and always *existent*) of the physical beings/phenomena referred to. Thus, in the case of the physical sciences, the duty of empirical analysis was to inductively ascertain where and in which contexts the premises of the mathematical demonstrations in the physical sciences ended up true and where they ended up false, based on the different natures and causal re-

lations of the objects of the physics study. (Cf. Aristotle, *Phys.*, II, 199b, 34 ff).³

The problem sprang from the fact that, both the Aristotelian philosophers of the 2nd century B.C., who fought with Hipparchos and his fellow astronomers, and those modern ones of the 16th century who fought with Copernicus and his followers, interpreted the correct assertions of both Gemino and Bellarmine, in a totally false way. The key to this false interpretation is linked to the statement by Gemino – also of Aristotelian origin – that with those hypotheses of a mathematical nature, it was important to «save the phenomena». For this reason, some Aristotelian philosophers both ancient and modern felt authorized to consider astronomical theories as *purely mathematical functions*, bereft of any physical reality. Vice versa, it was up to the cosmologists (and therefore the metaphysicians) to know the true causes of the movement of the heavens and their laws. The conclusion, well synthesized by Drake was that:

For almost two millennia, the systems of the astronomers were considered by philosophers as mathematical inventions, not as descriptions of the real movements of the sky. These took place truly and causally as Aristotle had said, uniformly in circles around a single fixed center of the entire universe (Drake 1990, 183).

Newton's rebellion concerning the ideological interpretation of hypothetical nature of physical laws

One can thus easily understand why Newton, using the famous phrase *hypotheses non fingo*, later refused to accept what he saw as the Gemino-Bellarmino «compromise» to which Galileo with his retraction after the trial of 1633, basically submitted. However, above all one understands how Galileo was also right when in the *Dialogue* he appealed to Aristotle in order to defend the inductive origin (and not deductive from the principles (axioms) of metaphysics) of the hypothetical principles of the physical sciences, such that these principles, where applicable, would result in true descriptions (*cinematic*) and/or explanations (*dynamic*), by way of the application of mathematical laws, of the real movements of the heavenly bodies. Here is a passage taken from the first day in the *Dialogue* where Salviati, spokesman for Galileo, defends himself with clearly Aristotelian arguments from the accusations of Simplicius, spokesman of those (pseudo-) Aristotelians who were persecuting Galileo.

SIMPLICIUS. Aristotle built his principle foundation on the *a priori* discourse, showing the necessity of the unchangeableness of the heavens by way of his natural principles, manifest and clear; and

³ Concerning the theoretical link between the hypothetical nature of the principles of physical science and the ontological doctrine of the «four causes» in Aristotelian thought in opposition to the apodictic determinism of a geometric type of Democritian (and modern) mechanism, cf. *infra* § 6.3.2, p. 431.

he established the same thing afterwards *a posteriori*, by the meaning and the tradition of the ancients.

SALVIATI. This that you say is the method with which he has written his doctrine, but I do not believe that it was the same he used to investigate, because I know for certain that he first tried, by way of the senses, experiences and observations, to be certain as much as possible of the conclusion and that afterwards he sought the means to be able to demonstrate it, for that is how one proceeds in the demonstrative sciences (...). The certainty of the conclusion helps not a little to find the demonstration.⁴

Lack of foundation
of attributing to
Aristotle the
derivation of
physical laws from
metaphysics

Drake comments correctly that «even Aristotle wrote his metaphysics after his physics, which can be seen by the name which was given to it». In that book, Aristotle examined the principles that he had used in the research that he defined as «scientifico» which dealt with both logic objects and their laws, as well as natural events and their causes. Aristotelian metaphysics is therefore much closer that it would seem (after millennia of instrumentalization by integralists of many races and cultural and religious extraction) to what we would call today a *metatheory* or a *theory of the foundations* of logical and physical sciences. Aristotle himself states at the beginning of his *Metaphysics*:

Given that in every field of research where principles or causes or elements exist, knowledge and science are derived from them, it is evident that also in the science of nature one should try to determine that which refers to the principles: from that which is least clear by nature (the being of the various types of physical beings, object of the different natural sciences, *Author's note*) to that which is most clear and knowable by nature (being in terms of being, of metaphysics, *Author's note*). (*Metaph.*, I, 1, 184a,10-15).

However, Drake continues:

During the Middle Ages, things became inverted; medieval philosophers considered the principles so absolutely established in the Aristotle's *Metaphysics*, that it was absurd to admit in physics something that was not directly deducible from them (Drake 1990, 206).

⁴ Quoted in Drake 1990, 207.

Thomas' defense of the hypothetical and inductive character of physical laws, against their derivation from metaphysics

However, no medieval thinkers were on this same wave length: only those of the Church of a backwards and integralist mentality which made an ideological use of the scholastic, neo-Aristotelian philosophy of nature. To see how right Galileo was to propose that his method not only captured if not the letter certainly the spirit of Aristotelian naturalism, it is enough to read this quotation where Aquinas comments the above quoted passage of the *Physics* of Aristotle (*Phys.*, II, 199b, 34 ff.), in which the hypothetical character of the demonstrations of physics is born out. Thomas' commentary makes explicit the correct epistemological meaning one requires when searching for the adequate premises: a stronger meaning than that defended by Salviati-Galileo in the quotation given above. In fact, from the heuristic procedure of grounding *a posteriori* the premises from theorems that one intends to demonstrate, Aristotelianism – and here we see the profound sense of the difference between the geometric approach to physical science which grounds logical necessity exclusively on the premises or on the initial conditions (mechanism) and the Aristotelian approach – provides an ontological justification by way of the notion of formal-final causality, as we will further develop in Chapter 6 of this volume (Cf. § 6.3.2.2).

In the demonstrative sciences, necessity is constituted *a priori*, as when we say that *if* the definition of a right angle is such, *then* it is necessary that a triangle be such, i.e., that it have three equal angles at two straight lines. From this, in fact, that which comes first (*ex illo ergo priori*) and that which is assumed as principle, the conclusion is necessarily derived (= if the premise is true, the conclusion is also true: *modus ponendo ponens*, of hypothetical reasoning, *Author's note*).

However, from this the inverse does not follow, i.e., that if the conclusion is (true), then so is the principle (=fallacy of the consequent, *A.n.*). Given that sometimes from false premises a true conclusion can be inferred (=material implication of the logic of hypothetical reasoning, in its most «scandalous» aspect, *A.n.*). It is still true that if the conclusion is false, the premise must also be, because falsity can not be inferred except from falsity (= *modus tollendo tollens*, *A.n.*).

In those things that happen because of some cause (physical, natural and technical sciences, *A.n.*), whether according to the technique or according to nature, *the inverse mentioned above does follow*: given that, if the final state is or will be, it is necessary that that which is before the final state either is or has been. If, however, that which comes before the final state is not, then neither can the final state be: and this is how it works in demonstrations, if there is no conclusion, then there will be no principle.

In other words, it is evident that in that which happens as caused by something, *the final state has the same order as the principle in demonstrative procedures*. And this is so because the end is also a principle: not of action, however, but of reasoning. *From the end, we begin to reason about things that are in relation to the end* (= procedure of inductive constitution of the law, as a premise of the consequent demonstrative procedures) and in the demonstrative procedures we are not interested in actions, but only in reasoning. Therefore, it is convenient that the end of the things that happen in relation to a final state have the place of principle in the consequent demonstrative procedures. For this reason, the similarity (between natural processes and demonstrative procedures) is from both sides, albeit with an inversion of the relation between the two which is derived from the fact that the end is last in action, which is not the case in demonstrations (*In Phys.*, II, xv, 273).

As will become clear, debating about the logical *consistency** of the Aristotelian doctrine of the four causes (Cf. §6.3.2.3), the «final cause» has an *epistemic* value, a non-aprioristic foundation of logical necessity, not *ontic* – as if the final state of a process were capable of influencing its initial conditions – , as neo-Platonism absurdly asserts and as in the late Middle Ages and in the Renaissance the authentic thought of Aristotle and of the most classical and «enlightened» scholasticism were interpreted. At the same time, the text from Aquinas helps us to see how, in his interpretation of Aristotle's doctrine, there is a defense of a notion of «cause» and of «existence» as very different things from either Aristotle or from the modern thinkers, particularly Leibniz (Cf. § 6.3.2.2).

But late medieval
thought was barely
influenced by
Aquinas

On the other hand, E. Gilson has amply shown how little late Medieval thought was actually influenced by Aquinas, despite what was previously thought. Particularly, we now know how little it was influenced by Aquinas' attentive and intelligent reading of Aristotle's physics and logics. From Aquinas' lessons, the medieval thomists and most of the moderns took only the apologetic part, more directly connected to theological questions which, separated from their original logical and epistemological context, made Thomas justifiably unpopular with lay and scientific thought. Vice versa, Aquinas was a long way away from the deductive vision of that physics defended by the Aristotelians who fought against Galileo that, not only did he define metaphysics as a *transphysicum* (*In de Trin.*, II, v, 1c), but he also asserted, in almost all the passages where Thomas speaks of his ideal of an «order of studies», that in order to be truly understood, metaphysics should be studied only after a

long and careful reflection on the physical and mathematical sciences: a suggestion that very few of Aquinas' followers in every period have put into practice!

Utility of a post-modern re-examination of Aristotelian philosophy of nature beyond opposing ideologies

Let it be very clear: by stating this, we do not intend to negate the profound theoretical differences between the modern approach to physical-mathematical sciences and the Aristotelian-thomistic one. It is precisely this profound difference that today becomes useful for a post-modern physics and mathematics which, after Gödel in logics and after the revolutions in physics in the 20th century (especially those more recent ones dealing with quantum mechanics and the theory of complex dynamic systems), can benefit from the Aristotelian notions to go beyond the various foundational *impasse*. The conclusion of this volume will be to highlight the fact that an adequate non-apologetic reading, but rather a simply logical and epistemological reading of the «complex» model of Aristotelian-thomistic causality (doctrine of the four causes), can be of great help for the reconsideration of the principle of causality in contemporary physical-mathematical science, given the crisis that the physics of complex systems, stable far from equilibrium, has produced in the Humian-kantian model and later in the Hempelian model of causality. Aside from these arguments which remain open, one thing remains clear: the attitude of those who condemned Galileo in the name of Aristotle and Thomist scholasticism not only went against logical and epistemological truth, but also went against the spirit and the letter of the Aristotelian and Thomistic texts on this delicate point.

0.1.1.2 Absolutizing Science

Against the ideological position of the Scholastics, defense of the apodictic character of physical mathematical science

*The second error that led to an ideological instrumentalization of science follows immediately from the first. The end result of the erroneous ideological interpretation of the hypothetical value of physical mathematical theories which saw the mathematical laws in physics as pure inventions of the mind in order to «save the phenomenon», forced the naturalist scientist into not being able to accept that the laws of movement described real things or events: a sort of false alternative between *essentialism* and *phenomenism*, between *Platonism* and that which will later be called *Kantism*.*

Two versions:
1. Neo-platonic essentialism

- ◆ *Essentialism*. If we accept the idea that only knowledge of the natures or of the essences can be true, corresponding to what is real because it is categorical (apodictic and not hypothetical), then the physical-mathematician, in order to flee from the interpretation of the mathematical hypothesis as an invention to save phenomena, will have no other alternative than to become a Platonist and anti-Aristotelian.

Koyré's
interpretation

That is to say, he will have no other choice than to assert that physical mathematical science, far from being limited only to phenomena (defining the mathematical law at the base of their ordered sequence), is better equipped than natural philosophy (metaphysics) to reach essences, the most intimate and profound structure of physical being. Such essences would therefore be of a mathematical nature, just as pre-Aristotelian, Pitagorian and Platonic metaphysics had asserted. This is the reading that Koyré provides to explain modern scientific revolutions and even of Galilean thought (except for his retraction under the influence of the Inquisition), a sort of victory (or defense) of the Platonism (and also of the Pitagorianism) of the ancient Greek physical mathematics over anti-mathematical medieval Aristotelianism (cf. *infra*, § 1.1).

If you claim a superior state for mathematics, and what is more if you attribute to it a real value and a dominant position in physics, then you are a Platonist. If, instead you see in mathematics an abstract science that has less value than physics and metaphysics which deal with real beings, if you affirm that physics does not need any other base than experience and it should be constructed directly on perception, and that mathematics should be content with a secondary and subsidiary role, then you are an Aristotelian. In this debate, one places doubt not on the certainty of geometrical demonstrations, but on Being; and not even on the use of mathematics in physics – not even the Aristotelians would have denied the right to measure that which is measurable and count that which countable – but rather the structure of science and therefore the structure of being. (...) It is evident that for the disciples of Galileo, as for his contemporaries and predecessors, mathematics means Platonism. (...) The *Dialogue* and the *Discourses* thus tell us the story of the discovery or better the re-discovery of the language spoken by nature. They tell us how to question it, i.e., they contain the theory of that experimental research in which the formulation of postulates and the *deduction** of their consequences precedes and guides observation. This later, at least for Galileo is a «factual» proof. The new science is for him an experimental proof of Platonism (Koyré 1980, 160. 163. 167).

Here it is clear that metaphysics and science run the risk of not appearing as two complementary disciplines, but antithetical: modern physics would be a truly new «philosophy of nature», antithetical to the medieval one. This is one of the roots (platonic, essentialist one) of the modern scientific deviation, just as Popper in his teaching has denounced many times, albeit in ways that often do not do justice to Greek philosophy. One thing is Plato, and another thing is Platonism, just as one thing is Aquinas and another thing is Thomism, or Aristotle and Aristotelian-

ism, etc. More often than not, the head of a school of thought would not recognize himself in the thought of those who make him their point of reference. Such thinkers often take from the Master those aspects which are not profound and essential but rather contingent and tied to the moment and therefore run the risk of turning his thought into an ideology if they are not aware of this limitation.

2. Newtonian-
Kantian
Phenomenism

- ◆ *Phenomenism.* Another way of fleeing from the false rationalist interpretation of the hypothetical nature of the physical mathematical theories as inventions to save the phenomenon is that of accepting the phenomenism of the Geminian and Aristotelian solution, while refusing 'hypothecism'. The apodictic nature of mathematical demonstrations of modern science would not be based on the presumed capacity of the mind to intuit the mathematical essence of physical reality beyond the phenomena, but rather on the modern principle of *evidence*.

'Evidence' here means empirical evidence for experimental measurements, rational evidence for mathematical laws and, synthesizing the two – as Kant will later teach – for the three fundamental laws of dynamics. These laws were interpreted as necessary and sufficient conditions for the applicability of the infinitesimal calculus to mechanical phenomena, according to the brilliant Newtonian (and Leibnizian) intuition which solved the centuries old problem of the framing of the curves of any form, a problem which Archimedes and all of Greek physical mathematics were unable to solve. This 'integration based on evidence', this 'apodictic character of mathematical demonstrations with the empirical nature of experimentation' that is the profound meaning of Newton's *hypotheses non fingo* (Cf. infra, § 2.1).

The Phenomena of nature teach us that such principles (the three laws of dynamics) really exist, even if their cause has not yet been investigated. The *laws* that we speak about are thus *evident* and only their *causes* can be called *obscure*. The Aristotelians and the Scholastics, however, considered obscure qualities not the properties in some way known, but rather other ones which they thought were hidden in bodies and constituted the unknown cause of the visible aspects. But then both gravity and electrical and magnetic force would belong to this category only if we were to presuppose that they are derived from the intimate nature of things which remains unknown to us, i.e., from an unthinkable and unapproachable substratum. These «qualities» are undoubtedly an obstacle for scientific progress and therefore they are refuted by *modern investigation*. The belief in specific essences of things comprised of specific hidden forces and therefore capable of producing certain sensible effects, is completely vacuous and without meaning.

However, to be able to derive two or three general principles of movement from phenomena, and explain how from them (as clear and evident presuppositions) all properties and manifestations of *all material things* should follow, would represent important progress for scientific knowledge, even if the *causes* of those principles were to remain completely unknown to us (Newton 1704, 326. My italics).

As Kant, known as Newton's epistemologist, will later explain the root of the modern revolution is in this substitution of the transcendental and foundational criterion of truth. One can be a phenomenalist, granting the mathematical and experimental character of the «new Galilean science» its full value without being an hypothesis. As Descartes will intuit and Kant will develop, the principle of self-knowledge (evidence is essentially a state of mind) is the transcendental foundation of the apodictic nature and of the truth of modern thought, instead of the being and of the essence of classical thought.

Anti-metaphysical
character of Kantian
phenomenalism

We are thus confronted with the other root of the anti-metaphysical modern scientism, i.e., Newtonian-kantian *phenomenalism*. Here, the polemics against the theoretical value of metaphysics is not decided in the choice between Aristotelian essentialism and a Platonic one; rather what is denied is the capacity of any type of rational scientific knowledge, whether it be of phantasm essences or natures of bodied beyond evidence, or of that which is mathematically and experimentally provable: a certainty based no longer on the investigation of «causes» which connect beings but of «laws» which connect phenomena, obtain by way of rigorous experimental measurements. In this way, «scientific» becomes synonymous with «mathematical» on the one hand, and with «experimental» on the other hand, such that natural metaphysics is thrown out of the modern encyclopedia of the sciences. Essences, reduced to mere «thinkable» objects, noumenical, pure «ideas» of the pure reason, from Kant on are destined to become – according to the method of evidence – self-evident postulates of reason in its «practical» use, postulates to ground ethics or «critical metaphysics», in the Kantian sense of the term. Phenomenalism becomes the other root of the 'scientistic' ideology of modern science, both anti-metaphysical and anti-realistic.

0.1.2 Post-modern Rationality

0.1.2.1 Irrational 'Involution'

Critique of the
principle of
evidence

In the rest of modern science, even though the phenomenalist direction contributed to the setting aside of the essentialist program, the evolution of mathematics and logics in these past two centuries has attempted to

place the very bases of phenomenalism and of its apodictic nature: *the principle of evidence*, in crisis.

Axiomatization
of mathematics and
the end of the myth
of self-evidence:
hypotetico-
deductive method
as scientific method

The axiomatization of geometry and arithmetic in the 19th century, the subsequent crisis of the foundations of mathematics and the axiomatization of the very theories about the foundations of mathematics (set theory, theory of classes and of relations), in order to avoid antinomies decreed the end of the modern dream of the apodictic nature of Cartesian evidence, i.e., they decreed the definitive recognition of the *apodictic nature* of scientific knowledge, beginning with its logical mathematic basis (cf. *infra* § 1.5). The framework was completed in 1931 with Gödel's discovery of the necessarily *incomplete* and *non-selfreferential** character of all formal systems, decreeing in this way that which I have defined «the scientific death of scientism» (cf. *infra* §3.1). The ideological dream of a *unique, immutable, complete* science (capable of demonstrating the truth or falsity of every theorem, rigorously formulated from within itself), and above all of a *self-consistent* science (capable of demonstrating its own consistency and truth in an autonomous way, independently of other form of language and of knowledge) was gone forever.

Impossibility of
absolute
demonstrations of
truth and
coherence: the
irrationalist risk

Although all of this favored the rigor of the scientific method which, unlike other forms of knowledge and of language, has been able to define its own limits, weakening the ideological infection that risked ruining its effectiveness, due to the 'law of the pendulum' which often occurs in history, today we are in risk of going to the other extreme. The fact that absolute demonstrations of truth and coherence can not exist, together with the awareness of the hypothetical nature of any scientific language, has produced a false consequence in contemporary man: no truth or consistency can therefore exist, however partial and perfectible, in demonstrations of any form of scientific language.

Ideological
interpretation in a
nihilistic sense of
the hypothetic-
deductive
epistemology and
return to anti-
galilean positions

Paradoxically, in the epistemology of some contemporary science, at least in a popular sense (due to a certain dominant culture with forms of anti-scientific irrationalism), we are now accepting the retrograde positions of the ancient and medieval Aristotelian naturalist philosophers who fought against Aristarchus and Galileo, even though those same contemporary epistemologists are the first to tear their vestments against the Inquisition.

These irrational positions are accepting to interpret the mathematical hypotheses of science as pure *inventions of the mind* without any relationship to a reality that they want to represent by way of various models which, thanks to those

hypotheses, can be constructed; yet, with a profound difference with respect to those pre-modern philosophers. While previous philosophers considered hypotheses of the physical mathematical sciences as mental inventions in opposition to the certainty and absolute truth of metaphysics, the modern thinker has nothing to oppose these supposed 'inventions'.

In an extreme synthesis and at the risk of making the wrong point: scientism has made the modern thinker lose faith in metaphysics in the name of that science. The contemporary logical-epistemological revolution has made the modern thinker also lose faith in science. The irrationalistic and nihilistic inclination appears to be the only reasonable alternative left.

It is a pity to have to quote an impeccable physicist, the Nobel Prize winner Richard Feynman, as an example of this attitude. A pity for those recognize the thought of this author and therefore understand the profound irony found therein which does not depict his profound convictions on the subject or the genius of his research. However, aside from his intention to popularize the base concepts of quantum electrodynamics, the quote speaks for itself:

Physicists have learned to live with this problem: they have understood that the essential point is not whether or not they like a theory, but whether it can provide predictions that correspond to experiments. The philosophical richness, smoothness and reasonableness of a theory do not matter. From the point of view of common sense, quantum electrodynamics describes an absurd nature, while perfectly agreeing with experimental data. I therefore hope that you will come to accept nature for what it is: absurd (Feynman 1989, 25).

Here, mathematical laws of nature become reduced to absurd hypotheses, purely mental inventions to save the phenomena, experimental measurements. This certainly seems like a serious 'involution' of modern epistemology, a return to the anti-Galilean philosophers' positions of the 16th and 17th centuries. Yet Feynman himself in another popular text offers a different interpretation and one that is scientifically constructive regarding this paradoxical situation and that of the formalism of «path integrals» which he developed (cf. *infra*, § 2.7.2).

I am always perplexed by the fact that, according to the laws as we understand them today, one would need a calculating machine and an infinite number of logical operations to calculate what happens in a spatio-temporal region, no matter how small. How can all that happen in such a small space? Why does one need an infinite quantity of logic to calculate what will happen in tiny piece of space-time? (Feynman 1964, 64)

Need for a profound revision of the mathematical and logical foundations of modern science

Here it is: a reference to the need of a true conceptual revolution, of a *finite* and *constructive** nature in logics and mathematics in order to solve the enormous problems regarding calculations which today afflict not only particle physics but all fields of applied science, both natural and man-made (one can think here of economics), forcing technology on an ultimately desperate path of the explosion of calculating times, by way of ever more sophisticated and costly *hardware* which increase the divide between developed countries and those which are developing.

The problem is not with the *hardware*, but with the *software*: a logical, theoretical problem. We need an authentic logical and epistemological revolution in the modern way of doing science, and above all, of doing mathematics. This is what Feynman courageously asserted forty years ago. This is the only real intellectual challenge facing us today, capable of giving a cultural identity to the period we are living and that until now we are forced to simply define «post-modern». Certain logical and theoretical frameworks of the past no longer work; we need others, but where to look for them?

0.1.2.2 Open Logical System

The end of the scientific myth and the need of a new foundation for knowledge

In his last book, *Return to Reason* (Toulmin 2001), Stephen Toulmin focuses on the inheritance left to us by the demise of the ‘scientific myth’. In the first paragraph of his Introduction, he explicitly asserts:

The intellectuals of the year 2000 – philosophers or social scientists, literary critics or economists – have inherited a series of problems about the idea of Rationality and its relations with the idea of necessity and certainty. However, they tend to ignore the most practical and complementary idea of Reasonableness, i.e., the possibility of living (as in pre-modern times) without absolute necessity or certainty.

The dissolution of the triple ‘scientistic’ myth of a *Universal Method*, *Perfect Language* and *Unique System of Nature* (Toulmin 2001, 67) has given rise to the epistemological awareness that:

No formalistic system can interpret itself; no system can validate itself; no theory can exemplify itself; no representation can represent itself; no language can predetermine its own meanings (Toulmin 2001, 81).

The need of going beyond formalism in logics and the notion of an ‘open’ logical system

Our approach differs from that of Toulmin inasmuch as we address the same problems from a logical point of view, not only by referring to our preceding work (Basti & Perrone 1996; Basti 1997), but corroborating it with what one of the leading Italian logical mathematicians, Carlo Cellucci, has affirmed (Cellucci, 1998). That which Toulmin asserted con-

cerning logical necessity, consequence of the discovery of the theorems of limitation for formal systems, above all that of Gödel, that post-modern science must be open to other languages and above all to a confrontation with that which is real, can be rigorously asserted with the need that science must work with *logically open systems* (Cf. *Infra*, § 3.1.5). In fact, one of the most noteworthy results of Cellucci's work is the demonstration that:

The non-determinism required by Gödel's results is that of *being able to introduce in each step new axioms in a non-algorithmic* (Cellucci 1998, 326.).

It is therefore clear why logical systems can not be formal systems: no formal or axiomatic system admits being able to introduce *new axioms* into itself. Vice versa, introducing «new axioms» during the course of scientific work is precisely that which characterizes «open» logical systems.

0.1.2.3 Methods: Analytic vs. Axiomatic

Rediscovery of the classical analytical method as a methodology for the invention of hypotheses vs. irrationalism

It is clear that if we want to overcome the limits of irrationalism and make logics once again useful for research, the introduction of new axioms *can not be arbitrary*, as it was for neo-positivists and for Popperian and post-Popperian falsificationists (Cf. *Infra* § 4.2.2). In other words, both Cellucci and we agree that post-modern epistemology needs to recover the pre-modern⁵ *analytical method* of Plato and Aristotle in order to construct new hypotheses, new axioms regarding different objects or parts of the same object to which a theory is applied. The rationalistic myth in metaphysics and the scientific myth in science are derived from the false emphasis on the *axiomatic method* as the only method in logics. However,

Inferences are not only useful for finding logical consequences from given axioms, *but also for finding hypotheses which are the best for solving a problem*. This is why finding a new demonstration produces new information: finding it means finding new hypotheses which are not implicitly contained in the conclusion and therefore establish new connections between the problem and the external world (Cellucci 1998, 332. My italics).

⁵ In discussing the analytical method, we will not take it in the sense of Pappo (4th century A.D.), taken by Descartes and becoming Newton's *analysis infinitorum* which we all know about. This sense of analysis is understood within the axiomatic method: divide a complex problem into simple parts, i.e., a complex enunciation into simple component ones. The analytical method here referred to by Cellucci and ourselves in the platonic and Aristotelian one as a methodology for the discovery and/or construction of hypotheses.

Characteristics of
logically open
systems

The demonstrations in these logically open systems (*analytical-deductive*) will be characterized by being highly *evolvable* – progressive adjustments can be made –, *plastic* – hypotheses are found along the way –, and *modular* – where each module has a specific role in the solution of a part of the problem, even if that role can change in relation to the other parts of the logical system and the environment in which the demonstration evolves.

0.1.2.4 Induction: constitutive vs. enumerative

Constitutive
induction (analytical
method)
vs. enumerative
induction
(hypothetical-
deductive method)

Toulmin's reference to pre-modern logics and life-styles, in which scientific myths about the absoluteness of logical-mathematical knowledge did not exist, is precisely located, thanks to Cellucci's analysis and that which we will expose further on. It deals with recuperating in a post-modern perspective that analytical method advanced by Plato and Aristotle and by the pre-modern Aristotelian scholastic tradition, more devoted to truth and rationality than to one's own convictions, and in particular the method of Thomas. Once again, it is thanks to Cellucci that Aristotelian syllogistic, far from being purely deductive based on an axiomatic method, as the rationalist and modern ideologies would have us believe, was also inductive. It included analytical procedures for the constitution of new premises, both for inductive syllogisms as well as for the fundamental procedure of the *inventio mediū*, i.e., the search for the middle term, essential for any deductive syllogistic (Cf. *infra* § 4.4.1) in the so-called *demonstrative syllogism*. In other words, a demonstrative syllogism, either categorical or hypothetical, is really «scientific» to the degree it is based on adequate (true) premises about the object. However, the constitution of these premises requires analytical procedures, *not demonstrative*, of the constitution of categorical and/or hypothetical premises, through the inductive syllogism, or more generally by way of the search of an adequate middle term.

Inductive syllogism
vs. demonstrative
syllogism

If, going against the dominating attitude of modern scholasticism, the centrality of the *inductive syllogism* and more generally the *analytical method* of the *inventio mediū* in the syllogistic procedure can be shown, then it is obvious the syllogism is much more than a mere *proof method* of propositions constituted as true in some other way. The goal of the syllogistic method is, in fact, for Aristotle to define a method

Which will tell us how we will always find syllogisms to resolve any problem and by what way we will be able to assume appropriate premisses for each problem (Aristotle, *An. Pr.*, I,27,43a 20-22)

This places the syllogism within the *method of discovery* and not only in that of proof, giving it a precise role in the *hypothetical-deductive* epistemology of modern science, as a set of rules for the constitution of true propositions, through the redefinition of generic hypothetical axioms (i.e., mathematical laws) over singular contexts, in order to construct *true models* of formal systems for well defined contexts of application. This is how Aristotle defined the relationships between deductive and inductive syllogism:

On the one hand, the syllogism which is constituted by the medium [the deductive syllogism, *NoA*] comes first by nature and is more evident. On the other hand, the syllogism which is developed through induction is for us the richest in terms of knowledge (*Post. An.*, II, 23, 68b, 35 ff.).

Thus, in the second part of this volume there will be an in depth look at the analytical method and particularly at the *constitutive inductive* procedure, as the procedure of the search for middle terms and more generally as the procedure for the constitution of premises for demonstrative syllogisms, either categorical and/or hypothetical. In this search, we will correct, in part, Cellucci's own interpretation concerning the use of the analytical method in Aristotelian syllogistic. In fact, according to Cellucci – who is well versed in the logical aspects of the problem – the analytical method of the *inventio medii* in Aristotle would be subordinated to the axiomatic one. According to this interpretation, the search procedure of the *medium* would anyway finish in self-evident, axiomatic propositions, thus including the analytical method in the axiomatic one, within a logic of «closed systems».

Syllogistic Inventio medii analytic method and non-tautological character of analytical propositions obtained with such inductive procedures

Such an interpretation, although respectable, does not deal with the question as well as medieval scholastic logic. In the chapters of the second part, therefore, we will go into depth into what we have termed constitutive induction, examining both the logical (metalogical) foundations as well as the ontological (metaphysical) ones. In fact it is true that goal of the procedure of the *inventio medii* is an analytic proposition, characterized by the convertibility of subject-predicate, as in the self-evident tautologies, so dear to Leibniz and modern logical-mathematical formalism. The difference, however, is that *not all analytical propositions are tautologies* and therefore *self-evident propositions*, as is the case in the procedures of the axiomatic method and as Cellucci and large part of the modern interpretation (and also of scholastic philosophy) seem to suppose concerning the *inventio medii* of Aristotle. Those non-tautological, analytical propositions are rather «one-of-one universals» of inductive origin, even (in the case of the natural sciences) of empirical origin. We are dealing with *connotations of singular terms*, whether they are singular individuals

(connotations of individual beings, as in the type «Giuseppe Garibaldi is the hero of two worlds») or singular species (connotations of species, like «reptiles are cold blooded animals»). Hence, we will quote some texts of Thomas Aquinas, commenting Aristotle's *Analytics* which will be very significant.

A text by Thomas
about the necessity
of induction

Here is a text about the inductive origin of all universals which places the Aristotelian-thomistic position miles away from the moderns view of evidence. Let us recall that only the meta-logical, first principles (i.e., the principle of non-contradiction) are evident and thus properly *axioms* for Aristotle and Thomas. The principles of the syllogistic demonstrations within other sciences are rather of inductive origin.

There are two ways of acquiring science: one through demonstration and the other by way of induction, as was stated in the beginning of this book. The two modes are different, because demonstration proceeds from universals, while induction from particulars. If, therefore, the universals from which demonstrations proceed could be known without induction, it would follow that man could acquire science directly from them, about which one cannot have sensate experience. *But it is impossible to speculate about universals without induction* (Thomas Aquinas, *In Post. An.*, I, xxx, 252.).

Analytical
propositions, goal of
the *inventio medi*,
as "one-of-one
universals"

Here is another text which shows that thanks to the inductive procedure of the *inventio medi* in Aristotelian syllogistic we are within an environment of «open logical systems» and not that of axiomatic systems. Given that in the minor premise of the syllogism an appropriate middle term is not induced or assumed which would be contained within the universal of the major premise, the conclusion is not implicit from merely the major premise. Thus, the syllogism increases knowledge, and leads to conclusions which are not implicit in the original axioms, which would happen if the analytical propositions in question were in fact tautologies:

In order to infer a specific conclusion, two propositions are needed, i.e., the *major* premise and the *minor* premise, given that, knowing the major premise does not imply knowing the conclusion. The *major* premise should be known first, not only in nature but also in time. Therefore, if in the *minor* premise something is *induced* and/or *assumed* [note here the difference between induction and assumption of a specific particular within a generic universal, *No.4*] which is contained under the universal which is the major premise, but concerning which it is not evident that it is contained under this universal, one still does not have knowledge of the *conclusion* because the truth of the minor premise will still not be certain [in other words: from the proposition «all men are mortal» I will not be able to deduce «Socrates is mortal» until in

the minor premise I have determined that «Socrates is a man». But to affirm this directly from the major premise, I will have had to enumerate the *entire* class of men, *NoA*]. If rather in the minor premise a term is assumed about which it has become evident that it is contained under the universal of the *major* proposition, then immediately the *truth* of the *minor* proposition becomes evident [the problem is resolved, *NoA*]. In fact, that which is placed under the universal is already known through which one has the knowledge also of the conclusion. [...] Exemplifying those things which are known also *temporally* before the conclusion [that is to say the major premises of each syllogism, *NoA*], Aristotle affirms: let us assume that someone, through knowledge acquired from a conclusion from a preceding demonstration, knows before hand the following proposition, i.e., that every triangle has three angles equal to the sum of two right angles. Now, *inducing* this further assumption, and that this object [singular, *NoA*] which is in a semi-circle is a triangle, at the same time he will know the conclusion, *given that this induced object has its own universality within which to be contained, in such a way that there is no need to search for a further middle term* [which would ground the belonging to the universal, *NoA*]. [...] It is in fact the term of a resolving procedure [i.e., *analytical* procedure, *NoA*], from the moment that the mediating propositions are always reduced to immediate ones. This can also be read as follows: the last *extreme* [i.e., the subject of the minor premise of a demonstrative syllogism, *NoA*] that is taken under the universal medium [the predicate of the minor premise, *NoA*] does not imply that it be known as being under that universal through some other medium. *And Aristotle immediately states which are those things that imply the immediate knowledge of their universal: only singular predicates* [«one-of-one universals», or «singular connotations», *NoA*] *which cannot be attributed to some other subject, because among singulars and their species that cannot be any intermediate term* (*In Post. An.*, I, ii, 21. My italics).

Problem:
metaphysical and
meta-logical (not
psychological) of
constitutive
induction

This quoted text is important because of another message it contains, i.e., the secret of the profound difference between *enumerative induction* (a proof procedure typical of the logic of propositions and which can only corroborate in terms of probability the truth of propositions already given axiomatically, as Hume and Carnap have taught us moderns (Cf. *Infra* 4.2.2.3)), and *constitutive induction* about which we are dealing here and for which (in modern times) an exclusively psychological justification is usually given – tying it to the theory of abstraction of the Aristotelian intellect –, completely ignoring its logical foundation. When it is dealt with in modern logic, it is spoken about in terms of a mysterious «psychological induction» tied to the even more mysterious capacity of the human intellect to penetrate (“Intuit” as *intus-legere*) objects, beyond the empirical data, to capture the logical universal in them. However in-

teresting that may be for philosophical anthropology, it is not helpful for scientific logic and epistemology, at least without falling into psychologism in logic.

We will attempt to fill this gap by treating the difference between induction as a *method of constitution of premises* either categorical or hypothetical (Aristotelian Thomistic logic) and that of induction as a *method of proof* (corroboration) in the contemporary hypothetical deductive logic.

Enumerative
induction in the
proof procedures of
hypotheses has
only collaborative
value

That which we will call *enumerative induction* is a procedure which belongs to the logic of propositions and to the hypothetical-deductive method of contemporary science. Its goal is to determine the degree of empirical corroboration which can be given about a certain premise, in function of the number of positive experimental controls of a specific empirical consequence, materially implied by the premises. A true consequence in fact can be validly derived even from a false premise and therefore the empirical controls will actually increase the degree of confidence in the truth of the premises, never verifying them in a rigorous sense. The enumerative induction belongs, therefore, to the *logic of proof* or of *justification*, specifically of the justification of *truth*, or more exactly of the *verisimilitude*, of axiomatic propositions already constituted, not to the *logic of creation*, of the *logical constitution*, methodical, of new axiomatic propositions with limited contexts.

Constitutive
induction as
procedure of a logic
of predicates and
not of a logic of
propositions (theory
of demonstration)

Constitutive logic (cf. *infra* 4.4.3), on the other hand – a term coined by us to distinguish it from enumerative logic and which translates the Greek term *ε/παγωγή* or the Latin, *inductio*, from the Aristotelian and Thomistic texts, respectively – is a procedure of a logic of predicates and not of propositions, and more exactly of syllogistic logic, through which one can constitute premises adequate to specific contexts of research – therefore *relatively* and *absolutely true* like axioms or first principles of pure metaphysics – of successive syllogistic demonstrations: demonstrations geared to connect in a necessary way subject and predicate in the proposition which functions as the conclusion of the demonstration, by way of connections between subjects and predicates that are supposedly already established in the premises (major and minor) of the demonstration.

Thomistic
adaequatio and
inductive
assumption of the
particular in the
universal,
adequating the very
universal

In treating the *inventio medii*, induction appears essentially tied to the procedure whereby the logical universal of the major premise (in the fundamental form of the syllogism *in Barbara*, the universal denoted by the predicate of the major premise, e.g., «to be mortal» in «all men are mortal») is given a specific, not implicit, content *assuming* within it that which is contained in the logical universal of the minor premise (the predicate of the minor premise, e.g., «to be man» in «the Greeks are men»), so that

in the conclusion all of this can be truthfully affirmed in prepositional form; that is to say, it can be affirmed that the particular content of the logical universal of the minor (the subject of the minor, e.g., «the Greeks» in «the Greeks are men») is contained also in the logical universal of the major (the predicate of the major «to be mortal» in order to conclude, e.g., «the Greeks are mortal»). With constitutive induction we are in the environment not of the logic of justification of the truth of the propositions already given axiomatically as premises of demonstrative procedures, but rather of the logic of the *constitution* of the very *premises*. We are fully within the analytical method, inasmuch as a procedure of discovery – even better, of *constitution*, seeing that here there is no universal already given platonically to be disclosed – of adequate hypotheses (i.e., limitedly and not absolutely true) for the solution to specific problems.

Analytical
immediacy and
one-of-one
universals

In the final analysis, therefore, the subject-predicate connection of the conclusion is a connection *mediated* by the subject-predicate connection of the minor premise. Thus, it is necessary that one arrive by way of the procedure of the *inventio mediū* to immediate subject-predicate *connections* (analytical propositions, characterized by subject-predicate convertibility, but not tautological, therefore capable of connoting their object in the form of definitions of essence or *quidditates*, certainly not complete or immutable) which would justify an entire syllogistic chain of demonstrations and therefore of mediated connections. We will return to the ontological justification, of a causal type, of the different degrees of *logical necessity* of certain properties? (predicates) belonging to specific subjects. Right now, however, we will concentrate on two other texts that deal with the *proprium* of the logic process which underpins the *inventio mediū*. Here is what characterizes the immediacy of the reciprocal subject-predicate belongingness in the «one-of-one» universals, final term of the resolving process of the *inventio mediū*.

It is necessary to know that here ‘universal’ is not understood in the sense of what is predicated of many subjects, but according to some *adaptation* or *adequation of the predicate to the subject*, with respect to which neither the predicate is said without the subject, nor the subject without the predicate (*In Post. Anal.*, I, xi, 91).

Difference between
deductive Kantian
assumption and
Thomistic inductive
assumption

Furthermore, this mechanism of progressive and reciprocal adaptation of a predicate to the singular elements of its *domain** is also at the base of the other inductive procedure characteristic of the syllogistic method and recalled by Thomas is a preceding text: the so-called «assumption» of the particular within the universal. This procedure is recalled by Kant in *Pure Reason* in «transcendental analytics», where he speaks about the «assumption» of the phenomenal particular within the *a priori* category

«empty» of content. The chief difference between the two procedures of assumption in Kant and in Aristotle-Thomas is that while Kant, slave to the axiomatic method and of the «closed» character of formal logical systems, is forced to leave the *a priori* unchanged and therefore to invent the «temporal schematism» to connect the the *a priori* with the *a posteriori* (without solving the problem as the critics of Kant have pointed out (Scaravelli 1990), this is not the case for Aristotle and Thomas.

Aristotelian-
Thomistic logical
system as an open
logical system

All of this confirms that the Aristotelian Thomistic logical system is an «open» one: what is more, to speak of an «open» logical system is the best way to translate the Aristotelian-Thomistic theory of truth as *adequatio* for a modern mind set. Furthermore, was not this the point of the «Copernican Revolution» of Kantian epistemology, i.e., that it was reality which should conform itself to the *a priori* of the mind? Evidently, therefore, the nucleus of the epistemology preceding the «revolution» was that which today we translate with the notion of «open» logical system. It is the *a priori* of the mind which should conform itself progressively to the *a posteriori*, to that which is real, known by way of empirical datum. This is the sense of the doctrine of truth as *adequatio intellectus ad rem* (the intellect conforming to the thing). Here is a text of Thomas, again from his commentary of Aristotle's *Analytics*, which shows how the principle of *adequatio* works: it seems that it is an iterative process, based on a concept of *recursiveness* which sounds absolutely «heretical» for the modern mind set. Here, the recursiveness is not limited to, for example, a translation of the value of a specific iterative *function** to step *n* depending on the value of the same to step *n* – 1:

$$x_n = f(x_{n-1})$$

In our case, it is the same *a priori* which comes iteratively, modified in function of the *a posteriori* which should be assumed and vice-versa, until reaching a certain stability. In terms of the logic of relations, here they are part of the same «field» or «space» about which the values of the variables, both of the domain and of the co-domain, are progressively defined to depend iteratively on the single value which should be defined above and vice-versa, in function (obviously) of the satisfaction of some finalistic criterion of local stability (e.g., progressive convergence in the computations). Hence, the text from Thomas' commentary from the *Second Analytics* where the series of single values, iteratively defined, is expressed in terms of classical syllogistic logic:

It seems that here Aristotle says that the *connotation** (*defined*) of the property (e.g., «to be man», *NoA*) is the medium in the demonstration. Yet one must add that *the connotation of the property in the medium can not be completed except by way of the connotation of the subject*. It is evident that

the premises which contain the connotation of the subject are premises that concern properties. Thus, the demonstration is not resolved in its first foundation (*first cause*), except by taking the connotation of the subject as its medium. In this way, it is necessary to conclude concerning the property which connotes the subject (e.g., «to be Socrates», *NoA*)⁶ by way of the medium of the connotation of the property that one wants to predicate of it (e.g., «to be man», *NoA*), and *iteratively* (*et ulterius*) it is necessary to conclude concerning the connotation of the property predicated relatively to the subject (e.g., «to be man» relatively to Socrates, *NoA*) by way of the connotation of the subject (e.g., «to be Socrates», *NoA*) (*In Post. An.*, II, i, 415).

0.1.3 Ontology of Open Systems

0.1.3.1 Real Distinction between being and existence

Metaphysical
foundation of
constitutive
induction vs. the
formalistic notion of
being as pure
belonging (being as
Kant's copula)

At this point, it is crucial to understand the *different ontology* that underpins this way of posing the logical problem (cf. §4.5). If we are within a modern ontology that begins with Kant and runs through Frege and Quine which identifies *existence* as *belonging to a class*, i.e., reduces «being» in a Kantian way to the copula between subject and predicate – to affirm that «*there exists* some playful Welshman» is reduced to «some Welshman is playful» – it is clear this principle of the reciprocal subject-predicate determination makes no sense. How can this be achieved without falling into a vicious circle by wanting to redefine the field of the domain of a predicate over an element which should pertain to it, if the entire being of the element consists in its very belonging? For the modern logician and mathematician, he will say, «Tell me over which field, space, set, class, collection... you are defined and I will tell you not only *who* you are – which is perfectly plausible – but I will tell you *what* you are, *that you exist*. All of this is inconceivable for the logic of open systems in an Aristotelian Thomistic point of view. The being of the quality (or the being of the essence, *entity*, the *quod quid est*, «the that which is»), designated by the predicate of the proposition, should be really distinct from the being of the existence («that which exists»), designated by the subject of the proposition with respect to which the property should be predicated.

⁶ It is clear that the property of «Socrates-ness» cannot be used as such: this would be the *heavitas* of Duns Scotus, that which individuates the single unit as such with respect to all possible contexts. Yet, there is no need for this: the same effect can be obtained by using, for each context, one of the properties that characterize Socrates, not absolutely, but sufficiently for a specific environment, e.g., in the context of humanity, his own individual «Greek-ness» by which «Socrates is Greek» is effectively the individual connotation «Socrates is Greek in-the-way-of-Socrates».

The subject has both a connotation (*definitio*), and its own *being* which does *not depend* on the property, yet its *own being* is known before hand with respect to the being of the property in it. Therefore, concerning the subject it is necessary to know not only the *quod quid est*, «the what is» [as with the property, *NaA*], but also the *an sit*, «that it exists». This is so essentially because the demonstration of the *medium* depends on the different connotation of the subject and of the property. (*In Post. An.*, I, ii, 15).

Subject-predicate conformity and difference between being of existence (common being) and being of essence (entità)

This mutual subject-predicate re-definition is at the base of that Aristotelian distinction, a true «cross» for experts in Aristotle, between *quod quid est* (το τι ἐστίν, «the thing that is» of a being, the specific predicate attributed to a being, e.g., «the whiteness of the horse» or «the whiteness of sugar») and *quod quid erat esse*, το τι ἦν εἶναι, «the thing that was the being» of a thing, the generic predicate before being re-defined over some specification of the being in question – and therefore, the other way around, the generic predicate as a term of an abstract operation of the mind, thanks to which one prescind from the specific notes – e.g., that through which the following can be defined *generically* and *univocally*: «white», «the horse», «sugar», «the ship», etc.

Causal explanation of the being of the essence as an Aristotelian “third way” between platonic realism and sophist nominalism in logic

Speaking about the being of the essence or *entity*, while etymologically one is reminded of the platonic οὐσία (*ousia*), it does has a different meaning in Aristotle and Thomas with respect to Plato, thanks to which one can be a realist in logic without being a Platonist, without believing in the existence of «a world of ideas» beyond the physical world. Ontologically speaking, the Aristotelian natures or essences of physical beings, the ultimate real foundation of the logical beings that as such exist only in the logical operations enacted about and through them, are not immaterial «substances» which exist «in themselves and by themselves» beyond the physical world. There are only the result, the effect, *the act* – in the sense of terms intrinsic in the being of a causal act (ἐντελέχεια, *entelecheia*) – of the set of causes necessary to «bring into existence» and to «bring into existence in such a way», with those properties, with that nature, a given being, physical, in a determined spatial-temporal placing of the universe. Such a notion is very similar to what we understand today in physics concerning biological systems with the concept of «species», inasmuch as intrinsically tied to a «ecological niche», necessary for the existence of that species. (Cf. *infra* § 5.4.4).

Being of existence and being of essence and Aristotelian distinction between primary and secondary substance

The Aristotelian essences or natures are a result of the effects of real relations (causes) needing a substratum which thus becomes determined. They are not, on the other hand, the result of something «in itself» of the substances, of subsistent beings of an ideal type. This is the meaning of the famous Aristotelian distinction, respectively, between *secondary substance* (essence which does not exist in itself, but only in the individuals which belong to that species of beings) and *primary substance* (subsistent individual, substratum or metaphysical subject (*subiectum*) of the qualities, existent in itself and by itself. Cf. *infra* § 5.4.1). «Primary substance» which is connoted by Aristotle with the famous expression τὸδε τι, a sort of a definition of the approximate meaning «this certain thing», which constitutes another «cross» for his interpreters and translators.

Absolutization of the axiomatic method in logic and impossibility of a foundation of logical truth and necessity

Aside from exegetical problems, the true theoretical problem of Aristotelian metaphysics, as a keen reader will have realized thus far, deals precisely with the metaphysical consistence of the notion of τὸδε τι, primary substance. What type of substratum are we dealing with? Where is its metaphysical consistence? These are questions that are not answered in Aristotle, which is why ancient natural philosophers as well as modern ones, looking to Aristotle as an authority, have advanced a monistic-materialistic interpretation of his metaphysics.

The lack of clarity regarding this point has led many followers of Aristotle to identify the primary substantiality of physical beings with the common material substratum of all physical entities, eventually making these entities (as in Spinoza) simple *modes* on one sole substance (cf. *infra*, §3.2.2.2). Such a doctrine is in direct contradiction of the rest of Aristotelian naturalistic metaphysics where: a) prime matter was anything but able to subsist without the 'form' (cf. *infra*, § 5.4.3); b) primary substance was anything but the only one in the physical world, but rather diversified according the species and the individuals. Every individual capable of subsisting for a more or less extensive period of time and of belonging to a determinate species of beings would be «primary substance» for Aristotle; e.g., primary substances would be a single cat, but so would single molecules of a stable composite, as would a single atom of some given chemical element.

Thomas' alternative: provide a causal foundation for the entire being of an entity, essence and existence

Thomas' alternative proposal to monistic materialism, which in our opinion is much more consistent with the rest of Aristotelian metaphysics, is to *provide a causal justification* not only of the being of the essence of some entity (as Aristotle had already done with respect to Plato), but also of *the being of the existence* of the same. In sum, not only is the form 'act', the result of agent causality with respect to the potentiality of mat-

ter in the constitution of the essence of physical beings, but also the being of the entity is 'act', the result of a particular form of causality (i.e., participation of the *to be*, distinct from the platonic participation) with respect to the potentiality of the being of essence. With respect to the act of being, the essence is potency inasmuch as the result of the action of causes (beings) which in turn, in order to exist and therefore to cause, must participate in being, ultimately through a non-caused being. It is clear that this participation is logically «perpendicular» to the chain of caused-causing beings (the «primary cause» is not the ultimate causing agent at the end of the series: that would be an inconsistent notion), inasmuch as it is that which gives metaphysical consistency to the entire causal chain and to each ring (contingent being). It is therefore a type of causality which is *independent of time and of becoming*, because logically and ontologically it contains and grounds them.

Notion of being as
act and real
difference between
being and essence

Although we are giving a partial illustration of the point, limited to the metaphysical constitution of physical beings, herein we find the heart of Thomasian metaphysics of *being as act* (*esse ut actus*): a doctrine based on the *real distinction* between *the being of the essence* and *the being of existence* of physical beings, inasmuch as the distinction is relative to two completely different genre of causality: physical causality («horizontal») and metaphysical causality («vertical») in the ontological constitution of the each physical entity.

We will go into more depth in § 5.5 concerning the novelty of Thomasian metaphysics of being as act with respect to Aristotelian and Platonic metaphysics, from which it is derived and with which it is often wrongly confused (with either the one or the other) by modern interpretations, incapable of capturing the theoretical novelty of Thomas' proposal. In § 5.6, by way of conclusion of the chapter dedicated to the reconstruction of the doctrine of being (metaphysics) in pre-modern, classical thought, we will discuss what is left of the Kantian inspired «Copernican revolution». If, in fact, the absolutization of the axiomatic method, typical of modern logic and epistemology, has wiped the bottom of the barrel giving us the best it could (the axiomatic *formalization** of analysis and calculation), today (both now and in the future) logics, mathematics, and especially the theory of effective computability must work with logical «open» systems: systems which therefore recover, in a post-modern way, various notions and methods forgotten by modern logics, epistemology and even ontology.

0.1.3.2 Truth and Causality

Absolutization of
axiomatic methods
in logic and
impossibility of a
foundation of logical
truth and necessity

Another fundamental limitation of the absolutization of the axiomatic method in modern logic and science is the *systematic impossibility* of that approach to account for cognitive *realism* and thus for the *truth* and *necessity* of scientific knowledge, and more generally, rational knowledge. It took one of the most renowned logicians and philosophers of the XX century, W.V.O. Quine, representative of logical neo-positivism within which a large part of the philosophy of contemporary science was developed, to point out the limits of logics based exclusively on the axiomatic method. The «inscrutability of reference» along with the impossibility of justifying (after Gödel) the idea of «necessity» in the logic of formal systems constitute two of the major contributions of the American logician to the logical and epistemological discussions of the twentieth century (cf. *infra*, § 4.3).

Root of this
limitation: the
impossibility of a
foundation of
reference once the
logical symbols are
constituted

Precisely because so clear, it becomes immediately evident where the limitation appears that renders the problem of reference of the assertions in logic (*semantics**) unresolvable and the same is true for the problem of realism in epistemological knowledge. Such a limitation consists of the fact that, once the logical symbol and/or linguistic assertion is constituted, it becomes systematically impossible to justify its eventual extra-linguistic reference. At most what can be done, as Quine correctly teaches, is to define the network of the eventual intra-linguistic equivalencies with other symbols and/or other assertions in different languages, without ever systematically being able to move «beyond the mirror» of the linguistic representations.

Whoever should thus conclude that the semantic problem of reference is *untreatable* in logic, and that the epistemological problem of *realism* would become a mere question of faith and/or personal conviction (as many illustrious philosophers of science enjoy doing in Italy) would certainly be seen as a «slave» of the modern «tyranny» of the axiomatic method. In other words, such a thinker reduces logic to the logics of demonstration, ignoring the construction of linguistic symbols; and logical method to axiomatic method, ignoring the analytical method of the constitution and re-constitution of symbols (sentences) in conformity to their object (cf. *infra*, § 5.5.3 and § 6.2). Naturally, we would be dealing with truths necessarily partial, limited and perfectible, yet at the same time *universally verifiable* through appropriate repeatable procedures.

Logical universality
and necessity vs.
their presumed
absoluteness

In terms of this difficulty, it will be appropriate to eliminate a common error. Many philosophers, even some Thomists, think that stop the irrational waywardness of contemporary cultural nihilism and relativism, it is necessary to defend the possibility that human knowledge not only be able to reach but also to *formulate knowledge in an absolute way*. In fact, when one makes them reflect, they are the first to recognize that absolute truth can be only one and as such ineffable, and all the truths knowable by man «participate in this one truth» (and are thus, by definition, *partial and perfectible*) and therefore it becomes obvious that what they were defending was not the *absoluteness of finite truths* but rather the *universality* of that truth, i.e., the possibility that «always and everywhere», whoever, applying the same method and in the same conditions, will arrive at the same results. «Universality» is not wed to «absoluteness» but to «relativity».

The modern position which, after the discovery of the theorems of limitation in the demonstrative procedures of formal systems, denies that there can exist «absolute» theories (against rationalism) because the validity of the very demonstrations is strictly tied to the limits of the various «logics» employed, leads to relativism only if one denies that there may exist an analytical method, i.e., that there can exist a set of logical procedures to prove in a repeatable way the partial truth of the premises of an argument with limited contexts.

In order to calm the spirits, it is worthwhile to quote Thomas Aquinas and right after comment in depth. Here Aquinas astutely (and within his «open» logical and metaphysical system) tries to account for the position of Anselm of Aosta – a thinker that many modern rationalists (from Descartes to Hegel), correctly or not, consider a primary source of inspiration:

The truth that is in the divine intellect is one only, from which many truths can be derived in the human intellect, like from one face in the mirror, many images can be derived. [...] The truths that are in things are multiple, according to their beings. [...] From this it follows that the truth about which Anselm speaks is that sole one because it is the measure of all true things [...]. However, the truth which is in the human intellect or which is in things is not in relation to things as an extrinsic measurement and common to all objects measured by it [like divine truth, *No. 4*], but is in relation either in the sense of that measured with respect to its measurement – as is the case of truth which is in the human intellect which must be varied according to the variety of things (*oportet eam variari secundum varietatem rerum*) – or in the sense of an intrinsic measurement, such as the truth which is in the things themselves. And it is also convenient that these measurements are many according to the multiplicity of that which is measured, in the same way as it is necessary that different bodies have different dimensional measurements [two bodies which would have the exact same

measurements in everything would be the same body, *NaA*] (Thomas Aquinas, *In de Ver.* I, 4c and ad 1um.).

Such is the case regarding logical *universality*. In terms of logical *necessity*, once again Quine indicates the only way left for the modern logician to reasonably affirm it. Quine states that after Gödel, the only way to justify *necessity* in the logic of formal systems is to accept in some way *Aristotelian essentialism*, i.e., that certain predicates are necessarily inherent in specific subjects, thus entering into field of modal logics, and therefore of *intentional logics* and not simply *extensional* ones (cf. footnote 83, Chapter Four).

Logical necessity
and Aristotelian
essentialism

As Quine is unwilling to end up an Aristotelian, he prefers to renounce necessity in logic and mathematics in the name of a simple tautological *analyticity*, a formalistic analyticity of modern thought in opposition to «inductive» analyticity of ancient thought.

Defending Aristotelian essentialism is not part of my goal. Such a philosophy is as much unreasonable for me as it was for Carnap and Lewis. Yet my conclusion, unlike Carnap and Lewis is the following: too bad for modal logic [...]. In fact, if we do not intend to quantify by way of the operator of necessity, it is not clear what advantage there is in using the operator instead of simply saying that the sentence is analytic (Quine, 1986, 145).

Causal foundation
of the essences in
Aristotle vs. platonic
essentialism

However, unlike Quine, we have highlighted various motivations that make it worth while to re-discover (in a post-modern way) some fundamental concepts of Aristotle's philosophy of nature. Above all, that which Quine does not even seem to suspect is that the Aristotelian approach to essences or natures of bodies is anything except an «essentialism», or an ideological defense of essences. Vice-versa, the heart of the Aristotelian theory of logical necessity is that its foundation is found in *ontological necessity*, both physical and metaphysical; i.e., the heart of Aristotelianism is the foundation of logical relations on *real or causal relations* among the various beings, the theory of a *causal explanation* of essences. If, therefore, some predicates appear more necessarily than others tied to the subjects of specific propositions, it is because the properties to which they refer are inherent in the respective beings – to which the subjects of those propositions later refer – inasmuch as they depend on the stability of the same causal relations thanks to which those beings are able to subsist throughout time. They are thus essential properties (*per se* as opposed to *per accidens* properties) due to the existence of respective beings. Everywhere those properties cease to be inherent in the respective beings because the causal relations cease, the beings themselves would cease to exist as such, becoming something else of a different nature.

Close link between
logical truth,
ontological causality
and foundation of
constitutive
induction

Logical truth and *ontological causality* thus appear intrinsically bound together in Aristotle's *meta-logic** and *metaphysics*, as any scholar of Aristotelian philosophy knows. Chapter Six is dedicated to the meta-logical and metaphysical analysis of the notions of *truth* in logics (cf. *infra* § 6.1) and of *causality* in ontology of physical beings, in the Aristotelian-Thomistic philosophy of nature and of epistemology. In synthesis, just as the meta-logic of constitutive induction is able to avoid the modern «problem of Hume» concerning the justification of induction (because we are no longer dealing with enumerative induction of the logic of propositions), so the metaphysics of the principle of cause is able to avoid Hume's criticism of the very principle itself: a criticism which was based on the inconsistency of enumerative induction, as a procedure of logical and ontological justification of the universality and necessity of causal laws in physics and mathematics.

Analogy between
open logical
systems and open
thermodynamic
systems, stable
beyond states of
equilibrium: both
augment
information within
themselves

Cellucci has already highlighted the strict relationship that exists between the theory of «open» logical systems and the theory of «open» thermodynamic systems, *stable beyond states of equilibrium*, or *complex systems* in physics, chemistry and biology. For these systems, the principles of statistic mechanics and linear thermodynamics – according to the hypothesis of molecular chaos of thermodynamics by Boltzmann – end up insufficient, forcing a radical revision upon the foundations of modern physics and mathematics (cf. § 2.3.4), much more profound than the same revision brought on by quantum mechanics: two revisions that in the near future may end up more profoundly bound together than was previously thought. The motivation behind the relationship between the two types of «open» systems in logics and in physics is quickly seen, because it was substantially anticipated by Aristotle himself in his criticism of geometrical determinism introduced by Democritus. This was why Aristotle hypothesized the existence of a *formal-final cause*, typical of his «complex theory» of physical causality, as well as the two «initial causes» (efficient and material) of Democritus' mechanics. Aristotle stated that, if in all physical processes, the final state were univocally predictable from the initial causes as Democritus affirmed, demonstrations in physics would be analogous to those in geometry, where all the theorems are univocally deducible from the principles. However, as Aristotle affirms, in the vast majority of physical processes, this is not the case. Hence, there is the need to hypothesize a more articulate causal component (cf. *infra* § 6.3.2).

Vice-versa, in closed thermodynamic systems, as in axiomatic logical systems, constant of information

The same holds for us today: just as in the axiomatic systems of logic and geometry all of the information is supposed to be in the premises, so in the stable mechanical systems of classical mechanics and statistical mechanics all of the information of the final state of the process is contained in the initial conditions. There is thus a profound theoretical motivation for the prevalence of the axiomatic method in the logic and epistemology of modern science, beginning with Newton's systematization of mechanics and dynamics: a systematization that, as all historians of science would agree to, sought its paradigm in geometry and in the Cartesian principle of evidence.

On the other hand, we are not the first nor the only ones to affirm that classical Aristotelian philosophy of nature, along with the modern type found in Whitehead (cf. *infra* § 3.2.2.2) seems to be an ontology much more convenient than modern mechanistic determinism, as that inspired by Spinoza or derived from Newton-Laplace in order to metaphysically characterize *complex systems* and more generally the physics of *stable systems beyond the state of equilibrium* (cf. *infra* § 2.7).

Analysis of the consistency of the Aristotelian doctrine of four causes and the theory of stable systems beyond equilibrium

That which is developed in an original way in this book is an analysis of Aristotelian ontology of physical causality, in continuity with the physical phenomenology of complex systems, in terms of self-organizing systems, capable of *generating information* not present in the initial conditions (cf. *infra* § 6.3.2.2). Following recent analysis by S. Galvan (Galvan, 2000), we are able to offer an initial analysis of logical consistency for this theory from a theoretical point of view, although not completely formalized (cf. *infra*, § 6.3.2.3). The crucial point of the logical consistency of the Aristotelian model of causality is that in Aristotle – in opposition to the neo-platonic interpretation of final causality (upon which the position of the *pseudo*-Aristotelians of the Renaissance depended in their struggle against Galileo, cf. *infra*, footnote 40, Chapter Two), «efficient causality» and «final causality» are different. In other words, in the neo-platonic and Plotinian model of final causality, that which determines the *necessity* (or high probability of statistical mechanics) of *existence* of a given being/event (the efficient component of the entire causality) is contradictorily identified with that which determines the *high improbability* of the same being/event (the finalistic component of causality). Vice-versa, as we will see, both in the Aristotelian model of physical causality and in the theory of complex systems (particularly of chaotic ones), these two components remain distinct. This confirms that the

model of Aristotelian causality can provide an appropriate ontology for the physical phenomenology of complex systems.

Correct
interpretation of
Aristotelian final
causality

It is in the causal interpretation of *intentional behavior* that the finalistic component acquires for Aristotle (and Aquinas) an explanatory value, yet only because the two components are connected by an *epistemic* factor (the awareness of the positive value of a certain desired being/event), even though in this case efficient causality would be *entirely* from the conscious, acting, subject.

Such an explanatory value of final causality is thus consistently operative in the theological interpretation of *metaphysical causality*, of *participation in being as act* in Thomas Aquinas, i.e., in the interpretation that identifies the Absolute which participates being in metaphysics with the *personal God* of Biblical revelation, thus transforming the famous five *proofs* of the existence of a First Cause of being into so many «ways» open to the believer for the knowledge of God – *et hoc dicimus deum* (*S.Th.*, I, 2, 3c), «and this we call God», as the «ways» conclude.

0.1.4 What metaphysics for post-modernity?

Post-modern
usefulness of a non-
apologetic re-
examination of
Aristotelian and
Thomistic
philosophy of nature

The final sections of Chapter Six are dedicated to the Thomasian doctrine of metaphysical causality, an analysis which we will conduct through the newly born discipline of *formal ontology* (cf. *infra*, footnote 76, Chapter Three) which has recently raised much interest in many fields of contemporary culture from philosophy to logic, computer science and cognitive science precisely because, utilizing *intensional logic* and symbolic formalisms therein developed, it allows for a rigorous approach to these queries, free from ideological presuppositions. It is not by chance, therefore, that in formal ontology scholastic metaphysics and particularly that of Aquinas constitute the most keenly studied and analyzed of objects⁷. In this way, our analysis of natural Thomistic metaphysics is not geared to an apologetic defense of the same, as if the answer to the question, «What type of metaphysics is suitable for post-modernity?» would be «the metaphysics of Aquinas». What we want to show is that, by illustrating the foundational journey of Thomas' metaphysics in the notion of being as act, both with regards to the problem of truth in meta-logics (cf. § 6.2), as well as with regards to the problem of causality

⁷ To appreciate this fact, it is sufficient to examine one of the most up-to-date and complete web sites dealing with this new discipline of contemporary analytical philosophy: www.formalontology.it.

in the ontology of physical being (cf. § 6.3), this is a model of a metaphysical theory that achieves a continuity with classical logic and ontology (especially Aristotelian). The usefulness of such a model for post-modernity and the renewed interest in ontological queries has already been highlighted.

Usefulness of
Thomistic
metaphysics to
resolve
misunderstandings
between cosmology
and theology

We will attempt to illustrate that the Thomasian model of the causality of being (doctrine of the participation of being as act) can be useful to resolve numerous misunderstandings that the recent debate about cosmological hypotheses concerning the origin of the universe has created with the theology of creation. Thomasian metaphysics of participation of being is, in fact, the only one capable of rendering reciprocally consistent metaphysical, foundational hypotheses of quantum-relativistic cosmologies about the eternity of the universe with the dogmatic definitions of Catholic theology concerning an absolute beginning «outside of time» of the same universe.

Thomasian metaphysics of being as act is perfectly consistent with both the hypothesis of the absolute beginning of the universe outside of time (as for theology) and with the hypothesis of its eternity (as for cosmology), just as Thomas asserted, precisely because time (and/or the 'becoming' which time measures) is «within» the universe – or eventually the series and/or set of many universes; i.e., time and becoming are notions which belong to the foundational, *meta*-physical, journey of the early thinkers of physical sciences, to which an authentic natural metaphysics should turn for its proper object of study. In this sense, the affirmation of the beginning or non-beginning of the universe is essentially an undecidable one (*decidability**) from within cosmological and metaphysical reflections. This is why one should always and only speak of *hypotheses* concerning this subject matter. Nor can any help come from the *meta*-physical or foundational notion of «first cause», inasmuch as it is often confused (in a modernist context) with the «absolute initial cause» of Cartesian metaphysical mechanics – the Cartesian «God» of the «initial shove» to the inertial system of the universe. Being can be participated through both a finite causal chain as well as an infinite one, in Thomasian metaphysics, as long as the later is not conceived as «closed» in itself ontologically; i.e., as long as it is very clear that the foundation of the being of the universe depends (ultimately *ab aeterno* and anyway independently of the course of cosmic time) on a causal principle «outside» the universe and time itself. Therefore, not only physics but neither metaphysics will be able to present sufficient argu-

⁸ These hypotheses are not (nor can they ever be) part of Galilean science, as has recently been recalled. What's more, they can not be bound by experimental control, neither in a collaborative sense nor in an inductive sense.

mentation to resolve the question concerning the eternity or non-eternity of the physical universe.

Theological character of the absolute beginning of the world vs. metaphysical character of the foundation of its existence

In this sense, the theological affirmation of creation as an absolute beginning of the universe outside of time, is not only consistent with Thomasian metaphysics of participation of being (and with its opposite), but also with cosmological models that hypothetically presume the eternity of the universe or set of universes. The reason for this paradoxical compatibility is stated by Thomas himself, shortly after the famous dogmatic pronouncement of the Lateran Council IV (1215) concerning the absolute beginning of the universe outside time as a truth to which believers in communion with the Catholic Church must assent.

"Open" character of Thomas' metaphysical system

This compatibility is essentially derived from the logical notion of an «open system», valid for all physical and metaphysical theories that can be reasonably constructed. If, in fact, the question about the absolute beginning of the universe is undecidable both for physics and metaphysics, nothing prevents a «supernatural revelation» – an ulterior axiom, outside of any procedure (both physical and metaphysical) of natural reason – from being chosen in an assertive way, apodictic and not merely hypothetical, from among the various alternatives. What Thomas underscores after the dogmatic definition of the Council (to be clear for both believers and non-believers) is that this truth requires a reasonable assent (not in contradiction with other rationally demonstrated or demonstrable truths.

That the world has had a beginning is an object of faith, which is not demonstrable or knowable [by reason alone, *NaA*]. It is convenient to reflect on this point so that somebody, trying to demonstrate that which is an object of faith, does not support his argument with reasons which do not prove anything, thus provoking non-believers to conclude that we believe certain truths not out of faith, but because of such false reasons (*S.Th.*, I, 46, 2.).

Hawking's cosmological model and the Thomasian theory of First Cause

If modern Theologians would have tried to understand and apply this interpretation (the model) – that the metaphysical doctrine of Thomasian participation can provide dogmatic assertions regarding the theology of creation – we would not have suffered at the beginning of modernity the «Galilean Question», nor would we had the more modest «Hawking Question» at the end of it. At the end of this book, we will be able to provide an interpretation, consistent with these metaphysical and meta-logical principles, of Stephen Hawking's own statement through which he tries to separate his work and proposal from the inadequate interpretations of both friend and foe. In one of his latest books, Hawking

ing answers a question by Sue Lawley from the BBC, who asked if he was able «to do without God» through his research on the foundation of cosmology:

All that my work has demonstrated is that one should not say that the way in which the world has begun was God's personal whim. The question still remains: *why does the universe bother to exist. If you believe, you can say God is the answer to this question* (Hawking 1993, 204).

Metaphysical
foundation of the
cosmos and its
possible theological
interpretation

Leaving aside what Hawking means by «God's personal whim» (cf. § 6.3.3.1), we should underscore the second part of his answer. The «ultimate question» to which Hawking refers is precisely *the meta-physical question* par excellence, to which physical science (if it wants to remain consistent) *can not give* an answer, because it deals with the very meta-linguistic and ontological foundations of the science itself. Aquinas gives an answer to this question as a meta-physician, an answer which is perfectly compatible with a physical science that recognizes its character as a formally and semantically «open» logical system. And even Hawking's subsequent conclusion – in which he correctly distinguishes between a metaphysical question (and answer) and a theological question (and answer) of the believer – is perfectly compatible with Thomas affirms in the conclusion of his famous five ways. Each of the five ways ends with the metaphysical «demonstration» of the foundation with the already recalled expression *et hoc dicimus deum* (*S. Th.*, I, 2, 3c). One thing is the answer to the metaphysical question that even a non-believer such as Hawking can and should ask, and another thing is the theological and also apologetic use which a believer can make of such an answer, to the degree with which a divinity (in which he believes) can be placed in continuity with the absolute being, arrived at through metaphysics.

Distinction between
theoretical and
sapiential
dimension of
metaphysics

This distinction between the *theoretical* dimension (foundational for metaphysical reflection) and its *sapiential* (and eventual religious) dimension is and should be one of the characteristics of this post-modern metaphysics we are talking about. After the errors of theology's integralist rationalism and ideological scientism of the last millennium, it is imperative that the distinction of the environments and reciprocal limits become very clear, without ideological confusions and dangerous popular exemplifications. Furthermore, it is the possibility of dealing with the metaphysical question and of rationally searching for compatible answers which eliminates one last misunderstanding about which the irrationality of contemporary «weak thought» has created much confusion in contemporary culture, especially on a popular level: the misunderstanding that the impossibility of whatever rational form of absolute knowledge would also lead to the *impossibility of the rational affirmation of the*

absolute, with a confusion between method and object of rational knowledge.

Thomistic notion
and modern notion
of absolute

To affirm that there cannot be any form of all-encompassing rational knowledge of a specific object (much less of the whole of reality) does not mean that all forms of rational knowledge of the absolute are precluded, unless one decides to identify the absolute, *tout court*, with the totality of reality. There is a form of theoretical knowledge of the absolute, partial, always incomplete, which will always contain affirmations about which it is impossible to verify the truthfulness or falsity – think of the rational *undecidability** of the thesis regarding the absolute beginning of the universe just mentioned – affirmations that always need further proofs as regards their consistency, without taking them for granted once and for all; in a word, a form of rational knowledge (always perfectible) of the absolute is completely coherent with the limits of post-modern rationality, as it was for pre-modern rationality.

The absolute, object
of metaphysics,
cannot be identified
with the totality of
being

In any case, that which a post-modern metaphysics should renounce is the idea that the absolute as object of a consistent rational knowledge could be identified with the totality of being, with the Spinoza-type «substance» or with Hegel's «entirety of being». The logical antinomies have taught us that inconsistencies begin when the infinite totalities to which one refers have the cardinality of V , the universal set. The inconsistency of Emanuele Severino, for example, finds its roots precisely in the fact that for him the absolute corresponds to the entirety of being, i.e., the totality of beings with graph of their relations completely defined (Basti & Perrone 1996, pp. 156 ff.). If, in spite of this, one still wants to affirm that the absolute is the totality of beings, one must conclude that the foundation is the *absence of foundation*, due to the resulting antinomies of the notion: one must conclude that affirmation and negation, «yes» and «no», coincide and therefore that the answer to the metaphysical question is the negation of the possibility of asking the question, just as is affirmed in those metaphysical systems at the base of the great Eastern Religions, so popular today among those who study logical and mathematical sciences.

Metaphysics and
the pre-religious
presence of the
Absolute

Such a position could even be acceptable, albeit painful for a Westerner, if it were the only alternative. But such is not the case: in order to avoid the radical metaphysical antinomy, it is enough to place its foundation «outside» of the totality of that which is real, and instead of weakening the graph of the relations in V , refusing to specify the entity of the elements that compose it, accept the multiplicity of meanings of the notion of being, refusing to identify existence with belonging to a class, even if

that class were V . That is to say, it is enough to accept that, by way of the distinction between being and existence and the mechanism of their reciprocal, progressive determination, one would guarantee that although all of the beings actually existing in V are perfectly specified, they do not all exist actually: some may exist in V in a virtual way, although the borderline between these two forms of existence, with all of their intermediate difference of degrees, may be continually mobile in the two senses, thus maintaining the system globally stable. In order to make the system consistent, it will be enough to «open it», presuming that its foundation would be «outside» of the totality of V .

The goal is a foundation in which being and existence coincide without anything left over, thus being actually infinite (without the antinomies) because it is Absolutely Simple, pure Actuality, without potentiality or differences. In a word, the absolute can be the object of rational theory as long as one accepts the idea that it is not identified with the *universitas rerum* (the totality of beings) and as long as one admits that one can never have a complete knowledge of this absolute; as long as one does not pretend that his theory can construct a logically «closed» system. This is why (due to its rational necessity of *transcending* the universe of beings and of the relative partial truths) we can designate this Absolute with a capital “A”, prior to any faith option and any religious belief.

Impossibility of an
absolute
metaphysics and
the possibility of the
search for the
Absolute

As a believer and Christian philosopher, I would like to close these introductory reflections by quoting a recent Encyclical of Pope John Paul II, *Fides et Ratio*. This quote complements the other one previously cited in this introduction (§ 51), where it was affirmed that no historically produced philosophy will be able to presume to constitute absolute knowledge, «to encompass the totality of truth, nor to be the complete explanation of the human being, of the world and of the relationship between man and God».

At the same time, the search for the Absolute seems intrinsically tied to the life of man in an undividable unity and consequentiality in the reciprocal distinction between theoretical and existential research, between metaphysical and sapiential searching:

No one can avoid this questioning, neither the philosopher nor the ordinary person. The answer we give will determine whether or not we think it possible to attain universal and absolute truth; and this is a decisive moment of the search. Every truth – if it really is truth – presents itself as universal, even if it is not the whole truth. If something is true, then it must be true for all people and at all times. Beyond this universality, however, people seek an absolute which might give to all their searching a meaning and an answer – something ultimate, which might serve as the

ground of all things. In other words, they seek a final explanation, a supreme value, which refers to nothing beyond itself and which puts an end to all questioning. Hypotheses may fascinate, but they do not satisfy. Whether we admit it or not, there comes for everyone the moment when personal existence must be anchored to a truth recognized as final, a truth which confers a certitude no longer open to doubt.

The search for truth, of course, is not always so transparent nor does it always produce such results. The natural limitation of reason and the inconstancy of the heart often obscure and distort a person's search. Truth can also drown in a welter of other concerns. People can even run from the truth as soon as they glimpse it because they are afraid of its demands. Yet, for all that they may evade it, the truth still influences life. Life in fact can never be grounded upon doubt, uncertainty or deceit; such an existence would be threatened constantly by fear and anxiety. One may define the human being, therefore, as *the one who seeks the truth* (*Fides et Ratio*, §§ 27, 28).

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FIRST PART

Philosophy of Nature
and of Science:
an Historical Outline
of the Issues

1. From the Origins to the 19th Century

From the absence of the distinction between Philosophy of Nature and the Sciences of Nature in classical thought, to the birth of the modern sciences in the 16th and 17th Centuries, to the eclipse of the Philosophy of Nature from the 17th – 19th Centuries, to the crisis of the foundations of modern mathematics in the second half of the 19th Century

1.1 Philosophy of Nature and Modern Science: the Origins

I C O N S	
	Important
	Definition
	Summary
	Glossary
	Bibliography

It is not easy to write a manual of the *Philosophy of Nature* which should also be (like this one) a manual of the *Philosophy of Science*. The difficulty stems from a whole series of historical and theoretical motivations that would require many volumes in order to explain, aside from being necessarily very specialized and therefore not easy to read.

Having to synthesize, therefore, we can say that in classical and medieval times of our Western culture, *physica* or *philosophia naturalis* included in its object of study not only the theoretical object of study which today we define with the term *philosophy of nature*, but also the experimental object of study of that vast and everyday more articulate group of scientific disciplines that in modern times we define with the term *natural sciences*¹⁰.



The philosophy of nature began to be considered distinct from natural sciences only during modern times, coinciding with the birth of natural sciences (especially with Galilean and Newtonian physics) during the 16th and 17th Centuries. Such sciences were progressively characterized by their specific *object* (phenomenological) and *investigative method* (experimental) as well as *demonstration* (initially apodictic-deductive and later hypothetical-deductive) and a specific *formal language** (mathematical), completely different than the object, method and language of ancient metaphysics and natural philosophy. We will soon define (cf. § 4.1) the

¹⁰ As can be seen by the *key* of symbols next to the text, we will use different icons to highlight the various parts of the exposition in order to facilitate the reading and understanding of the same.

notion of philosophy of nature as part of metaphysics in order to understand the general sense of these statements.

The origin of physical and mathematical science in Ancient Greece

On the other hand, the hegemony of modern science and its mathematical method of investigation have led to the triumph of the philosophy of nature of *platonian* inspiration with respect to that of Aristotle of the late medieval period. We know that Pythagoras' (6th century B.C.) doctrine, which made mathematical entities the essence of physical reality, was used by Plato (4th century B.C.), developed by the exceptional axiomatic work of Euclid of Alexandria (3rd century B.C.) and systematically applied to the study of physical realities and their laws by Archimedes of Syracuse (3rd century B.C.)

Progressive separation between metaphysics, mathematics, and physics, and the birth of logic

Even in ancient Greece, however, this Pythagorean-Platonic framework underwent its first radical alterations. Initially, Aristotle's criticism (4th century) of the formal inconsistencies of Plato's system (the theory of numbers and of ideas) and of his doctrine of «formal participation» led to the autonomous development of mathematics of the Aristotelian metaphysical system. This misconstrued the mathematics of the system as having a foundational character for natural sciences, and at the same time, by granting mathematical entities only a mental, abstract reality, it bound the philosophical investigation of being – which Aristotle will later define as «metaphysics» – directly to the physical sciences and not to the mathematical ones, as it was in the Pythagorean-Platonic tradition. Thus, we see the first systematization of formal logic – and with this the *theory of demonstration* and of the *deductive method* – completed by Aristotle, based on the propositions of ordinary language and not on those (more rigorous) of mathematical language, which later Euclid will accomplish in his *Elements*. On the other hand, through Euclid, mathematics will be completely separated from metaphysics, although through a different road than that taken by Aristotle. Not only arithmetic (as it already was for Pythagoras) but even geometry will become with Euclid a purely formal and deductive science, unattached from any moorings with reality, and therefore no longer an ideal representation of it.

The first 'shock' of Greek mathematics and overcoming it through the principle of exhaustion

The use of the empirical method in mathematics and particularly in geometry returns with Archimedes, but as a *heuristic method* for the discovery of new theorems, about which a formal demonstration would be provided, usually through the argument of absurdity. And it is precisely in this way that the Pythagorean-Platonic program encountered its most serious obstacle towards a generalization of the geometrical representation of physical reality, empirically verifiable.



Eudoxus'
demonstration as
one of 'absurdity'

We know that the genius from Syracuse, among the greatest contributions that he bestowed on mathematics, was the calculus of the volume of a sphere, of the cylinder, the square of a parabolic segment, as well as the very value of π by way of the application of the *principle of exhaustion*, invented by another of Plato's ingenious disciples, Eudoxus of Cnidus (4th century), perhaps the greatest mathematician of ancient Greece. He helped Greek mathematics overcome its first great shock of history: *the discovery of incommensurables*, i.e., that there were not only natural and rational numbers (which sprung from relations between commensurable geometrical sizes), but also irrational numbers that sprung from relations between incommensurable sizes (e.g., $\sqrt{2}$ as the relationship between the side of a square and its diagonal).

Such was a discovery that placed the Pythagorean vision of numbers in crisis as a finite relationship and therefore all of mathematics as the science of harmony and absoluteness, perfectly static. The genius of Eudoxus was teaching the Greeks to not be afraid of infinity – and irrational numbers are characterized precisely by the infinite extension of their decimal «tail» – even though such numbers are not represented by those static, harmonious numbers of Pythagoras, but as sizes which vary and can approximate exact values, just as in the classical representation of the method of exhaustion (cf. Figure 1-1), multiplying to infinity the sides of a polygon, one can see that *they cannot help but to coincide* – thus the demonstration through 'absurdity' – with the circumference in which the polygon is inscribed. It was thus Archimedes (because we have almost no information about Eudoxus) who showed the fruitfulness of the principle discovered by his predecessor at Plato's Academy, as a method for calculating areas, surfaces and «incommensurable» relations like those accrued between a circumference and its diameter (the famous value of π).

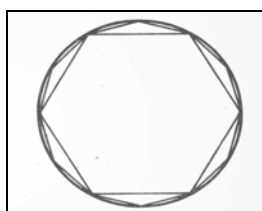


Figure 1-1. Intuitive illustration of the *principle of exhaustion*. Increasing the sides of the inscribed polygon to infinity, it cannot help but coincide with the circumference.

The second shock
(not overcome) of
Greek
mathematics: the
problem of the
square of a circle

However, just as Archimedes attempted to generalize this method of calculus, he came up against the problem of the *square of the circle*, i.e., the problem of calculating the background area of a curve of any form by way of associated numerical equations. In order to sidestep this calculus problem, Archimedes formulated his «mechanical method» of solving mathematical problems, a real predecessor of modern physics-mathematics. First, following Eudoxus' method of double negations, he demonstrated, *ad absurdum*, the non-contradictory character of the existence of a solution. Then he «calculated» the problem's (approximate) solution by means of a physical experiment, by measuring the related physical magnitudes and corresponding variations. In so doing, he discovered famous laws, such as the law of levers or that of bodies' buoyancy, etc.

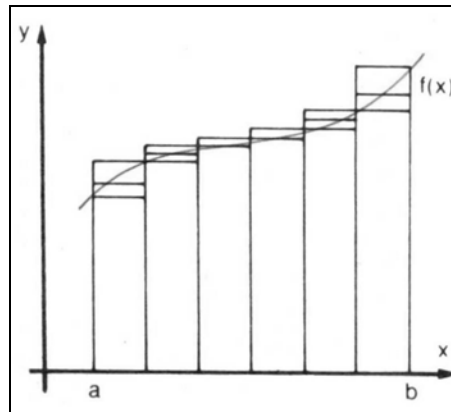


Figure 1-2. Modern representation of the problem of calculating the background area of a curve (=the «square of the circle») through the calculus of the definite integral of a given interval of that curve.

The solution of the
problem of the
square of the circle
coincides with the
birth of modern
science

We will then have to await the Modern era to be able to find the mathematical solution to the problem of the square of the circle or of the calculus of integral forms. In fact, from the point of view of the history of mathematics, one could say that if the beginning of the Modern era coincides with the birth of modern science, the birth of modern science coincides, in turn, with the solution to this calculus problem. Indeed the problem of the square of the circle was solved two thousand years later, during the 17th century, thanks to the formulation of infinitesimal calculus by Newton and Leibniz. In this way, calculus provided the Galilean «new science» of nature, born a century before, with the capacity to predict the temporal evolution of physical magnitudes that

Nature should not be observed, but rather interrogated through our mathematical hypotheses

can be represented geometrically, which was the basis of the power — hence of the success — of at least early modern science and technology.

Galileo had indeed used the ancient Greek idea of the mathematical and, above all, geometric nature of physical laws. However, he had integrated it with two new methodological principles:

- ◆ Nature should not be observed, but rather *interrogated* through our mathematical hypotheses, on the basis of which we design *experiments*.
- ◆ The experimental observation of nature does not develop through the five senses and ordinary experience, but rather through a rigorous experimental method, intended as the *measurement* of physical magnitudes (See *infra*, § 4.2.1).

Lack of generalization in Galileo's method, due to the lack of a universal calculation method, able to solve the problem of the square of the circle

Yet, Galileo's famous *law of falling bodies* that, in a revolutionary way, did not link one body's fall to its weight, and therefore to its nature — contrary to Aristotle's common sense and physics — but only linked it, *kinematically* (see *kinematics**), to the «geometric variable» of the body's relative height (hence *dynamically* (see *dynamics*), after Newton, to the influence of the force of gravity) was mathematically represented with the parabola equation, which had also been known to the Greeks. Without Descartes' invention of *analytic geometry* and without the discovery of *infinitesimal calculus*, which made it possible to associate algebraic equations to curves of any shape, Galilean science would have never become the modern science that we all know.

Conversely, thanks to calculus, which is at the origins of the success of modern science, Galileo's intuition could develop all its potential. Even if, at least in the early days of modernity, this entailed a non-necessary *absolutization* of the concept of number, linked to the use of the principle of evidence in *algebra** and in calculus, first of all with Descartes and Leibniz. This absolutization is indeed at the origins of modern 'scientism'.

In any case, the essentialist interpretation, which Galileo adopted to account for the cultural revolution of the «new» science (see above, p. 20), confirms that Koyré was correct in describing the fundamental turn of the birth of modern science as the supremacy of the Platonic system of thought over Aristotle's.



If you claim a superior status for mathematics, if you accredit it with a real value and a dominant position in physics, then you are a Platonist. If, conversely, you see in mathematics an abstract science, whose value is therefore lesser than that of the sciences that deal with real entities — physics and metaphysics; if, in particular,

you state that physics needs no basis other than experience and that it must be built directly on perceptions, and that mathematics must be content with a secondary and ancillary role, then you are an Aristotelian. In this debate, what is in question is Being, not the certainty of geometric demonstrations. Neither is the use of mathematics in physical science under discussion; not even Aristotelians would have denied the right to measure what can be measured and to count what can be counted; rather, the structure of science and, therefore, the structure of being are put into question. (...) Clearly, for Galileo's disciples, as for his contemporaries and predecessors, mathematics means Platonism. (...) The *Dialogue* and the *Discourses* thus tell us the story of the discovery—or, better, the re-discovery—of the language spoken by nature. They tell us how to interrogate it; that is, they contain the theory of that form of experimental research in which the formulation of postulates and the deduction of their consequences precede and guide observation. This is, at least for Galileo, an «actual» proof. The new science is, for him, the experimental proof of Platonism (Koyré 1980, 160.163.167).

The non-philosophical character of Galileo's «Platonism» and his refusal to use algebra in the calculations of the «new physics»

This statement of *de facto* Platonism in Galileo's approach, however, should be nuanced in light of the recent discoveries of the strictly experimental, and absolutely non-philosophical, nature of Galileo's 1604 formulation of the law of the falling bodies, advanced by historians such as Stillman Drake (Drake 1990), who worked directly on Galileo's notes. This formulation derived from Galileo's accurate and daily measurements, which convinced him of the common relationship existing between the distance covered by a body in motion and the square of the related time, detected in various experimental contexts. As Drake underlines, Galileo made these discoveries and measurements without ever writing an algebraic equation, even if he was clearly aware of *algebra** in the form of equations that was gaining ground during his times as a teacher of mathematics in Pisa. In his notes, Galileo simply used, in a rigorous manner, the notion of «number» as contained in Book V of Euclid's *Elements*, intended as the ratio and proportionality between continuous magnitudes; thus, he only worked with whole numbers and whole fractions of numbers. This made Galileo's calculations extremely complex, so that their translation into algebra by Descartes and Leibniz was a simplification that, as already mentioned, was at the roots of the diffusion and success of modern science. At the same time, Galileo's calculations were, at their time, absolutely accurate, given that the creation of calculus in the modern sense will have to wait for the 19th century and Dedekind's work on the concept of «real numbers» (see ch. 2).

The calculus limit of Galileo's physics at the roots of its post-modern value

The calculus limit of Galileo's physics-mathematics —judged in hindsight, from the point of view offered by post-modern mathematics after Gödel's theorems— becomes a *de facto* advantage from the theoretical point of view. In 1616, at the time of his first accusation, before the ecclesiastical courts, it allowed Galileo to accept Bellarmine's advice to attribute *hypothetical* value to his demonstrations, as well as to those of Copernican physics-mathematics in general —even if not in the sense of “mental fictions” as employed by his Aristotelian enemies. This hypothetical value did not undermine the *absolute* value of the statements of faith. Indeed, as Drake wisely notes,

Galileo, and Newton after him, took care to avoid possible pitfalls by reasoning from ratios and proportionality (defined by Euclid as «sameness of ratio») only, *relations without commitment to numbers as absolutes*. Leibniz, and Descartes before him, did not exercise the same caution, and thereby enhanced ease of calculation at the cost of mathematical rigor in procedure (at least until the axiomatization of calculus of the 19th century, *Editor's note*) (Drake 1990, 8).

1.2 Modern obfuscation of the philosophy of nature

Modern science and the recovery of Democritus' approach

The relationship between the philosophy of nature and the incipient natural sciences, as they had developed during the first three centuries of modernity, was later made difficult by the fact that the natural sciences —because of the unfortunate opposition they encountered by Aristotelian philosophers of nature during Galileo's trial (See § 0.2.4.) — portrayed themselves as the bearers of *absolute*, rather than *hypothetical*, certainties, which would replace ancient natural philosophy and even ancient Scholastic metaphysics (Koyrè 1961)¹¹. Hence, the so-called re-

¹¹ The title of the text quoted here is self-explanatory: «From the World of Approximation to the Universe of Precision». The fact that the formal rigour of contemporary scientific disciplines (both natural and mathematical) has reached levels of refinement that were unknown to classical physical and mathematical thought is undisputable. At the same time, the fact that this rigor is methodically insufficient to be extended, as such, to fields of thought such as classical logic and metaphysics —which, from the point of view of content, were much richer and, in ancient times, had reached a degree of formalization that was noteworthy— is just as undisputable. (Bochenski 1956). The late-modern (or «post-modern») thought in which we now live is then called to incorporate the contents of classical logical and metaphysical thought in the formal rigour typical of modern reflections, thus enlarging and deepening its formalism, until now limited to physical-mathematical sciences. This is the challenge on which depends the fate of humanistic and scientific thought —which can no longer be separated or, even worse, ideologically juxtaposed, as was the case in the modern age—, both in the present times and in the

covery of the Pythagorean-Platonic inspiration in the study of nature did not denote the corresponding recovery of the idealist and spiritualist content of Platonic metaphysics. To the contrary, it corresponded with a given vision of the world gaining ground, which made explicit reference to a mechanicism with a geometric basis of a Democritean kind, interpreted in anti-metaphysical and later entirely anti-philosophical terms.

Modern science
and the
advancement of
scientist ideology:
Enlightenment and
Positivism



Modern obfuscation
of the philosophy of
nature

This is the fundamental core of those *Illuministic* (18th century) and *Positivist* (19th century) «world visions» or «ideologies*», characterized by «scientism», that, together with the *Historicist* world vision, or *ideology**—that stood in opposition to and somehow followed them—led Martin Heidegger to define the Modern age as the «age of world visions» (Heidegger 1950). The modern crisis of the philosophy of nature has then coincided with the crisis of philosophy *tout court* facing the advancement of natural sciences, of their experimental methods and mathematical language. In modern culture and mindset, sciences have increasingly become—for peoples and individuals, for rulers and their subjects, first in the West and then in the East as well—the bearers of those certainties of knowledge, of that «well-founded knowledge», indeed called «scientific» (the *epistēmē* of the Greeks and the *scientia* of the Latins) that, in ancient times, were conveyed by metaphysics and theology.

As a general cultural phenomenon this has become so true that, in modern culture, the term *science* has become synonymous with *natural science* and *mathematical science* (or «logical-mathematical», starting from the end of last century). Of course, this has also been bolstered, in a sort of intellectual suicide, by the fact that philosophical and metaphysical reflection in general have progressively lost the demonstrative rigor they had in classical and medieval times, becoming fruitlessly caught in a discipline half way between aesthetics and rhetoric. This intellectual suicide has profoundly weakened modern culture and thought, by depriving it of a fundamental component. In academia, philosophy of nature and metaphysics with a naturalist basis of a Scholastic kind have thus survived, in modernity, almost exclusively as ancillary subjects in the *curricula* of Catholic theological universities, often ideologically, hence fruitlessly, juxtaposed to modern scientific subjects.

immediate future. This challenge has been fully taken up by the contemporary «analytic school» of metaphysics, particularly by formal ontology. The latter, however, should be considered as in profound continuity with the Scholastic tradition. For an up-to-date treatment of these issues, see Basti *et al.* (1999).

1.3 Hegel's philosophy of nature

The philosophy of nature in Hegel's system



In the «secular» cultural language of modern academia, the «philosophy of nature», as a subject distinct from natural sciences, temporarily regained ground during the first half of the last century, thanks to the 'meteorite' of Hegel's philosophy and «historicist» world vision.¹² Through the *historical-dialectical method* of enquiry, philosophy indeed tried to regain some role and a specific space in modern culture that, during the past century, had been affected by the advancement of a 'scientist' ideology, particularly the «positivist» one. Therefore, the crisis of Hegel's historicist idealism as an all-embracing metaphysical system, and its regression to a mere political philosophy—in the two opposed ideologies of the Hegelian «right» and «left»—coincided with another obfuscation of the philosophy of nature in modern culture. We will have to wait for the 1930s to witness a re-birth of the philosophy of nature on grounds different from those of the Hegelian system (See § 3.2.2.).

The demise of the Hegelian system and the birth of the human sciences



After the demise of Hegel's philosophy of nature, with its fruitless claim to represent an ideological alternative, or «world vision», opposing Galilean-Newtonian natural science and method, the same attempt to recreate philosophy as a «science of the spirit» through the historical-dialectical method was inevitably overshadowed by the power of the *operational**, experimental-mathematical method, typical of the Galilean model of natural science. In this way, Hegel's philosophy of history as a «science of the spirit» split into a series of empirical disciplines of research, the so-called *Geisteswissenschaften*, whose objects are Man, his history and activities. With time, then, *psychology*, *sociology*, *economics*, *historiography* and the related subjects were born. Compared to the Hegelian root from which they originated, these subjects lost the historical-dialectical method of enquiry, and also the labelling, shifting from the high-sounding «sciences of the spirit» (*Geisteswissenschaften*) to the much more modest and factual *sciences of Man*.

¹² We should recall that, in Hegel's *Encyclopedia* of philosophical sciences, the *philosophy of nature* represented the second stage (the «antithesis») of the historical-dialectical process of self-awareness of the absolute spirit. It represented the moment in which the «idea in itself», the object of *logic*, «alienated itself», «negated itself» in the «for itself» of material nature, the realm of obscure and blind necessity, to «recover itself» in the «synthesis» of the varied natural and historical forms of life of the *spirit* («subjective», «objective») up to the full self-awareness of the «absolute spirit». This moment of synthesis represents the realm of the «in itself and for itself» of the spirit's meta-individual liberty, which is expressed in the three subsequent historical forms of *art*, *religion* and, finally, *philosophy*, as the highest and definitive «science of the spirit». This science, of course, was at its apex in Hegel's philosophy.

The sciences of
Man and the
Galilean model of
science



Indeed, once the post-Hegelian German historicism of the 19th century had failed to endow *Geisteswissenschaften* with a rigorous method of enquiry and of *formalization** that would oppose the *operational* (or experimental-mathematical) *method** of natural sciences (*Naturewissenschaften*), the sciences of Man also started to be increasingly permeated by mathematical language and quantitative experimental tools of enquiry. Rather than fruitlessly and ideologically opposing the sciences of nature, therefore, the human sciences turned out to give a valuable and still present impetus to the revision of the foundations of modern logical and mathematical thought, which characterized the end of the 19th century and the first half of the 20th century. Indeed, the logical-mathematical formalisms developed for the study of *mechanics** during the modern age proved entirely inadequate for the rigorous treatment of *complex* phenomena, both in physical, chemical and biological sciences (See *infra*, § 2.6); even more so, they proved inadequate for the study of psychological, social and economic events, the very focus of those human sciences.

1.4 Origins of Illuministic scientism

1.4.1 Apex of the Illuministic program

Development and
systematization of
mathematical
analysis

The parabola of the Hegelian meteorite coincided with a deepening of the foundations of modern mathematics, which perhaps was, at the time, at its apex. This was the moment in which, thanks to the work of high-powered mathematicians such as Giuseppe Luigi Lagrange (1736-1813), Jean Le Rond d'Alembert (1717-1783), and Augustin Cauchy (1789-1857), the key of the success of modern science, calculus, and its rigorous application to mechanics and dynamics seemed to have reached a final systematization, through the deepening and development of the notion of *function** within a finally rigorous framework offered by a new subject, *mathematical analysis**. This development will culminate — in the second half of the 19th century, after the first rigorous definition of the concept of *real number** in the work of Richard Dedekind (1831-1916)— in the formal definition of the notion of *limit**, by German mathematician Karl Theodor Wilhelm Weierstrass (1815-1897), within an approach to mathematical analysis that was finally freed from any reference to geometric intuition —contrary to its beginnings, particularly with Newton— and aiming for its full arithmetization (arithmetic of real numbers).

From the
systematization of
analysis to the crisis
of the foundations

Ironically, this development of the queen of sciences in the «century of the Enlightenment», *mathematical analysis**—that, through its completion and rigorous formalization, was meant to signal the final victory of the «Illuministic enlightenment» and of its scientist program—contained in itself the seeds of the end of that same program. This demise was spurred by George Cantor's reflections on the foundations of analysis and the translation of the latter into arithmetic. Before delving into a brief account of the main turning points of the reflection on the foundations of mathematics and of the theoretical reasons that led to it (Cf. § 1.5.2), however, we will briefly examine the theoretical origins of the Illuministic 'scientistic' program.

1.4.2 The absolutization of evidence

The presumed
absolute self-
evidence of the
postulates of
Euclidean geometry
and of the laws of
Newtonian physics

The solution of the problem of the square of the circle in the algebraic form of the modern integral calculus, and the success of modern Galilean-Newtonian physics-mathematics that were associated with these extraordinary discoveries, strengthened the conviction that the postulates of Euclidean geometry, which were at the basis of the mathematical representations of that type of physics, were *true*, that is, adequate formal representations of physical space. Rather than an empirical intuition, the basis of this ascription of *truth* to these postulates would have been their absolute *self-evidence*,¹³ identical to the purely formal—self-evidence that was the basis of modern algebra and its analytical tautologies. All Illuministic scientism of the 18th-19th centuries was grounded on this implicit conviction. In other words, it was grounded on the claim that, because the physical-mathematical «new science» was grounded on the *absolute rigour* of deductions founded on notions that are true because they are evident—Euclid's axioms in geometry and the three laws of *dynamics** in physics (on the centrality of evidence as the criterion of truth in a physics built along the lines of geometry, see Newton's quote in the *Optics*, § 2.2)—, it could replace ancient metaphysics and philosophy of nature, downgraded (together with theology) to the status of pure superstition.

The absolute
mechanistic
determinism and
«Laplace's demon»

The two following anecdotes, better than any commentary, reveal the spirit of scientism that dominated parts of modern culture at the end of the 18th century. The first, which is the most famous, relates to the dialogue between the great physicist and mathematician Pierre Simon de Laplace (1749-1827) and Napoleon, concerning the latter's comment to

¹³In particular, this conviction is at the roots of Descartes' doctrine that identified the geometric notion of extension (or *res extensa*), with *physical matter*. In fact, Descartes made this presumed *self-evidence* the second «clear and distinct» (or self-evident) idea of his system.

Laplace's monumental work, the essay *Le Système du Monde* (1797). Faced with Napoleon's surprised observation that all reference to God was missing in this treatise of cosmology, Laplace spitefully replied that he «had not needed this hypothesis». This statement was linked to the principle of *absolute determinism* of a mechanistic kind in the sciences of nature, popularized as the «principle of Laplace's demon», whose most complete statement is in a famous excerpt of Laplace's other essay on the philosophy of nature: the *Essai philosophique sur les probabilités*:



An intelligence that, in any given moment, could know all the forces that animate nature, and the situation of the beings that make it up, and that, in addition, were great enough to be able to submit this data to *analysis*, could embrace, in the same formula, the movements of the biggest bodies in the universe and those of the lightest atom: nothing would be uncertain for it, the future, just like the past, would be in front of its eyes. *The human soul offers, in the perfection that it was able to give to astronomy, a weak sign of such an intelligence.*¹⁴

Stated another way, if it were possible to know, with a precision increaseable at will, perhaps infinitely—as it would be possible for a «demon»—the initial conditions of the movement (position and quantity of movement) of all particles that make up the physical universe, it would be possible to know (deduce) with the same, absolute precision, the evolution of the whole universe, solely by means of the laws of dynamics. Modern astronomy, in the systematization given by Laplace himself in his masterpiece *Mécanique Céleste* (1798-1825) on the basis of the Newtonian laws of gravity, offers therefore, according to the French scientist, an example of that paradigm of absolute science of nature to which this excerpt refers.

Ideological exaltation of the new physics as a model of absolute knowledge, replacing the philosophy of nature

The second anecdote—the only thing it can be—is reproduced in E. Cassirer's *History of the Modern Thought* (Cassirer 1978, v. II, t. 2, p. 444), and was taken from *The Principles of Mechanics*, published in London in 1773, authored by a little known popularizer of Newton's ideas, S. Emerson. While less famous than the first one, this anecdote is a clear example of the state of Dionysiac exaltation that was provoked, in some cultural milieus, by the myth of the absolute certainty of the new scientific conquests¹⁵. In addition, this excerpt well reflects the conviction

¹⁴ Quoted in Ruelle (1984, 16).

¹⁵ What is more, people started calling Newton «the new Moses»: just as Moses had given humanity the absolute certainties of the Ten Commandments in morality, so Newton had given the absolute certainties of the three Laws of Dynamics in physics. This was also implicit in the famous Kantian saying: «the starry sky above me [referring to the

that the «new» physical-mathematical Newtonian science was not another form of knowledge, complementary to metaphysics and the philosophy of nature, but rather a new and ultimate philosophy of nature, absolutely incomparable with those that had preceded it and insuperable by any other that would follow.

Newtonian philosophy, that is, the only true philosophy existing in the world, is equally founded on mechanics. (...) Some have ignorantly countered that Newtonian philosophy, as any other that preceded it, will grow old and will be overcome by some new system (...). Such an objection is utterly false. Indeed, no philosopher before Newton used his system. While philosophical systems are nothing but hypotheses, opinions, inventions, speculations, fantasies, that were invented at will with no backing in the nature of things, he built, by himself, a completely different basis. Indeed, he only admits what he obtains through accurate experiments and observations; anything that is then built on this basis is deduced according to rigorous mathematical reasoning (emphasis added).

Kantian philosophy as the epistemological justification of this conviction and the criticism of metaphysics

This same conviction that, through the analytical extension of Euclidean geometry (in mathematics) and with Newtonian mechanics (in physics), one had finally reached a «hard core» of *absolutely certain* knowledge, that had nothing in common with the uncertainties and obscure points of the old metaphysics and philosophy of nature, is also the departure point for Immanuel Kant's work (1724-1804). Indeed, this conviction constitutes the foundation of the three *Critiques*, in particular the first: the *Critique of Pure Reason*. This work intended to discover which conditions make mathematical and physical sciences «pure» (non-applied), endowed with those characteristics of absolute necessity and universality, considered the characteristics of authentic science, that any metaphysics introduced in the history of thought instead lacked. In the Introduction to this work, Kant asks:

*How is a pure mathematics possible? How is a pure physics possible? Of these sciences, given that they really exist, one would wonder how they are possible, because that they are possible is proved by their actual existence [N.B.: Some could still doubt that pure physics exists. But one need only look at the different propositions that one encounters at the beginning of physics proper (empirical), such as those of the permanence of the same quantity of matter, of inertia, of the equality of action and reaction, and so on, to convince himself that these constitute a *physicam puram* (or *rationalem*), that well deserves to be described separately, as a special science, in all its extension, be that large or small]. As for metaphys-*

Newtonian law of universal gravitation, Editor's note], the moral law within me», pointing out the two spheres of absolute certainty.

is, its progress has been, so far, quite unhappy, because of none of the metaphysics formulated to date, concerning its fundamental aim, one can say that it really exists, which must lead each of us to doubt that it is possible (Kant 1787, 55).

The progressive defeat of the scientific myth during the 19th-20th centuries

What happens in the immediately following years in the sanctuary of scientific certainty (Euclidean geometry) and what happens throughout the 19th century and the first thirty years of the 20th century to the mathematical and physical sciences, will provoke a conceptual earthquake of such proportions that it destroys the presumed unwavering edifice of the absolute nature of these certainties: an earthquake whose aftershocks have not ended and to which we will have to grow accustomed at the beginning of this third millennium.

1.5 The 19th century crisis of the foundations of mathematics

1.5.1 Axiomatization of mathematics and truth

1.5.1.1 Birth of non-Euclidean geometries

Euclid's fifth postulate and its various formulations

The presumed self-evidence, hence the presumed «absolute truth» of Euclid's five postulates was an idea that, in the past, had raised doubts for both philosophers and mathematicians. Among the first, Aristotle, first and foremost, rejected this idea given that, according to him, only *axioms* — that is, the primary principles of metalogic and metaphysics, such as the *principle of non-contradiction* (p.n.c.) — are self-evident. Among mathematicians, the fifth postulate is the one that provoked more reservations. According to this postulate, «If a straight line falls on two straight lines in such a manner that the interior angles on the same side are together less than two right angles, then the straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles». Stated otherwise, two parallel straight lines never meet, not even if drawn *ad infinitum*.¹⁶ An equivalent formulation of the same axiom, that will be useful later in this account, is the following: «only *one and one only parallel straight line* can be drawn through a point external to a straight line».

¹⁶Indeed, the sum of the angles adjacent the intersection of any straight line cutting two parallel lines, in each of the two directions, always equals two right angles, whatever direction they may take.

Difficulties concerning the evidence of the fifth postulate and need to demonstrate its truth

The idea that ancient thinkers had found difficult to grasp was the fact that many so-called «asymptotic» lines, lines that do not meet in finite space, meet instead in infinite space (for example, a hyperbola is asymptotic at the axes: see figure 1-3).¹⁷ Given that the fifth postulate did not seem «self-evident» at all, many mathematicians, both ancient and modern, felt the need to demonstrate its truth, by deducing it from the other four postulates.¹⁸

Failure of the attempts to demonstrate the fifth postulate

In the modern age, a number of high-standing mathematicians devoted themselves to the problem of demonstrating the fifth postulate, all of whom failed in their attempts. Among them, one should recall Italian Jesuit Girolamo Saccheri (1667-1733), Swiss German Joan Heinrich Lambert (1728-1777) and French Adrien Marie Legendre (1752-1833). These failed attempts strengthened the conviction, among scientists of the time, that the fifth postulate should be considered true because self-evident, despite its complexity. This conclusively confirmed the Cartesian-Kantian principle of evidence and destroyed the Aristotelian and Scholastic idea of the difference between *axioms* and *postulates*, that is, between the primary principles of metaphysics and those of the various sciences, particularly the physical-mathematical ones. The latter, in turn, were confirmed as the new «absolute sciences» of the «century of the Enlightenment», able to replace obsolete Scholastic metaphysics. Philosophical reflections taken aside, the presumed self-evidence of Euclidean postulates justified the modern use of language, which considered the notion of «axiom» equivalent to that of «postulate».

Gauss' doubts on the absolute character of Euclidean geometry

Therefore, when, around 1820, Friedrich Gauss (1777-1855) —the *princeps mathematicorum*— grew convinced that the impossibility to demonstrate the fifth postulate —particularly in light of Saccheri's failed demonstration *ad absurdum*¹⁹— was not a matter of fact, but hid the possibility that a geometry without the fifth postulate could be consistent —hence the possibility that «non-Euclidean» geometries could exist, as the

¹⁷ Imre Toth's work (Toth 1997), dedicated to the commentary of non-Euclidean parts in the *corpus aristotelicum*, shows —thanks to a great quantity of documentary evidence— that the issue of the Fifth Postulate, hence of Euclidean and non-Euclidean geometries, was present throughout all Western thought, from Plato, Eudoxus, Aristotle and Euclid, up to modern mathematics.

¹⁸ These are: 1) a straight line segment can be drawn joining any two points; 2) any straight line segment can be extended indefinitely; 3) given a center and any distance from it, a circle can be drawn; (NOTE: the correct English version of the postulate would be the following: “Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as centre”. I have left the literal translation in the text); 4) all right angles are congruent.

¹⁹ Saccheri had unsuccessfully tried to demonstrate the truth of the fifth postulate through the demonstration of the absurd character of its negation.

mathematician himself first defined them— he did not make this idea public, in order to avoid the disapproval that it would have engendered. The demonstration of this idea, however, was “floating in the air” or, to use a Hegelian terminology, it was «in the spirit of the time».

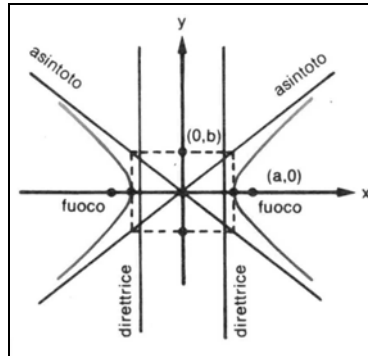


Figure 1-3. Hyperbola in the customary formulation $x^2/a^2 - y^2/b^2 = 1$, that is, a plane curve defined as the locus of points for which the difference of distance from two given points, called foci, is constant. One can see, from the figure, that the arms of the hyperbola overlap the related asymptotes only when extended to infinity. Only at infinity, indeed, the distance from the foci equals zero.

A few years later, in 1829, Russian mathematician Nicolaj Ivanovic Lobachevski (1793-1856), Chancellor of the University of Kazan and, in 1832, young Hungarian mathematician Janos Bolyai (1802-1860), reached the same conclusion. Contrary to Gauss, however, they published the results of their theory and —uselessly— looked for public support.

Lobachevski's
fundamental
demonstration of
the logical
impossibility of
demonstrating the
fifth postulate

Lobachevski, in particular, proved the impossibility of demonstrating the fifth postulate in the equivalent formulation recalled above (that is, that only one straight line can be drawn through a point external to another given straight line). On this basis he created what he called an «imaginary geometry», given that, without the fifth postulate, it was a geometry entirely removed from common perception. However, it was a fully consistent geometry: the first non-Euclidean geometry, of a hyperbolic kind, to be successfully advanced in the history of humanity.

Bolyai's «absolute
geometry»

If possible, Bolyai went even further, by showing that it is not contradictory to admit not only that more than one parallel straight line can be drawn through one point, but that an *infinite number of straight lines parallel* to a given straight line can be drawn. On this basis, he built an «absolute» non-Euclidean geometry —which he called «absolute science of

space»—, given that it was founded on a more comprehensive hypothesis than that used by Lobachevski. However, it is only with Bernhard Riemann (1826-1866) that the conceptual revolution started with the discoveries of these three authors reached its peak.

The meaning of this conceptual revolution

Before describing Riemann's theory, even if only briefly, we should stress a fundamental epistemological point, which will help us realize the momentous change that these discoveries implied. E. Nagel e J. R. Newman, for example, describe it as follows:



The traditional conviction that the axioms of geometry (or the axioms of any other system) can be proved by their seeming self-evidence was then radically destroyed. Furthermore, little by little it became clear that the real task of a pure mathematician is to *derive theorems from postulated hypotheses*, without being concerned, as a mathematician, with deciding if the introduced axioms are actually *true* (Nagel & Newman 1993, 21).

The end of the modern illusion that self-evidence can be used as the absolute criterion of truth

The claim of all modern philosophy after Descartes to replace the conformity to being with the evidence and/or self-evidence of some propositions as the criterion of truth, proved then unfounded. We have already recalled that this conviction was so deep-rooted—and it still is, especially in countries like Italy, which lag behind in terms of scientific popularization and updating—that the most important modern mathematician, Gauss, had to avoid talking about his discovery in public in order to save his reputation. As Boyer recalls in his history of mathematics, «in a number of letters to his friends, Gauss praised Lobachevski's research, but he never acknowledged it in his works for fear of provoking the laughter of 'idiots' (*sic*)». And Boyer continued:



Lobachevski is considered the «Copernicus of geometry», the one who transformed this domain of mathematics by creating an entirely new branch (...), *by showing how Euclidean geometry was not the exact science, repository of absolute truth, that it had been previously considered*. In some sense we could say that the discovery of non-Euclidean geometry gave a mortal blow to Kantian philosophy, that was comparable to the consequences that the discovery of incommensurable magnitudes had had on Pythagorean thought (see *supra* § 1.1). Lobachevski's work made it necessary *to radically modify the fundamental ideas on the nature of mathematics* (Boyer 1968, 621ff, emphasis added).

1.5.1.2 Riemann's formalization

The axiomatization of geometry after Riemann and the purely formal character of mathematical sciences

The consequences mentioned by Boyer became clear only after Bernard Riemann, in his famous thesis of qualification — *On the Hypotheses which Lie at the Foundations of Geometry*, which he defended in Götting, before Gauss, in 1854— brought the notion of «non-Euclidean geometry» well beyond Lobachevski's and Bolyai's hyperbolic geometry, and created a new elliptic geometry, based on a new postulate of the parallel lines. In his geometry, *no* parallel line can be drawn through a point external to a given straight line. However, this is only part of the importance of his discovery. With Riemann, geometry (hence, all of mathematics) becomes completely *axiomatized*. The postulates of any science should *never* be taken as absolute truths, but only as *hypotheses*. In addition, at least in some domains of science such as «pure» mathematics, the very descriptive content of primitives and axioms —hence of the whole axiomatic system derived by them— must be relinquished. With Riemann, for the first time in the history of thought, mathematical science gives up any denotative content of objects —the so-called «mathematical entities», so dear to the Pythagorean-Platonic tradition— in order to become a pure science of the *syntactic relations between the symbols* of mathematical language.

The formalization of geometry and the end of mathematics as the «science of quantity»

Stated otherwise, despite the paradoxical nature of some conclusions, one can no longer talk of a geometry of «spaces», «straight lines», «shapes» or «points» as understood in our daily experience. Geometry, particularly after Riemann, only deals with «*n*-tuples [relations among symbols, such as pairs, triples, etc., Editor's note] that are grouped according to precise rules».

Mathematics is then no longer the «science of quantity», as it had been for over two thousand years, and becomes a «science of relations», the formal science *par excellence* that draws conclusions that are logically implied in any consistent set of axioms. This point should be clearly understood, because it is fundamental to grasp the process of *formalization* of scientific languages that is characteristic of contemporary science.

Steps of the progressive formalization of mathematics



In Greek mathematics, geometry, intended as the mathematical science of space, exclusively dealt with spatial shapes that are accessible to intuition. For this reason, for example, it did not go beyond three-dimensional plane space. The first step towards formalization was made by modern geometry, thanks to the algebraization of geometry, in particular when, starting from Descartes, the correspondence existing between *geometry** and *algebra** became clear. After Descartes' *Géométrie* was published, the idea that any geometric figure represents the solution of the related algebraic equation (polynomial) in spatial form finally became

part of mathematical thought. The birth of mathematical analysis and of infinitesimal calculus would have been impossible without the algebraization of geometry.

The first step:
Descartes'
algebraization of
geometry

In particular, thanks to the symbolism for algebraic expressions developed by Descartes —the use of the «+» and «−» signs, of the first three letters of the alphabet to denote constants, of the last ones to indicate variables and, above all, of the exponential notation to indicate power— geometry could overcome the limits imposed by the overly strong link with spatial intuition. For example, if we take the calculus of the square of the following binomial:

$$(a + b)^2$$

Analytic geometry
and intuitive
geometry: the
example of the
square of the
binomial

Each of us could learn in school how to calculate it, thanks to a simple *algorithm** that allows us to obtain the resulting polynomial, through a sequence of simple operations and with no reference to the related geometric form. First, we write down the *variables*, linking them with the exponent, in increasing and decreasing order:

$$_a^2 + _ab + _b^2$$



The empty spaces of the *coefficients* are obtained, for the second power of the binomial, from the second line of the so-called «Tartaglia's (or Pascal's) triangle», in turn obtained through another simple *algorithm*∗:

$$\begin{array}{c} 1 \\ 1 \ 1 \\ 1 \ 2 \ 1 \\ 1 \ 3 \ 3 \ 1 \\ 1 \ 4 \ 6 \ 4 \ 1 \\ 1 \ 5 \ 10 \ 10 \ 5 \ 1 \\ 1 \ 6 \ 15 \ 20 \ 15 \ 6 \ 1 \end{array}$$

.....

Usually, however, one does not talk about the geometric structure associated with the algebraic calculus of the square of the binomial $(a + b)^2$ —a structure that, incidentally, can be defined as a *geometric model* of this calculus, which makes it *evidently* true. The associated structure is a square resulting from the sum of two smaller squares, with side a and b , and two rectangles with sides $a \cdot b$.

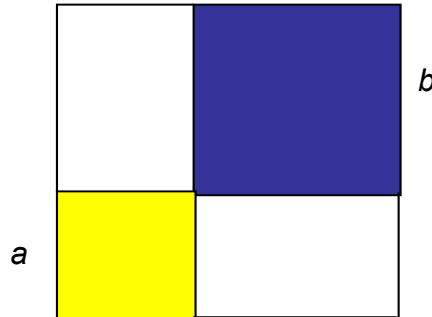


Figure 1-4. Geometric-intuitive equivalent of the calculus of the square of the binomial

The potential of the algebraic method in analytic geometry and the limits of the intuitive method in classical geometry

Now, using the same algorithm, one can easily calculate the third, fourth, fifth,..., the n -th power of a binomial and, with a similar rule formulated by Newton, of any polynomial.

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

...

Conversely, when using the classical intuitive method, one could not go further than the square of the binomial: it is indeed impossible to imagine a space with more than three dimensions. Thanks to the algebraization of geometry, then, it was possible to extend Euclidean geometry to plane spaces with not only three, but n dimensions.

More generally, Descartes' algebraization of geometry allowed one to turn Euclidean postulates of plane geometry into as many algebraic expressions. The «point» corresponds with a pair of numbers; the «line» with a (linear) relation between numbers described by a first degree equation in two unknowns; the «circle» with a relation between numbers described by a second degree equation of a specific form, etc.

The second step: Riemann's formalization of geometry

Riemann's geometry introduced a further level of abstraction from intuitive contents. The algebraic expressions of elementary algebra still refer to magnitudes (numerical and/or spatial) that they denote in a symbolic form. After Riemann, the algebraic expressions used in his geometry are constructed in such a way that they denote nothing, they are expressions devoid of any (referential) meaning, but that can take on different (referential) meanings through a specific *interpretation**. With Rie-

mann, geometry becomes a *formal system** properly considered, in the modern sense of the word. Stated otherwise, geometry becomes a deductive science, able to represent abstractly, with its symbolic formalism, *relations* and *structures**, rather than relations among magnitudes. These relations and structures can be applied, through subsequent interpretations, to multiple types of entities. In themselves, however, they are devoid of any denotative content, of any referential meaning, or any reference to objects.



In practice, it was recognized that the validity of mathematical deductions does not depend in any way on the particular meaning that can be associated with the terms or expressions contained in the postulates. It was then possible to see that mathematics is much more abstract and formal than it was traditionally thought: more abstract because, in general terms, *one can make mathematical statements on any thing, rather than on intrinsically circumscribed sets of objects or properties of objects* [i.e., quantitative properties, Editor's note], because the validity of mathematical demonstrations rests on the structure of the statements, rather than on the particular nature of their content. (...) One should repeat that the only issue that a pure mathematician is concerned with (as opposed to a scientist that uses mathematics to study a particular object) is not whether the postulates that he admits and the conclusions that he draws from the first are true, but whether these conclusions are the *logical and necessary conclusions*, given the hypotheses he started from (...). As long as we are dealing with the essentially mathematical task of exploring the purely logical relations of dependency among the various statements, we must ignore the customary meanings of the primitive terms [the terms with which the starting postulates are constructed, editor's note], and the only «meanings» associated with them are those assigned by the axioms they enter. This is the meaning of Russell's famous saying: pure mathematics is the science in which we do not know what we are talking about and whether what we are saying is true (Nagel & Newman 1993, 23ff.)

Mathematics as a purely formal science and the replacement of *truth* with *consistency* as the criterion of validity of mathematical statements

Strumia—who was as careful as we are in capturing the differences and the relations between modern science and classical metaphysics—was then correct in stating that, in this way, mathematics reached the point of greater distance from metaphysics, much beyond that imagined by the Greeks. It was no longer related to *truth*, but only to the *coherence* of its statements and, therefore, to the *consistency** of the formal system to which they belong. In this way, mathematics increasingly resembles *formal logic*, the only science that, in classical times, did not deal with the content of statements, but with their *form* as well as with the form of the reasonings and/or arguments associated with them; that is,

of the arguments constructed starting from those statements (if these operate as the premises of a reasoning), or constructed to arrive at them (if they are the conclusions of a reasoning).

Formalization of mathematics and birth of mathematical logic

Indeed, it was not a coincidence that mathematical logic, intended as a pure symbolic logic concerned with the formal correctness of languages (coherence, consistency) rather than with the truthfulness of their content, while foretold by Leibniz, was only developed after G. Frege's axiomatization of mathematical sciences (end of the 19th century). This was the axiomatization begun by Riemann's axiomatization of geometry around the mid-19th century, and continued with Italian mathematician Giuseppe Peano's (1858-1932) axiomatization of arithmetic at the end of the 19th century²⁰.

How to assess the consistency of statements without first being able to assess their truth?

This, however, created a difficult problem. How could one assess with absolute certainty the *consistency* of, hence the *reciprocal compatibility* between, the statements of a non-Euclidean geometry—which is entirely counter-intuitive—if there was no means of assessing the truth of these statements? In ancient Euclidean geometry this was easily done: given that one could ground the *truth* of given statements on intuitive evidence, and given that it was believed that true statements are always compatible with one another²¹, from the evident truth of these statements, their reciprocal compatibility was derived and, as a consequence, the *consistency* of the system. This reasoning, however, was incompatible with the counter-intuitive abstract quality of the new geometry.

Riemann's solution: to create a Euclidean model of his geometry: the geometry of curved space

Riemann found a solution that was partial from the theoretical (logical-formal) point of view, given that it opened the way to the use of *non-finitary methods* of demonstrating the truth and consistency of *formal systems*; yet, it made non-Euclidean geometry accessible to intuition, taking it far from pure formalism, which is accessible to a small elite. In more



²⁰Peano's system, which was the first example of a completely axiomatized mathematical theory, entirely expressed in the symbolic language of the new mathematical logic, was first published in the *Arithmetices principia nova methodo exposita*, in 1889. As primitives, this formal system took the terms «zero», «whole number» and «ordering relation». These primitives satisfied (that is, they took on a purely syntactic «meaning» for) the five following axioms: 1) Zero is a number; 2) If n is a number, then the successor of n is a number; 3) Zero is the successor of no number; 4) Two numbers whose successors are equal, are themselves equal; 5) Each property that is possessed by zero and by the immediate successor of any number that has that given property, belongs to all numbers (axiom of mathematical induction). Ordinary arithmetic is a *model** of this *formal system*, which makes it *true* for this particular application. Conversely, Peano's formal system is an axiomatization of ordinary arithmetic even if today, after Gödel, we know that it is an *incomplete* axiomatization.

²¹ Today, however, after Gödel, even statements such as these are not foregone. We will come back later to this point.

technical terms, Riemann proposed a *Euclidean model** of his non-Euclidean elliptic geometry—which Italian mathematician Eugenio Beltrami (1835-1900) extended to Lobachevski's hyperbolic geometry—as a *geometry of curved space*. This model could give a partial but intuitive meaning to the statements of the formal system under examination (in this case, of the various formal systems under study). Therefore, it was able to make the *consistency* of the paradoxical (apparently absurd) or counter-intuitive statements of that given system evident²². In this Euclidean model, the surface of a sphere is *interpreted** as a «plane» and the *maximal circle** as a «straight line». In so doing, the «consistency» of non-Euclidean geometry was brought back to the truth of the Euclidean one, hence, to the presumed consistency of its statements. As mentioned, in light of Gödel's work, this solution was debatable. However, it was able to offer an important result: to make the abstract character of non-Euclidean geometries intuitively «evident» and usable also by non-mathematicians. Indeed, in this model, Riemann's elliptic geometry coincides with a geometry described on the positive curvature surface of the Euclidean sphere, while Lobachevski's hyperbolic geometry coincides with that described on the negative curvature surface of the Euclidean (pseudo) sphere.

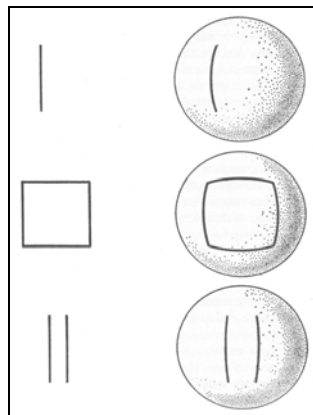


Figure 1-2. Two points on flat space (left side) become two antypodes on Euclid's sphere (right side); two pairs of parallel straight lines become two maximal circles; two parallel segments become two arcs of *maximal circle**. If

²² This method of demonstration of consistency, as much as partial from a formal point of view, given that it is founded on logically solid conditions (such as the use of infinitary demonstration methods), still has its equivalent in the logic of ordinary language. In order to make abstract reasonings evident, it is essential to give examples. One can immediately grasp that this principle can be valid for an indefinite (potentially infinite) number of similar cases, that is, that formally possess the same *structure**.

prolonged, these meet: clearly, this is contrary to what is stated in Euclid's axiom of parallel lines.

Intuitive character of the Euclidean model of Riemann's and Lobachevski's geometries

In this way, some paradoxical statements of the non-Euclidean geometries immediately become evident. The statement of Riemann's geometry according to which no line parallel to a given straight line can be drawn through a point external to it becomes evident, as soon as one realizes that two parallel line segments on Riemann's elliptic space correspond to two arcs of maximal circle in Euclid's sphere. If prolonged, the segments meet: obviously, going against Euclid's axiom of the parallel lines (see Figure 1-5).

Example of the sum of the internal angles of a triangle in Euclid, Riemann and Lobachevski

Similarly, the statement that the sum of the internal angles of a triangle is greater than 180° in Riemann's elliptic geometry, or smaller than 180° in Lobachevski's hyperbolic geometry, becomes evident if these are described on the surface of Euclid's sphere—respectively, on the positive and on the negative curvature surface—rather than on Riemann's or Lobachevski's plane (see Figure 1-6).

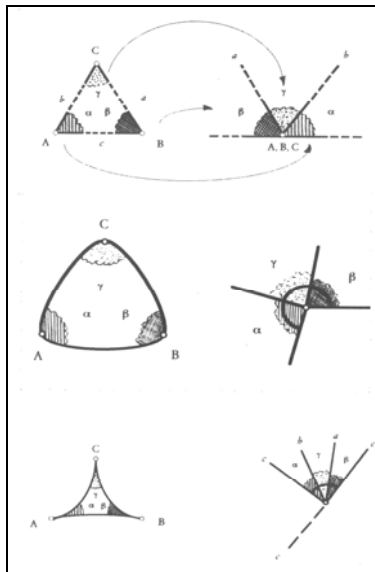


Figure 1-3. Intuitive description of the triangles, respectively; in Euclidean flat space (top), in Riemann's elliptic space (centre), in Lobachevski's hyperbolic space (bottom). Beside each figure, the projections of the three angles obtained by superimposing the adjacent sides of the original triangles show that, in the first case, the sum of the internal angles equals two right angles (180°); in the second case, it is greater than 180° ; in the third, it is lesser.

1.5.2 The discovery of antinomies

1.5.2.1 Antinomies in Cantor's theory

Limits of indirect demonstrations by means of the models of consistency of a formal system

Earlier we made reference to the limits of the *indirect* demonstration of consistency of non-Euclidean geometries by means of a Euclidean model of the same, such as the demonstration of curved space. Indeed, this demonstration consists in reducing the problem of the consistency of non-Euclidean geometries to that of the intuitive (evident) truth of Euclidean geometry, hence to the presumed consistency of the latter.

On the other hand, one could try a *direct* demonstration of the consistency of non-Euclidean geometries by using the same model, taking advantage of its evidence. Unfortunately, however, when dealing with infinite and non-finite models of a given formal system, as in our case, the evidence criterion —the Cartesian principle of «clear and distinct ideas»— is subject to terrible mistakes and contradictions, the so-called *logical antinomies*. These surface as soon as one tries to analyse, with *constructive methods**, the consistency of similar formal theories. That is, they emerge as soon as, with the principles of mathematical theories, one does not stop short of stating the existence of given entities, on the basis of the presumed evidence of the related notions, but tries to define, demonstrate or calculate the mathematical entity presumed as existing, starting from simpler entities and/or operations.

The problem of infinity in mathematical sciences

As ancient thinkers were well aware of —and the moderns had to relearn at their expense— infinite objects —particularly those that are «too infinite» such as those that are necessary in a theory of the foundations of a science aspiring to *all-inclusiveness*²³ in relation to its object, and *autonomy* in relation to other forms of language and knowledge— are incompatible with the use of constructive methods. One cannot think of building an infinite object «piece by piece».²⁴ On the other hand, as be-

²³ Technically, sets whose cardinality can be compared to that of V , the «universal collection» of all objects of a given discourse universe, are «too infinite». In practice, what we have learned from Cantor's and Russell's antinomies is that, for a science, the rigour of an extreme formalism, such that only a constructive approach can guarantee it, can only be used with a subset of the objects of that science; that is, by limiting the dimension of the constructible sets, those whose existence can be demonstrated, and not only presumed through a specific axiom (see Hallett 1984).

²⁴ Thus spoke P. J. Cohen in the Introduction to his book on the *Continuum Hypothesis*, which still constitutes the last chapter written so far in the modern theory of the foundations of mathematics (Cohen 1966). In Aristotelian terms, to think that it is possible to have an infinite «in potency» completely shift to «in act» is absurd. This does not mean that, for this philosophy, the notion of actual infinity *tout-court* is absurd, such that this notion can only be used in theology as well as by a «negative theology» of a mystic, hence a non-rational kind. The Scholastic doctrine of actual infinity is much richer than these simplifications. Clearly, this cannot aspire to the rigour and richness of notions and

lieved by those who try to use mathematics to solve problems (rather than simply contemplating it), constructive methods, if built on a *finitary* base, are the only ones which allow us to deal with and solve specific problems, and which do not reduce mathematics (and logic) to an abstract intellectual exercise, which (rightly so) is less and less popular today.

A purely abstract science is certainly fascinating; however, after the discovery of antinomies, such a science is forced to make only *generic* statements, precisely in order to avoid falling into contradictions and because it cannot use constructive methods. Other than being often incomprehensible for non-experts of that given subject, these statements are almost always impossible to apply concretely.

Scientific barbarism is the risk : separation of applied from theoretical science and subordination of the former to technology

The growth of *scientific barbarism* is then the consequence. Culture is less and less «made» by scholars of theoretical mathematics and of theoretical subjects generally and increasingly developed by «scientific meddlers» that create *ad hoc* models for the solution of specific problems, with no guarantee of the truth or consistency of what they propose. Above all, these models are formulated with no guarantee that the rest of the community may verify that what is proposed as the «scientific solution» of a given problem is not merely the result of the logic of technological exploitation (for the solution of this problem in the notion of «open» logical systems, see Cellucci (1998) and *infra* § 3.1.5.)

Cantor's attempt to create a constructive analysis of the foundations of mathematics and the infinitist prejudice that affected it

Turning back to our historical account, the issue of the foundations of mathematics imposed itself as a pressing issue starting from the second half of last century. This was due to the extraordinary discoveries that were made at the time, of which we only mentioned the most revolutionary, which overturned certainties held for centuries. German mathematician Georg Cantor (1845-1918) was the first to try, with great intellectual honesty and through a *constructive method*, to deal with the re-founding of mathematics after the intellectual earthquakes of the first fifty years of the 19th century. He did so by dealing directly with the problem of *actual infinity* that, according to a typical modern prejudice linked to a realist and Platonic interpretation of infinitesimal calculus, seemed indispensable to guarantee the *absolute* consistency of mathematical theories, in particular of the «queen» of modern mathematics: analysis in its arithmetic form, as developed by Dedekind and Weierstrass. This realist interpretation of infinitesimals was a prejudice that was only eradicated thanks to the development, in the 20th century, of

methodologies that today, after Cantor, we can have when dealing with this problem. The results that we have reached, however, do not contradict those obtained by Scholastic thought. We will come back to this point in § 5.5.5.

an *intuitionist* mathematics (Brouwer 1908; 1954) and of *non-standard* analysis (Robinson 1974) —entirely equivalent to classical versions (Gödel 1933)— which demonstrated its unfounded nature (Nelson 2002). Cantor himself formulated the problem in the following way:



Undoubtedly, we cannot relinquish *variable* quantities in the sense of potential infinity; from this, one can demonstrate the necessity of actual infinity. For a variable quantity to exist in a mathematical theory, the «domain» of its variability must be known beforehand, through a definition. However, this domain must not be, in turn, something variable, otherwise any sound basis for the study of mathematics would founder. Therefore, this domain is a defined, actually infinite set of values (Cantor 1886, 9).

Cantor's discovery of antinomies, during his attempt to formulate an exclusively constructive theory of sets

Cantor dealt with his constructive task by using a notion that was both exceptionally simple and inclusive for all different mathematical sciences: his definition of *set*²⁵. Indeed, if one could no longer consider the fundamental notions of mathematics as self-evident (e.g., the concepts of number, shape, function, ...) one could demonstrate them starting from a much simpler notion, precisely that of “set”. Intuitively speaking —as Weierstrass had already suggested— a number can be defined as a set of units, a geometric figure as a set of points, a function as a relation between ordered sets.... While Cantor's theory was very fruitful for a number of discoveries concerning the foundations of mathematics — particularly for the notion of infinity— it inevitably ran into unsolvable antinomies. This happened when Cantor tried to constructively ground objects that are «too infinite» —whose cardinality, in other words, is approximate to that of the universal collection V — such as the *universal set* (in the foundation of all-inclusive, infinite sets, based on cardinal numbers) and the *maximal set* (in the foundation of all-inclusive, infinite sets, based on ordinal numbers).

1.5.2.2 Russell's antinomy

The discovery of Russell's antinomy as a logical antinomy

Bertrand Russell's (1872-1970) discovery, in 1902, of the antinomy that carries his name had a much greater cultural impact. This antinomy, while substantially equivalent to Cantor's, was found in G. Frege's attempt to develop a *logist* foundation of mathematics.

Resonance of the discovery:

The cultural resonance obtained by Russell's antinomy is based on two reasons, one contingent, and the other substantial:

²⁵ This is not the appropriate place to delve deeper into Cantor's set theory. For a more detailed account, see (Basti & Perrone 1996) and, above all, (Hallett 1984).

1. For the importance of the individuals involved

- ◆ The first, contingent, reason is linked to the prominence of the individuals involved. These were Frege, who was the most famous mathematician and philosopher of mathematics of his time (the end of the 19th century) and Russell, who would have shortly become the most famous, at the beginning of the 20th century, after his historic work was published between 1900 and 1920 —*Principia Mathematica*, which he wrote with Alfred North Whitehead (1861-1947). In this work Russell presented to all mathematicians not only his antinomy, but also the solution he proposed for it —the *ramified theory of logical types*— which is also not immune from important formal problems²⁶.

2. For the demonstration that the problem of antinomies was also logical-syntactic rather than simply logical-semantic, as it was believed for thousands of years

- ◆ The second reason was the fact that the discovered antinomy was of a *logical-syntactic* type. First of all, it was not only of a *mathematical* kind, as the antinomies of Cantor's set theory seemed to be. Because it related to classes and predicates, it was a *logical* antinomy, which, then, included predicates and classes of mathematical objects as a particular case. In addition, while being of a logical nature, this antinomy was *logical-syntactic*, rather than *logical-semantic*, contrary to the various logical antinomies that had been known in the history of logic for thousands of years, starting with the famous «liar's paradox». The newly-born formalist approach, then —that is, the rejection of truth in the name of pure consistency, in order to give rigour to science while avoiding all references to contents— turned out to be as weak and vulnerable to inconsistencies as the approach it intended to replace²⁷.

²⁶ The *simple theory of logical types*, proposed by Frank Plumpton Ramsey (1903-1930) in 1926, proved much more practical than Russell's solution. This theory distinguishes the various logical types through specific *primitive terms*. In so doing, it can avoid referring to logics of a higher order with predicates whose argument is an infinite set of predicates of a lower logical order, whose existence, in Russell's theory, is justified through the debatable «reducibility axiom». Ramsey's solution, however, still presents some problems. By «multiplying entities», indeed, it imposes very complex assessments of the consistency of the resulting formal system. In any case, the principle of the *unrestricted theory of types* remains confirmed: in order to avoid antinomies, the propositions of a given formal language must be built with predicates such that, when their arguments are totalities of elements (e.g., predicates with variables linked by a universal quantifier: «for all»), these elements belong to a lower logical type. That is, collections of elements that can be defined as existing only by presuming the totality of the collection itself (principle of the vicious cycle in impredicative definitions) are not admissible. An infinite totality of elements can be defined as such only from «outside» the totality itself.

²⁷ This does not mean to deny the progress that is implied in the modern formalization of mathematical science. However, it is important not to believe that this progress consists in the definition of a method proposing infallible theories, contrary to what the scientific ideology would have wanted. On the contrary, it goes in the direction of propos-

The great French mathematician, Henri Poincaré, who proudly opposed formalism, and was the founder of the «intuitionist» school of the foundations of mathematics, thus sarcastically commented the announcement that Russell's antinomy had been discovered: «formalism has finally proved productive: it has created antinomies».

The logical-syntactic character of Russell's antinomy

But what are the characteristics of Russell's antinomy and of its logical-syntactic quality, which made it so intimidating? The problem that it highlighted—thus showing the great value of the formal method as one favouring rigour and transparency in demonstrations, against the traps of evidence—can be summarized as follows. First, we will examine the *logical*, other than mathematical, character of the antinomy; second, within the former, we will examine its *syntactic*, rather than semantic, character.

The logical, and not only mathematical character of the antinomy

The logical character of the antinomy and its refutation in Frege's logicist attempt to found mathematics

As recalled, Russell's antinomy stopped Frege's *logicist* attempts at the foundation of mathematics, that is, his idea to demonstrate the substantial *equivalence** between logic and mathematics, thus grounding the latter on the former²⁸. Incidentally, Frege successfully pursued the rigorous construction of a *mathematical logic* (or *symbolic logic*), even if the symbolism he used was too obtuse, so that Peano's and Russell's much more intuitive symbolism—which was made famous in the *Principia*—prevailed in the use of mathematicians of the 20th century. In any case, thanks to his complete symbolization of classical formal logic, Frege conquered an immortal place in the history of logic, mathematics and philosophy.

Constructive character of Cantor's set theory

With his discovery of antinomies in Frege's class theory, Russell struck a mortal blow to the logicist theory of foundations, as well as to the great German mathematician's psychology²⁹. Frege was convinced that the

ing a methodology that does not «hide its mistakes» under a plethora of words, but rather makes them explicit. Formalism helps transparency and intellectual honesty, it does not guarantee infallibility.

²⁸ Because of his idea, Frege has been defined as the last great «Aristotelian» logician. By asserting the pre-eminence of formal logic over mathematics, he moved in the same theoretical direction as the Stagirita.

²⁹ This is also explained by the circumstances in which Russell conveyed his discovery to his German colleague. This was while the proofs of Frege's work on the foundations (about which Frege asked for Russell's opinion) were being printed. After Russell's discovery, the whole book, which represented the crowning accomplishment of Frege's career as well as the ultimate aim of his whole life's work and of his creation of symbolic logic—the unification of logic and mathematics—had to be abandoned. However, this was not the case with Frege's great discoveries, in particular his formulation of the notion of *propositional function**, core of all of modern symbolic logic. This notion is certainly one of the greatest intellectual achievements of Western thought in the field of logic, second only to Aristotle's invention of formal logic. At the same time, this invention did

antinomies discovered by Cantor were linked to his particular notion of «set» and its characteristic constructive nature, which made it a notion of strictly mathematical applicability. Indeed, in Cantor's theory, sets are not brought into existence through hypothetical axioms of existence — as it will be done in the subsequent «axiomatic» theories of sets³⁰ — but through a wholly constructive, even algorithmic, demonstrative procedure, which was based on the p.n.c.. Any set is indeed constituted as the subset of its power-set, that is, of the set of all possible combinations of the subsets of the original set³¹, and so on, *ad infinitum*, both upwards and downwards, without assuming «primary» sets as axiomatically given, on which the whole edifice is based. It is then natural to think that, by replacing the property of recursive constructiveness that always makes any set the subset of some other set, one also eliminates the risk of producing antinomies³².

not serve the purposes identified by its author. Intellectual humility and moral detachment from one's own discoveries are fundamental also in science. We are here to serve truth and science, never to be their master!

³⁰ Following (Hallett 1984), today recognized as one of the most authoritative accounts of Cantor's theory, based on original texts, we distance ourselves from the theory's «official» interpretation, made popular in the teachings of Abraham Fraenkel (1891-1965). Indeed, we owe to this mathematician, who worked with Ernst Zermelo (1871-1953), the substantial formulation of an axiomatic theory of sets (the so-called Zermelo-Fraenkel theory) that is universally presented and taught as «the» theory of sets, in any mathematics textbook. According to Fraenkel's interpretation, Cantor's would be a «naïve» theory of sets, based on an implicit «inclusion axiom» for the construction of sets that is entirely equivalent to the principle of unrestricted abstraction that was in force in Frege's class theory (see fn. 35). The peculiarity of Cantor's theory is, conversely, that it aims to be fully constructive, taking the consistency of sets — that is, the p.n.c. — as the only axiom for set construction. In addition to the already mentioned work by Hallett see, on this point, (Basti & Perrone 1996, particularly Ch. V).

³¹ For example, given set A , made of four elements $\{a, b, c, d\}$, it can be considered as a subset of the power set of A , $\wp A$, made of sixteen elements. $\wp A$, in other words, will be made of subsets constituted with all the possible combinations of A 's elements: none, one by one, two by two, three by three and, finally, four by four, that is, $A: \{\{\emptyset\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$, in total, sixteen elements. In general, therefore, the «power», or «cardinality» (the cardinal number of elements) of the power-set of a set with n cardinality is 2^n .

³² It appears clear that an antinomy such as that of the *universal set* immediately derives from this constructive characteristic of sets. For a set to be *universal*, it must contain *all* sets, hence also itself. However, a set, in order to be defined as such, according to Cantor, *must* be contained in another set with greater power, or cardinality. Therefore, *it cannot be contained in itself*. The universal collection, as a consequence, cannot be a set. In this case, however, the constructive plan of reducing mathematics to a construction based on one simple notion falls through. Since the beginning, Cantor was aware of this antinomy, related to a notion of set grounded on cardinal numbers. For this reason, he tried to «close» his system through the notion of *maximal set*, based on ordinal numbers (there is a bi-univocal correspondence between the two kinds of numbers), as the limit of a se-

Naïve character of
Frege's theory of
classes



From this derived the so-called *unrestricted abstraction axiom*, that is, Frege's idea to ground mathematical sets through the logical notion of *class*, by defining a *universal property* characterizing all classes. This axiom states that, given a *property* and the *predicate* that linguistically expresses it, the class of all elements having that property (hence that *satisfy* that predicate, that is, that make it «true», thus constituting the *domain** of that predicate³³) is also brought into existence. For example, to define the predicate «being red» means to construct the «class of all red objects» and, with it, the *domain** of application, or *extension**, of that predicate, which makes it true, by univocally defining it within the language. Because of this characteristic of grounding the consistency of the notion of «class» —that is, the consistency of a predicate's domain in a given language— on the presumed «truth» of the predicate it is associated with, Frege's theory (contrary to Cantor's, see footnote 30) is defined as the *naïve theory of classes*.

Distinction between
ordinary and non-
ordinary classes

The antinomy hidden in this construction is immediately revealed as soon as, with Russell, we distinguish between two types of logical classes:

- ◆ Ordinary classes, which are not members of themselves, contrary to the majority of logical classes in any kind of language (for example, the class of men is not in turn a man, because it is constructed on a predicate, being-man, that does not apply to itself). In particular, the great majority of classes that make up mathematical objects, starting with numbers, are ordinary classes³⁴.
- ◆ Non-ordinary classes, that is, those that contain themselves as an element, because they are built on predicates that can be applied to themselves (for example, the class of polysyllables belongs to itself,

quence of transfinite sets in increasing order, once the so-called «continuum hypothesis» —according to which the continuum (or set of real numbers) is the set whose power immediately follows that of the numerable (the set of natural numbers and of the sets that are equally powerful to it). It was Italian mathematician Cesare Burali-Forti (1861-1931) who first demonstrated the paradoxical character of the notion of maximal ordinal infinite set, thus signalling the failure of Cantor's project. On the question of antinomies in Cantor's theory, see (Basti & Perrone 1996, particularly Ch. V).

³³ An equivalent yet more binding definition of this principle can be found in (Lombardo-Radicce 1981, 87) and is given in footnote 35.

³⁴ In Frege's and Russell's approach, each natural number 0,1,2,3... is defined as the class of all classes that contain, respectively, zero, one, two, three, ... elements. Clearly, then, numbers are all classes that *do not* contain themselves. Let's now turn to the class of all numbers, which is the object of arithmetic. This will be the class of all classes that do not contain themselves. Will it or will it not contain itself? If it does, the foundations of arithmetic (and of mathematics) should be found elsewhere...

The paradoxical nature of the notion of «total class» as applied to ordinary classes

because «polysyllable» is polysyllabic; contrary, for example, to «monosyllable»).

Clearly, as long as we deal with non-ordinary classes, to think of «total classes», that is, of «classes of all the classes that contain themselves» is not problematic. Such a class will also contain itself; therefore, it will be total. However, as soon as we attempt a definition of «total class of normal classes» —those from which, in Frege's system, mathematical objects are made, starting with numbers— we encounter an insurmountable antinomy, as in Cantor's theory. In order to understand this, it is sufficient to ask whether the class of all classes that do *not* contain themselves will or will not contain itself. If it *contains* itself, then it *does not contain* itself —because, by definition, it can only contain all classes that do *not* self-contain. If it *does not contain* itself, then it *contains* itself —given that, again by definition, it must contain *all* the classes that do not self-contain. So, if *it is so*, then *it is not*; if *it is not so*, then *it is*, and this is the paradox. Thus it can be clearly seen that the notion of mathematics that we normally use, as a «class of all numbers», is not evident and foregone at all: to the contrary, it hides many pitfalls.

The passionate letter with which Frege replied to Russell is meaningful. Frege was indeed well aware that the discovery of the logical, rather than mathematical, character of the paradoxical nature of all-inclusive objects-collections, whether sets or classes, implied the death of a dream shared by an entire cultural time period, the dream of 'scientism'.



The contradiction you discovered caused the greatest surprise in me, I would even say dismay, given that it shook the basis on which I intended to build arithmetic³⁵. (...) Still today I cannot understand how arithmetic can be grounded scientifically (...) if it is not possible (...) to move from a concept to its extension. Can I, in any case, speak of the extension of a concept, that is, can I speak, in any case, of a «class»?

Solatium miseris, socios habuisse malorum. This consolation, if it is one, assists me as well. Indeed, all those that used conceptual extensions, classes, sets, in demonstrations, are in my same situation. What is in question here is not my particular foundation method, but the possibility of a logical foundation of arithmetic in general (quoted in Lombardo-Radice 1981, 87).

³⁵ As we already mentioned this basis is generally defined as the *unrestricted abstraction principle*: «each property determines the set of the elements that satisfy it (it determines its 'extension')».

The syntactic character of the antinomy.

Russell's antinomy is independent from the problem of truth and is solely linked to the notion of membership

Frege's last remark adequately conveys how Russell's discovery went well beyond the limits of his own attempt to build a logicist foundation for mathematics. Indeed, Frege's theory still moved within the classical approach to logic and to mathematics, which stipulated that consistency should depend on truth. This was the same approach for which, before Lobachevski and Riemann, one took the «evident» truth of Euclidean geometry for granted. For Frege, both logic and mathematics deal with statements that are true, rather than consistent. The same abstraction axiom recalled above makes it evident that, according to Frege, the syntactic structure of a class is grounded on the truthful nature of the statement: the class is made up of the relationship between the predicate and the domain that satisfies the predicate, that is, that makes it *true*. The antinomy discovered by Russell, to the contrary, exclusively deals with the syntactic level of the relationship of class *membership*, and does not touch on the issue of the adequacy (truth) of a predicate to its domain. In fact, the antinomy «precedes» this issue of truth³⁶.

This also eradicated a centuries old prejudice, affirming that antinomies are of an essentially semantic nature

This characteristic of Russell's discovery is important, because it eradicated another prejudice that had taken roots throughout the centuries. According to this prejudice, logical antinomies exclusively originated outside *syntax** and outside the form of argumentation; in other words, they were only linked to the content of the argumentation itself, and could be of a merely *semantic*, rather than *syntactic*, kind.

The paradigmatic case of the liar's paradox, of Sophistic origin

The famous «liar's paradox», known since ancient times —at the time of the Megarics— with which Aristotle dealt in his *Topics* and in *On Sophistical Refutations* (25, 180b2-7) and, possibly, Plato before him in the *Euthydemus* (283a-286a)³⁷— is a paradigmatic example in this respect. In its simplest form, this antinomy started with the following statement «the Cretans state: 'all Cretans are liars'». It then asked whether what the Cretans stated about themselves was true or false. Clearly, this is a semantic antinomy: it deals with the truth or falsity of some statements over other statements, as the need to use two kinds of quotation marks makes it

³⁶ As Aristotelians and Thomists we are convinced, along with Frege, that the problem of truth precedes that of consistency. The error is that the problem of truth is at stake before the relation of «to satisfy», which presumes the *domain** of an already constituted predicate. Indeed, the truth is at stake at the level of the *constitution* of the domain itself —hence of the related class or set. That is, at the level at which we *constructively set into being* the existence of the given logical and/or mathematical object —be that an element or a set (see above, fn 23, p. 79).

³⁷ The traditional ascription of such an antinomy to Epimenides (VI century), also based on the biblical reference in Paul (Epistle to Titus, 1,12), is more doubtful. The ascription to Eubulides —one of the founders of the Megaric school (IV century) and contemporary of Aristotle— made by Diogenes Laertius is on the other hand more likely.

clear from a grammatical point of view. Due to the mistaken assumption of the exclusively semantic character of any logical antinomy, following the model of the liar's paradox, rationalists of all times, both ancient and modern, insisted on the need for formalization, while pursuing the myth of the absolute character of their demonstrations. Russell's discovery suddenly destroyed such an approach. There exists a syntactic, purely formal root in antinomies, which formalism in itself cannot avoid.

But then, where do antinomies come from, if they do not solely originate in semantics?

Herein lies the problem: if antinomies originate neither in the particular nature of some mathematical notions, such as that of set, nor in the use of particular semantic terms, where do they come from? If antinomies have a clear syntactic origin, and not only a semantic one, what does this origin consist of? Does it descend from some anomaly in the linguistic use of formal languages themselves, which has never been noticed, but which could be revealed and, therefore, avoided?

Gödel's theorems will give an answer on the syntactic, logical-formal structure of any antinomy

The (negative) answer to this question will only be developed in the following century —starting from 1931 and from the *incompleteness theorems** of Peano's formalized arithmetic (see fn 20, p. 76)— by the Austrian mathematician Kurt Gödel, with a demonstration that Turing and Tarski subsequently extended to all formal systems. These theorems will make it clear that the risk of contradiction is inherent in the same notion of formal demonstrability. In this sense, therefore, it is unavoidable, at least in an absolute sense (See § 3.1.3).

Before dealing with this last step on the issue of foundations, however, we will give a brief summary of the revolutionary discoveries that, in the meantime, came about in the physical sciences between the end of the 19th and the first thirty years of the 20th century, and which ushered in the concept of a «new physics». This is the object of the next chapter.

1.6 Conclusion: the metaphysical value of antinomies

Metaphysical value of antinomies: weakness of the notion of «being» in axiomatic logic and mathematics

In order to insert the preceding discussion in the context of the meta-logical and metaphysical issues dealt with in the first volume of this work, we should stress the metaphysical aspects of the question of antinomies. While recalling, from the logical point of view, the syntactic, rather than semantic character of antinomies, we should not forget that antinomies also have a precise *metaphysical* value, relating to the notion of «being» and «existence». Indeed, both in the case of Cantor's set theory, and in that of Frege's theory of classes, the contradiction appears as soon as the question of *existence* is dealt with in a metaphysically reduc-

tive way. That is, it appears as soon as one tries to reduce the notion of «being» (being of existence, of a metaphysical subject of properties and relationships) to that of «essence» (the being of essence, of a set of properties and relationships that determine a subject), without adequately distinguishing between the two, while maintaining their radical unity and complementarity with a more adequate and inclusive notion of «being» (being as act).

Essentialist limit of the notion of being, reduced to pure «inclusion» and/or «membership» in Cantor's and Frege's constructionism*

Both in Cantor's «constructionist» approach, and in Frege's «naive» approach, the notion of a logical-mathematical entity's existence is *reduced*, respectively, to the *consistency* of the recursive relation of inclusion between a set and a subset, or to the *consistency* of the relation of class membership. This is 'being' reduced to the simple «copula» between a subject and a predicate that Kant spoke of, as we will see in Chapter Four (see § 4.3). In Cantor's theory, indeed, the existence of an object (a set) is reduced to the recursively demonstrable consistency of the *relation of inclusion* of a given object X as a subset of its set-potentiality Y (in symbols, « $X \subset Y$ »). In Frege's theory, the existence of an object x (element and/or class) is reduced to the consistency of the relation of membership of x to a given «class» y . That is: « x exists», in that theory, can be meaningfully considered if and only if one can demonstrate that this statement is equivalent to the other « x is y », in the sense of « x belongs to (is a member of) the class y » (in symbols, « $x \in y$ »).

Risk of impredicativity in the construction of sets and classes well highlighted by Poincaré, before the discovery of antinomies

As Poincaré first realized, in both cases this reductionism that, rather than considering being as the irreducible *primum cognitum*, so that any other concept—including those of truth, consistency, membership, inclusion, etc.—must be reduced to a particular articulation of the all-inclusive notion of «being», tries to reduce the latter to the former, ex-posing the two theories to the risk of impredicativity and of the vicious cycle in the very definition of the two fundamental notions, respectively, that of «set» and of «class». The definition of an object is impredicative if the terms used for it in some way presume the definition itself. More radically, an impredicativity gets stuck in a vicious cycle and, therefore, in a contradiction, if, in order to justify the existence of a given object, one must presume the existence of the collection that contains it, which, in turn, presumes the existence of the object itself in order to exist. This is what Husserl defined as the impredicativity of the notion of 'part' and 'whole': the existence of the «part» presumes that of the «whole», and vice versa.

The doctrine of being-as-act avoids impredicativity by preserving constructivity, because it guarantees a rule for the construction of existence in individuals that is independent from the rule of construction relating to membership collections

Obviously, only when the foundation of the individual's (part) being is separated from the foundation of the being of the collection to which it belongs (whole), one can break the vicious cycle. Indeed, as Aquinas recalls, it is possible to state that the whole *cannot exist* without the part, but the reverse is not true: a part *can exist* without the whole. Obviously, in this case, it will not exist as a part, but as an *individual* (see Aquinas, *In de Trin.*, II, I, 3c). However, in order to admit this, one must assume that an individual's existence cannot be reduced to class membership, without renouncing the constructive nature of individual existence; that is, without entrusting the existence of the fundamental components (or *Ur-element*) of the theory to acts of «logical faith», hypothetical axioms of existence, but rather to some «explanatory principle». In this sense, for Aquinas, existence has a «constructive» character. For example, it has a «causal» explanation in his metaphysics: the doctrine of the participation of being «according to» the essence. However, this constructive character is not reduced to that of a mechanism of membership or of a more or less recursive mechanism of inclusion. Briefly put, *existence* (being of existence, or *common being*) is not the *being something* (the being of essence, or *entity*) even if one determines the other according to two reciprocal, but not symmetrical, relations. The core of the Thomistic metaphysics of *being-as-act* lies in the *real* difference and also unity between being and essence, as we will see in Chapters Five and Six.

Conversely, in subsequent axiomatic theories of sets and classes, the existence of the *Ur-element* (from above or from below) is left to existence axioms

Conversely, due to the lack of this distinction, the theories of sets (and of classes) that followed the discovery of antinomies are called *axiomatic theories of sets* (and of classes) given that, through appropriate *existence axioms*, one can eliminate the risk that the impredicativity of the notions of set and/or class turn into vicious cycles. This is the case with the set theory of the «axiom of the power set» by John Von Neumann (1903-1957), through which the existence of those sets that are «too infinite» (e.g., the mathematical continuum, see Cohen 1966) —thanks to which the other, «smaller» sets can be built as their subsets, incurring no risk of vicious cycles— is axiomatically guaranteed. Concerning the theory of classes, it is also the case with Frank P. Ramsey's «theory of simple types», which we referred to above (see fn 26, p. 82) and in which, thanks to given existence axioms, one can guarantee the existence of «individuals», as 'primitives' that can guarantee the stability of the consequent constructions of classes and types. In both cases, as E. Zermelo correctly points out, in order to avoid contradictions, one must axiomatically guarantee the existence of the *Ur-element*, of primitive elements, on which one can ground —from «below» or from «above», depending

on the case— the subsequent construction of the theoretical edifice of classes and/or sets.

The limit of axiomatic theories: they separate constructivity from consistency, thus condemning logic and theoretical mathematics to being generic and useless

To repeat the same idea in the words of Hallett, after the discovery of antinomies, all foundational theories must, in some way, contain axioms that limit the dimension of constructible sets (or classes) (Hallett 1984). As Hilbert correctly pointed out, however, due to this axiomatization, mathematics ousted itself from «Cantor's paradise», almost as if it had committed a terrible «original sin». That is, it forestalled the possibility of combining *rigour* (constructivity) and *consistency* (coherence), condemning itself to *generic* theories, which are as consistent as they are non-constructive, hence sterile (Cellucci 1998).

Importance of the doctrine of being-as-act

A real metaphysical thinker, who knows well that «being» 'punishes' all those that pretend to build theoretical edifices without using «being», even presuming to be able to reduce it to some «more fundamental» (sic!) logical-formal notion, would smile at all this. At the same time, the same occurrence would push a Thomistic metaphysical thinker to meditate.

The Thomistic metaphysical thinker's reflection is bound to be less satisfactory. He would think of this as of another missed opportunity (for Western thought and culture) to make use of the Thomistic doctrine of the *real difference* between «being» and «essence» and, at the same time, of the *deep unity* of these two constituent principles of the metaphysical structure of any entity (= doctrine of *being-as-act*, that is, of being intended as *actuality* of a correlated essence intended as *potency*), including logical-mathematical entities, as with this case.

Frege's and Cantor's constructionism is flawed by essentialism, as much as the axiomatic approach is flawed by existentialism

Indeed, if Cantor's and Frege's constructionism can be accused of essentialism, having reduced the being of existence (of an individual) to that of the essence (inclusion and/or membership of a set and/or of a class), the axiomatic approach can be accused of existentialism. By guaranteeing, in an axiomatic form, the existence of the «fundamental ingredients» of theory with a kind of «mathematical fideism», this approach makes the being (of existence) too independent from the (being of) essence, thus splitting and weakening logical-mathematical scientific knowledge into a myriad of models and theoretical constructions, of which one necessarily no longer can encompass the comprehensive view and general consistency. Assuming as existing all the objects of the universe V of the theory —both the «fundamental» ones (through axioms of existence) and the «constructed» ones (through the application

of construction and inference rules)—the only way of avoiding contradictions is to weaken the diagram of relations among the objects within V , i.e., to limit the demonstrative and affirmative value of the theory to purely generic statements. Only in this way, one can avoid «fallacy», even using impredicative definitions (see Longo 1999). In this way, however, one greatly diminishes the effectiveness of calculus (both logical and mathematical)—that is, its capacity to find rigorous solutions to specific and well-defined problems. Conversely, if we want this type of solution, we are obliged to weaken the rigour of demonstrative procedures (Basti & Perrone 1996). Rigour is therefore constrained in the abstract quality of formulas that look beautiful but are useless, while the «useful» science, that of applications—which affects all our lives—becomes the realm of arbitrariness, of *ad hoc* models, which are always prone to the technological and economic exploitation of «strong» powers.

Aquinas' theory of being-as-act as a middle way between essentialism and existentialism, Platonism and Aristotelianism

Unfortunately, space constraints prevent us from going deeper into the Thomistic alternative to this state of things in the theory of the foundations of logic and mathematics. Aquinas' theory is a sort of *middle way* that avoids the excesses of essentialist *formalism* as well as of existentialist *axiomatism*, and we have developed elsewhere this aspect (Basti & Perrone 1996). While with different guises, these two trends in the contemporary ontology of logical-mathematical entities have their metaphysical predecessors in, respectively, Platonism and Aristotelianism. Indeed, if metaphysics is flawed by essentialism in Platonism (because it reduces the being of existence to that of essence: Cf. § 5.3), the Aristotelian way of avoiding the contradictions inherent in Platonic essentialism presents a problem typical of any axiomatic approach. This relates to the abstract and generic character given that, after distinguishing essence from existence (primary from secondary substance), Aristotelianism is unable to give a foundational theory of the latter in relation to the former, contrary to what it had been able to do when dealing with essence, through the distinction between act and potency, as applied to form-matter (See § 5.4). Also from this point of view, then, Aquinas' thought performed a synthesis of the theories of the two great metaphysical thinkers of classical times, Plato and Aristotle and, more generally, of the two great tendencies of metaphysics of any age: essentialism and existentialism, idealism and empiricism.

We will come back later to these issues, delving deeper, on the one hand, into the empiricist ontology of logical neo-Positivism (See § 4.3), and, on the other, into the Thomist doctrine of being-as-act (See § 5.5.2).

1.7 Summary of Chapter One



In this first chapter we gave a short historical account of the birth of modern science, of its developments until the end of the 19th century and of the influence that these events had on the classical philosophy of nature. In § 1.1 we briefly described the birth of modern scientific thought, during the 16th and 17th centuries, as the reformulation of the Greek physical-mathematical approach to the study of nature, grounded on new epistemological bases. The beginning of modern science, in particular, coincided with the development of Galileo's *new mathematical-experimental approach* (nature should not be observed, but interrogated through our mathematical explanatory hypotheses) and *the birth of infinitesimal calculus* by Newton and Leibniz, with the solution of the problem of the square of the circle, at which the development of the Greek approach to the physical-mathematical study of nature had stopped.

In § 1.2 we recalled the modern obfuscation of the philosophy of nature, due to the incorrect interpretation of modern science as a new metaphysics of natural entities, that is, as a new philosophy of nature, rather than as something very different. From this derived the success of *scientism* in its various forms, particularly the Illuministic and the Positivist.

In § 1.3 we briefly referred to the meteorite of the philosophy of nature of Hegelian inspiration and to its role in the birth of the *sciences of Man* (human sciences) as distinct from the *sciences of nature*.

In § 1.4 we tried to identify the roots of *Illuministic scientism* that, opposed to *philosophical rationalism*, gave birth to that harmful ideological juxtaposition between the «two cultures», the scientific and the humanistic, the consequences of which are still felt today. First, in § 1.4.1, we briefly recalled how the scientific apex of the Illuministic program was reached with the axiomatization of mathematical analysis and of the concept of 'limit' between the end of the 18th century and the first half of the 19th century. Then, in § 1.4.2, we identified the origins of the modern scientific ideology in the claim to build a natural science on the presumed *self-evidence* of the laws of Newtonian dynamics, following the model of the postulates of Euclidean geometry. A typical example of this ideology was the hypothesis of *absolute mechanist determinism of the physical realm*, laid out in Laplace's work, and the exaltation of Newtonian physics as an «absolute science». That is, as an *apodictic* (= the consistency of the demonstration linked to the truth of the axioms) rather than as a *hypothetical*

(= the consistency of the demonstration independent from the truth of the axioms) form of knowledge, which would replace metaphysics and, more generally, philosophy. From this originated the program of Kant's critical philosophy, as a program of justification of the universality and necessity of «pure» mathematics and physics. According to Kant, these are not based on the presumed knowable character of the nature of things, but on the self-evident, absolute character of the formal principles of the mathematical and physical science, which is a product of self-awareness, of the meta-subjective «transcendental-I». According to Kant, therefore, the two queen sciences stand in opposition to the contradictory and sterile traditional metaphysics, at this point seen only as an unfounded dogmatism. But, as in any historical parabola, the apex of the Illuministic exaltation of modern science also marked the beginning of its decline, which was linked to the so-called «crisis of the foundations», first of all of the mathematical and epistemological basis of modern science: geometry and mathematical analysis.

In § 1.5 we therefore explained at some length the *crisis of the foundations of mathematics* of the 19th century. The defeat of the scientific myth started with the axiomatization of mathematics (See 1.5.1) and the resulting affirmation of the *hypothetical*, rather than apodictic, *character* of scientific theories. The first step was the creation of non-Euclidean geometries by Bolyai e Lobachevski (See § 1.5.1.1). From this moment on, the queen of modern apodictic sciences, geometry, was downgraded to a hypothetical science, and evidence ceased being considered as the foundation of the (now presumed) absolute truth of mathematical postulates, which undermined the Kantian paradigm. The enlargement of this approach to the new non-Euclidean geometries came with the first axiomatization of geometry by B. Riemann (See § 1.5.1.2). Because of the complete formalization of this approach, geometry, and mathematics more generally, came to resemble more and more formal logic, particularly due to the replacement of the notion of truth with that of consistency as a criterion of validity of mathematical statements. Riemann gave, finally, a Euclidean model of his elliptic geometry, as a geometry of curved space—which Beltrami extended to Lobachevski's hyperbolic geometry—thus giving, at the same time, a unified, almost intuitive model of the new geometries, as well as a direct proof of their consistency, which then proved equivalent to that of Euclidean geometry.

The deep overturning of the convictions that had guided mathematicians for thousands of years led to the need for a deep reflection on the foundations of mathematics. At that point, one could not rely on evidence any longer, and all that had appeared as foregone until then (that is, evident) now had to undergo a careful rational examination (See §

1.5.2). Within this systematic effort, the problem of antinomies emerged. In his non-axiomatic theory of sets, Cantor was the first to face the paradoxical nature of all-inclusive notions such as that of *universal set* or *maximal set* (See § 1.5.2.1). Indeed, particularly after the axiomatization of the notion of limit by Weierstrass, the notion of «set» had started to be considered as a general one, common to all mathematical concepts, from those in geometry (figure = set of points), to those in arithmetic (number = set of units) and, therefore, to analysis itself (function = relation between ordered sets).

B. Russell's discovery of a similar antinomy (the antinomy of *total class*) in Frege's *logician* attempt to build a theory of the foundations of mathematics based on the logical notion of *class*, rather than of set, had a much greater cultural echo. The importance of the discovery was due to the fact that the antinomy was of a *logical*, other than mathematical kind; also, it was a *syntactic*, rather than semantic antinomy (it was based on the syntactic notion of «membership», rather than on the semantic notion of «truth»), contrary to the other logical antinomies known in the history of Western thought, starting from the famous «liar's paradox» in Greek thought. One of the central reasons for abandoning the substance of classical metaphysical thought, being and truth, thus disappeared: the presumed certainty that logical antinomies were only of a semantic nature.

Finally, in § 1.6, by way of conclusion, we analyzed the metaphysical value of antinomies and the limits of the axiomatic solution of these, in the foundations of the contemporary theories of sets and classes. By limiting, in each theory of foundations, the collection of constructible sets, and by entrusting to specific axioms the existence of the other sets that create problems—as it is done in the axiomatic theories of sets after Cantor (for example, in the ZF theory of Zermelo–Fränkel, or in the NGB theory of Von Neumann–Gödel–Bernays)—one falls into excessive existentialism, just as Cantor's and Frege's constructive approaches were flawed by an excessive essentialism. Hence, the usefulness of the Thomistic doctrine of being-as-act that, by distinguishing, within being, between the being of essence (entity) and the being of existence (common being), included both of them, so that being and essence determine each other, similarly to act and potency in the construction of the existence and entity of any individual.

1.8 Bibliography of Chapter One



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2. Scientific Revolutions of the 20th Century

The conceptual revolutions of the “new” physics — thermodynamics, relativity, quantum theory, complexity — and the paradigm changes that they imply for the epistemology of Humean-Kantian modern science, in particular with respect to the “«transcendentalist”» (logical) foundation of the principle of causality and the determinism-aleatority dichotomy in the study of physical processes.

2.1 The Birth of «new physics»

Unlike what we have just done with the revolutions in mathematics at the end of the 1800s, to which we shall not return in this work, the discoveries of the «new physics» will constitute, carefully examined from the point of view of the philosophy of nature, the subject of the Third Part of this work. Thus, in this section, we shall confine ourselves to recall them briefly, in order to complete the picture of the scientific revolution developed in the last one hundred and fifty years, which has radically modified centuries-old conceptions and, in the case of mathematics, millenary ones.

2.2 Reconsideration of Newtonian Physics

Pretension of the omnicomprehensiveness of Newtonian mechanics

The point of departure is the *pretension of the omnicomprehensiveness* of Newtonian mathematical physics regarding the study of *all* physical phenomena that could be the subject of study of modern natural sciences. A pretension that is based on the supposed *self-evidence*, and thus on the absolute *truth*, of the three *laws of dynamics*³⁸, very similar to the one supposed for the postulates of Euclidean geometry³⁹. Here is, then, a quota-

³⁸ We recall them summarily: 1) *principle of inertia*; 2) *principle of proportionality of the force to the product mass $\times$ acceleration* ($\mathbf{F} = m \times \mathbf{a}$, a fundamental law of dynamics); 3) *principle of action and reaction*. The mathematical character of such principles is derived from the fact that they are a *necessary and sufficient condition* for the applicability of calculus to the study of dynamics. Indeed, it is only by stopping, according to the order of derivation, at the second

tion from Newton's optics, where these ingredients of his theory clearly emerge:



The phenomena of nature teach us that such principles (= the three laws of dynamics) really exist, even though their cause has not been investigated yet. The *laws* of which we are talking about are then *evident* and only their *causes* could be said to be *obscure*. The Aristotelians and the Scholastics, instead, had considered as obscure qualities not only certain properties somehow already known but rather other ones that they thought to be hidden in bodies and constituted the unknown reason of the visible aspects. However, both the gravitational and the electrical and magnetic force would belong to this category only if we would suppose that they derive from the intimate (to us unknown) nature of things, that is to say, from an unthinkable and unknowable substratum. Such «qualities» are undoubtedly an obstacle for scientific progress and are thus rightly rejected by *modern inquiry*. *The belief in specific essences of things endowed with specific hidden forces*, and thus adequate to produce certain specific sensible effects, is completely empty and meaningless⁴⁰. Instead, to derive from phenomena *two*

derivatives (see *derivative**), not supposing a variation in the acceleration, that calculus works. For this reason, it is necessary for the system to be isolated, that is, that the principle of inertia could hold, or that, in any case, by appropriately acting to reduce the complexity of the studied system, the situation could be realistically reduced to such a fundamental paradigm.

³⁹ It is historically proven that Newton — precisely to attack the *hypothetical* character that Galileo, under the advice of Cardinal Bellarmine, accepted to admit for his mathematical science of nature, on the occasion of his first accusation by the Inquisition in 1616, which however was not enough to avoid his condemnation in 1633, based on the erroneous conviction that in his *Dialogue* he wanted to *demonstrate* the Copernican theory (Drake 1990, 183; 214 f.) — had instead intended to provide an absolutely apodictic character (*hypotheses non fingo*) to his *Philosophiæ naturalis principia mathematica* of 1687, entirely analogous to that which, at that time, could have been ascribed to Euclidean geometry (Cf. on this point Boyer 1968, 450 f.; Koyré 1965, 31–43).

⁴⁰ It is worth noting — always to emphasize the points of connection, that, at the level of the metaphysical foundations, these points can be highlighted between modern science and genuine Aristotelian-Thomistic philosophy of nature — that on this matter Thomas Aquinas had been even more drastic with certain «Aristotelians» of his time than, more than four centuries later, Newton was in this and other texts — and before him Galileo — with the descendants of those Aristotelians. Indeed, Thomas not only defines as «empty» and «meaningless», but even as «ridiculous» the idea of understanding the «nature» or «specific essence» of bodies as a «hidden force» that produces particular mechanical effects. Obviously, in the 1200's these Neoplatonic contaminations of genuine Aristotelianism, which later, unfortunately — also because the teaching of Thomas was confined to mere theology — gained ground in the Renaissance, contributing in this way to the crisis of credibility that had afflicted the Aristotelian-Thomistic philosophy of nature up to our days, were quite alive. Thomas, literally, affirms: «Ridiculous are those Aristotelians (*deridendi sunt*) that, intending to correct Aristotle on this point, pretend to define nature as something *absolute* (not relative to the action of the agent-causes upon matter, and thus as an *active* and not *passive* principle, as in fact it really is, *No.4*.) claiming

or three general principles of movement, and then explain how out of them, as clear and evident presuppositions, there should follow all the properties and manifestations of *all material things*, would already be an important progress of scientific knowledge, even though the *causes* of such principles would remain completely unknown to us (Newton 1704, 326).

Hypothetical
character of
Newtonian physics

After the discovery of non-Euclidean geometries, it is clear that also the pretension of the apodictic nature of the Newtonian laws has to be replaced with a more sound and objective *hypothetical* one. This revision was not only linked to the change of cultural atmosphere, but also to a series of fundamental scientific discoveries in the field of physics, which have greatly enlarged the universe of the physical sciences and dynamics itself.

Partial character of
Newtonian physics,
limited to the study
of macroscopic
phenomena

While waiting to face these subjects more deeply in the Third Part of our work, let us limit ourselves to highlighting the more exciting discoveries which have radically modified the view of physical reality, under the light of modern sciences, by considerably enlarging its contents. Hereinafter, Newtonian mechanics will not be «*the mechanics*» anymore, but it will identify itself with a subset of it, «*classical mechanics*», which has a well-defined domain of application — the so-called *macroscopic* mechanical phenomena (those of ordinary experience, so to say). In the *mesoscopic* field (molecular aggregates) the principles of *thermodynamics* hold (*linear* and *non-linear*). In the *microscopic* field (from the molecule, to the atom, and then to the sub-atomic and to the sub-nuclear) the principles of *quantum mechanics* and *restricted relativity* (*quantum electrodynamics* and *quantum chromodynamics*) hold. In the *megascopic* field, at the level of phenomena at a cosmic scale, the principles of *relativistic mechanics* (*general relativity*) hold.

As we can see, the destiny of Newtonian mechanics in the history of modern scientific thought is very similar to that of its mathematical paradigm: the *Euclidean geometry*. The birth of non-Euclidean geometry has not made Euclidean geometry «obsolete». It has only reduced the importance of the false and non-scientific ideological pretension that it was the *only* geometry that there was and thus was an *apodictic* theory, based on absolutely *true* postulates. It is instead a *hypothetical theory*, thus with well-defined limitations of truth, validity and applicability. In the same way, the birth of new physical theories with new axioms, new laws irreducible to those of classical mechanics, has only meant a revision of the false pretensions

that nature is a force hidden in things (*vis insita in rebus*) or something similar» (Thomas Aquinas, *In Phys.*, II, i, 145). For the Aristotelian-Thomistic explanation of the nature of a determined species of bodies as *passive principle*, as a result of a determined physical causality inside the cosmos, cf. *infra* § 5.4.4 and § 5.5.2.

of the absoluteness of Newtonian mechanics, but it has not meant, in any way, its «obsoleteness» by the «new» physics.

The most innovative aspects of these new theories with respect to classical mechanics can be synthesized in this way:

Innovations:

1. Thermodynamics and arrow of time
 - ◆ The birth and development of thermodynamics as a statistical theory of molecular aggregates that introduce a temporal *irreversibility* in such physical phenomena. An irreversibility that, in thermodynamics systems, goes in the direction of an increase of *disorder* inside the system.
2. Quantum mechanics and indeterminacy
 - ◆ The birth and development of quantum mechanics with its principles of *quantitization, indeterminacy, exclusion, complementarity*, which have no equivalents in classical mechanics.
3. Development of restricted relativity
 - ◆ The birth and development of the *special theory of relativity*, with its fundamental law $E = mc^2$ that shows the reciprocal transformability between mass and energy for bodies accelerated to velocities close to the limit-velocity of electromagnetic radiation (light). This is an idea that, combined with the principles of quantum mechanics, has shown to be quite fruitful in the domain of the physics of microscopic, quantum systems, under the form of *quantum electrodynamics* and *quantum chromodynamics* (Cf. Third Part).
4. Development of general relativity
 - ◆ The *general theory of relativity* which provides an explanation of the *force of gravity* G that Newtonian physics did not have, even if Newton had described its mathematical form, by means of the *law of universal gravitation*, for the first time in the history of humanity:

$$G = g \frac{m_1 \cdot m_2}{d^2}$$

where g is the constant of universal gravitation, m_1 and m_2 the masses of the bodies among which force is exerted, and d their distance. In general relativity, this force, for the first time in the history of humanity, finds a mathematical explanation, in the domain of a non-Euclidean geometry of a Riemannian type, as a curving of space-time due to the action of the masses of the bodies that exert this force. So, in this scheme, the gravitational force would have a completely different nature with respect to, for example, the electromagnetic force. That is, it would not be linked to the interaction among bodies and to an exchange of matter (mass-energy) among them, but to the capacity of the masses of bodies of modifying the structure of the surrounding space-time.

5. Development of the theory of complex systems

- ◆ The *new science of complexity*. The whole of modern physics, statistical thermodynamics, quantum and relativity included, is colored by a plain *reductionist* theory. This «reductionism» can be displayed both in a *synchronic* and *diachronic* sense. It is shown in a *synchronic* sense if, considering a physical system in its static nature, one pretends that the final state of a dynamical system could always be reduced to the sum of the dynamic of the constituent simple elements. It is shown in a *diachronic* sense if one pretends that the final state of a dynamic system is always unambiguously determined by its initial conditions. This double reductionist principle is countered in its absolutist pretensions by the study of *irreducibly non-linear* physical systems or, precisely, *complex systems*. The principles of this conceptual revolution were introduced at the end of the past century by Henri Poincaré's discovery of the *intrinsically unpredictable* behavior of «three-bodies» dynamical systems. From the second half of the '60s, due to the computational simulation of the dynamic of systems of such a sort, we have moved on to the systematic study of such systems. They constitute, in fact, *dynamic models** of real systems, such as physical, chemical, biological and neural, but also social and economical systems: real systems that have thus far escaped systematically — due to the reductionism that pervaded modern science — the mathematical methods of classical dynamics, which had their paradigm in the *linearity* of the fundamental equations of Newtonian mechanics.

2.3 Thermodynamics

2.3.1 Classical thermodynamics

Reversible character of all the equations of motion in Newtonian physics

One of the fundamental characteristics of the Newtonian approach to mechanics was the temporal *reversibility* of the equations of motion. In other words, in the equations of motion of mechanics, given a process which has taken the system in time $t_0 \rightarrow t_1$ from A to B , it is enough to reverse the sense of the direction of the velocity to bring back the system from B to A , that is, *exactly* where it was at t_0 . The *temporal irreversibility of phenomena* — that is, the fact that it is impossible to go back in time —, basic for our experience, but also, as Einstein observed, basic for the possibility of speaking, more in general, of physical systems that *exchange information*, is something which has no sense for the laws of Newtonian mechanics. Being perfectly deterministic systems and, hence, reversible according to the model of geometrical systems (from the initial conditions it is possible to pass deductively to the final ones and vice versa), an increase or diminution of the information in the modifications that characterize them does not exist. We could claim that

the heart of the eighteenth-century's contrast between *naturalism* and *historicism* can be found at this point: the time of *nature*, the time enclosed in the equations of Newtonian mechanics, is not the time of history, the time of experience and of human affairs.

One can understand, then, the shock produced in the scientific mentality and in the philosophical culture of the time, by the discovery that also in physics, more precisely in *statistical mechanics*, — that branch of mechanics that studies the behavior of physical systems composed of a great number of particles according to the mathematical principles of the *theory of probability* — irreversible phenomena occur. More precisely, those phenomena occur every time that energetic transformations are verified: the transformation of energy into work or the transformation of a certain form of energy (e.g. mechanic) into another form of energy (e.g. electric: think about the work of a dynamo), since these transformations are always associated with the production of a degraded form of energy, not transformable anymore into work, that is, into *heat*.

2.3.2 Statistical thermodynamics

Development by
Boltzmann of
statistical
mechanics

This very genuine cultural revolution — the first of a long series that will take place in physics between the end of the XIX century and the first thirty years of the XX century — is essentially the work of an Austrian physicist, Ludwig Boltzmann (1844-1906), one of the creators of the kinetic theory of gases. Due to him and to his famous equation, *Boltzmann's equation*, the molecular theory of matter — the so-called *mesoscopic* level of the study of matter — has rightly entered in to the study of physics. His equation, based on the hypothesis of *molecular chaos*, defines, indeed, the behavior of a collection of particles on conditions of non-equilibrium. It claims, in substance, that different factors contribute to the variation of a function that expresses the probability of finding a particle in a particular unitary volume of the *space of phases**, factors such as external forces, the effects of diffusion, and the collision between particles.

Application of the
principles of
statistical
mechanics to
thermodynamics

By applying these principles to the study of *thermo-dynamical* phenomena, that is, dynamic phenomena where exchanges of energy take place, Boltzmann gave birth to a new branch of physical science, the *statistical thermodynamics* of equilibrium systems or *linear thermodynamics*. What determined the result (which is the title of this sub-section), is the application of the above mentioned principles of statistical mechanics to a particular concept of classical thermodynamics, introduced at that time by the German physicist Rudolf Emanuel Clausius (1822-1888): the concept of *entropy*.

2.3.3 The notion of entropy

However, in order to properly illustrate the concept of entropy, it is necessary to take a look at classical thermodynamics — that is, before its Boltzmannian interpretation within statistical mechanics — and to the enunciation within it of the first two principles of thermodynamics. The *first principle* of thermodynamics is nothing more than an expression of the more general *principle of conservation of energy* in the domain of the thermodynamic systems:

First principle of thermodynamics: conservation of energy

First principle of thermodynamics: the quantity of heat Q that a thermodynamical system exchanges with the outside is obtained by the sum of the work L that it exerts upon the outside (or that, from the outside, is exerted upon the system) and by the variation of the internal energy U of the system itself⁴¹.

The second principle (presented for the first time by Clausius himself) in its classical formulation already introduced a sort of irreversibility. In its formulation it indeed asserts that:

Second principle of thermodynamics in its classical version (Clausius)

Second principle of thermodynamics (Clausius' postulate): if heat flows by conduction from a body A to a colder body B, then it is impossible to operate a transformation whose only result is that of passing heat in the opposite sense, that is, from B to A⁴².

⁴¹ In more intuitive terms, given a cyclical thermo-dynamical transformation in which the system returns to the initial state (e.g., the complete cycle of a steam machine) the quantity of work exerted by the system L (e.g., the complete circuit of the piston of our steam engine), paid in terms of the decrease of internal energy U (e.g., the decrease of pressure of the steam in the boiler), must be compensated by a quantity Q of heat introduced in the system (e.g., through the combustion in the boiler's furnace), according to the relation $L = JQ$, where J is one unit of heat or calorie expressed in energetic units (joules). Obviously, this is linked to the fact that heat is a form of energy; indeed it is, for the second principle, the most universal form of energy, to which all the others can be reduced.

⁴² This property of heat — which in practice claims that it is impossible to make all the heat emitted by a source «come back» to the source itself — is extremely important for the operating of all the cooling systems, since it is related to the other property of heat regarding space: that of preferring, among all the directions of motion, the centrifugal ones. In other terms, heat renders the space around it *anisotropic** for all the dynamical phenomena associated with it; indeed, it makes certain directions of motion *preferential* with respect to others (because of this, for instance, in all the heat-dispersing systems, of air refrigeration of a machine and/or of heating of an environment, the edges of the ir-radiant surface are multiplied). We shall see in the Second and Third Part how much this property of heat, empirically well-known also to the ancients, was important for Aristotle, a careful observer of nature (in particular biology), since the lives of organisms were closely associated with exchanges of heat with the environment. For him it was exactly the irreducible presence of heat in all (not only organic) physical nature, with its property/capacity to make the space anisotropic, the *physical* reason to reject the Platonic-

In the other classical, equivalent, formulation of the second principle, due to Lord William Thomson Kelvin (1824-1907), heat is considered a degraded form of energy, for a *complete transformation* of heat into work is impossible. That is, in Lord Kelvin's formulation of the second principle, it is asserted that:

Second principle of thermodynamics in its classical version (Kelvin)

Second principle of thermodynamics (Kelvin's postulate): it is impossible to bring about a transformation, the only result of which is the transformation of heat drawn from a unique source with uniform temperature into work⁴³.

So, if one takes entropy S in Clausius' definition, as a quantity that measures the relation between an infinitesimal variation of the quantity of heat dQ and the absolute temperature T :

Definition of the notion of entropy

$$S = \int \frac{dQ}{T}$$

it is clear that the quantity S will not vary in each reversible transformation, i.e., in each transformation in which the system will receive enough heat from a second source, in order to return to the initial conditions. Vice-versa, in an isolated thermo-dynamical system, left to its own, where there is no addition of heat from the outside of the system, the quantity S will irreversibly increase.

Pythagorean use of geometry in the study of natural entities. And, at the same time, it was the reason for asserting, on purely physical bases, the presence of a *final causality* which introduced certain irreversibilities and certain preferential directions of the motion of terrestrial physical bodies. It was precisely this property which, according to Aristotle, differentiated the motion of terrestrial bodies from that of the celestial ones. The motion of celestial bodies, due to their presumed circularity, were *reversible*, always equal to themselves, that is, endowed with fundamental *symmetries*, and hence mathematically representable and predictable, as was already done in ancient times, i.e., Assyrian-Babylonian and Egyptian astronomy.

⁴³ Or, «it is not possible to construct a machine, the only result of which consists in producing work by absorbing heat from a unique source». For example, part of the heat and energy produced by a steam-machine will not be transformed into work, but will be dispersed under the form of thermal irradiation, or transformed into friction and into the consequent irreversible degradation of the mechanical means, etc. In other terms: no real physical process is completely *reversible*. In every physical transformation (of energy into work or into other forms of energy) there is always some production of heat, or of degraded energy, no longer transformable into work.

Application to thermodynamics of the principles of statistical mechanics for the study of the mesoscopic level of the structure of matter

All this has received a more than convincing explanation through the statistical interpretation of thermodynamics and of its laws by Boltzmann. In such an interpretation, in the *mesoscopic* analysis of the physical systems — that is, at the level of the behavior of the molecular and/or atomic aggregates —, the *variables of state* typical of thermo-dynamical systems, *volume* V , *pressure* P and *temperature* T , assume a precise statistical meaning in terms of statistical magnitudes associated with the motion of the extremely great number of particles that compound the system. So, in the case of a gas in a recipient, for example, pressure P will be proportional to the average number of collisions of the particles over the walls of the recipient, as also temperature T will be proportional to the average velocity with which the gas particles move. By increasing temperature T , the average velocity will be increased and, hence, pressure P will likewise increase, because it will also increase the average number of collisions that the molecules of gas will have with those that form the internal surface of the recipient.

If, instead, an external source of heat does not exist, through their collisions the particles that compound the system will progressively and equally divide the kinetic energy available until they completely degrade the total energy U of the system⁴⁴ that will reach, in that way, the state of minimal energy or «equilibrium state» that will be constituted by the *maximal* possible configurations of the system.

Statistical measure of entropy.

From this, the *statistical measure of entropy* defined by Boltzmann as:

$$S = k \log \pi$$

where k is Boltzmann's constant⁴⁵ and p the probability of one state of the system, defined as the number of possible configurations that give rise to the thermodynamic state itself.

In the case of the Boltzmannian interpretation, what is asserted is that an isolated thermo-dynamical system goes in the direction of an increase of the *global disorder of the system*⁴⁶ and, given the irreversible character of

⁴⁴ That is, inside U the *free* energy $F = U - TS$ (where T is the absolute temperature) will diminish, i.e., the heat that can be transformed into work. Such energy depends on the fact of not being equally distributed among the particles of the system, that is, on the presence of a differential of potential, in our case, on the presence of a clear distinction between «hot» and «cold» particles, dynamically, «fast» and «slow» particles. On the basis of the above mentioned F 's formula, it seems indeed clear that, at a constant absolute temperature T , the more S increases, the more F diminishes, that is, the part of non-degraded energy of total energy U diminishes.

⁴⁵ This is one of the *fundamental constants of nature* and has the value $k = 1.380662 \approx 10^{-23} \text{ J K}^{-1}$, it has thus the dimensions of a minimal energy in relation to the quantity of heat.

⁴⁶ We need to make clear what the term «disorder» means. Let us consider the case of the classical experiment of the two cylinders perfectly thermally isolated from the out-

the process, the disordered final state — or state of maximal entropy, or of minimal free energy, or of «thermal death» of the system — will be defined as the most probable state towards which *each thermodynamic system will inevitably tend*. In the more general language of statistical mechanics, such a final state — or *attractor** of dynamic — will be that one towards which each macroscopical physical system composed by a very great number of particles (molecules or atoms) will tend, provided that the system has enough time to reach it (= *thermo-dynamical limit*). From this, the formulation of the *second principle of thermodynamics* in terms of statistical mechanics (Boltzmann's formulation):

Second principle of thermodynamics in Boltzmann's version

Second principle of thermodynamics: every isolated physical system tends to transform itself so that its own entropy, that is, the disorder of its components, increases.

It is clear that, in this more general formulation, the second principle approaches the realistic meaning of the same principles of classical mechanics, not only by introducing an irreversibility into them, but by indicating as a *universal law* of all the physical systems — the physical universe included, to the extent that it is understood as an isolated physical system — an irreversible destiny of disorder and «thermal death».

Philosophical consequence: end of the Enlightenment myth of progress

However, at the level of cultural mentality, there could not have been a more profound shock than the statistical formulation of the second principle of thermodynamics, in that it destroyed, all at once, the whole scientific myth of the illuminist idea of progress, as the destiny of the *physical*, before than human, *world*, a destiny grounded on the optimism of reason «enlightened» by science. The destiny of the universe, instead

side, but in communication between them through a conduct, and containing a gas of molecules, one cylinder at high temperature and the other at low temperature. In that case, at the beginning of the process, the number of the possible configurations of motion of the particles of the total system was limited. So, two sets of equations were enough to represent the only two possible configurations of motion of the system: «fast» (hot) particles and «slow» (cold) particles. In this state, the global system will possess the maximum of *order* (only two possible configurations) and the maximum of free energy, given by the difference of thermal potential between the two recipients. But, as soon as, due to the communication between the two recipients, the collisions between the hot particles and the cold particles increase, the «fast» and «slow» particles will tend more and more to distribute themselves equally among the two recipients that will tend to reach the same temperature, T , the average of the two initial ones. In that way, the free energy F of the system will diminish, the available configurations will increase, at most they will become one for each one of the infinite number of particles that compose the system. From an ordered system with a low degree of freedom, we moved on to a system of a very high degree of freedom, hence, to a disordered, casual system. The maximum of entropy will be achieved when the system will reach the equilibrium state and the particles will distribute themselves equally and casually (probability $\frac{1}{2}$) between the two containers.

of being that of an indefinite progress, proclaimed by the men of the enlightenment and, in the biological sciences, by the theorists of evolutionism such as Lamarck and Spencer, is that of disorder and thermal death!

Nietzsche's
example

Is it not a matter of chance that the most well-known prophet of the *crisis of the classical and modern rationalist optimism*, Friederich Nietzsche, had studied carefully and precisely the boltzmannian thermodynamics. And perhaps, it is not a matter chance that the three most lucid «prophets» of the «crisis of the European sciences» — as Husserl will later define it (Husserl 1954), as a crisis of an epoch itself, that of the rationalist and scientific instrumentalization of science — Cantor, Boltzmann, and Nietzsche himself, had all died in a condition of abandonment and terrible solitude, and with serious mental diseases. We are in the presence of a human and cultural drama, that these great men lived in their own lives before others, because they understood before others what was happening. It is the drama of the absence of absolute certainties — neither those «metaphysical» ones of the classic age, nor those «scientific» ones of the modern age — that today, unlike then, penetrates a large portion of contemporary culture and mentality.

Coming back to the birth of modern thermodynamics — just to complete the picture with *the third principle of thermodynamics* — if we introduce considerations concerning quantum mechanics (sub-atomical physics), that is, if we consider the equilibrium state of an atom — or a fundamental state of it —, given that such a state is constituted by *only one possible configuration* — all the electrons are at the lowest level of energy of those allowed — such an equilibrium state will be characterized by a state of *minimal entropy* or of *maximal disorder*. Now, all the matter at absolute zero (-273°C or 0°K) *crystallizes*, so that the atoms that compose it go to the fundamental state. But, at the same time, as soon as temperature descends, the variation of entropy ΔS also diminishes in a much greater proportion. And thus one could define the *third principle of thermodynamics* in harmony with the two above mentioned principles. This principle was formulated for the first time, in 1906, by the German physico-chemist Walther Herman Nernst (1864-1941), who received the Nobel Prize in chemistry in 1920 for this discovery.

Third principle of
thermodynamics

Third principle of thermodynamics. At absolute zero the difference of entropy ΔS among all the states of a system that are in thermal equilibrium is null. In other words, what is claimed is the unreachability of absolute zero by a physical system, at least in a finite time.

But the last chapter in the recent history of modern thermodynamics is not that of Nernst's theorem, which is at the basis of the mentioned

third principle. The last chapter concerns the «non-isolated» or *open* thermodynamic systems (von Bertalanffy 1965) — that is, systems that exchange energy and matter with the outside, as in the majority of chemical systems and of the totality of living systems.

2.3.4 Stability independent of equilibrium

Thermodynamics of stable systems far from equilibrium: thermodynamics of the living organisms

The systems we are talking about are systems that, *by consuming «energy» from the environment, produce order, «information» at their inside*, thus opposing themselves, however limitedly and provisionally, to the destiny of the energetic decay common to all compound physical bodies. In general, these systems, which exist in nature, are characterized by a high *non-linearity*, unlike the thermo-dynamical systems studied by the classic boltzmannian statistical approach, which are rarefied systems, like the rather low-density gases. The study of stabilities *far from the thermo-dynamical equilibrium* in these systems reveals to science new forms of dynamical order, completely different from the *geometrical* one, something «dead» in terms of matter crystallized at absolute zero, which, however, the third principle of thermodynamics allows to exist, even at the 0 energy state. Ilya Prigogine (1917-), Nobel Prize winner in chemistry in 1977, gave the name of *dissipative structures* to these forms of organization of matter at the *mesoscopic* level. These are structures that organize and maintain their internal order, independently of the initial conditions, dissipating heat and then consuming energy from the environment (Prigogine & Stengers 1979; Prigogine 1981).

Extraordinary confirmation of these principles in the recent discoveries about the human genome

Certainly, these discoveries will allow us to write new and more convincing pages in the history of the notion of *evolution* in biology, by finding in the principles of *dynamical instability* (Cf. § 2.6.1) physico-molecular mechanisms of the variation of the genetic equipment of organisms which are much more efficacious and physically reliable than the much less realistic stochastic mechanism of casual mutation, typical of the Darwinist approach to natural selection (Kauffman 1992; 1995; 2000). The recent discovery, dated February 2001, of the limited number of genes (approximately 30.000) in the human genome and, hence, of the prevailing role that the non-linear dynamics of molecular synthesis of proteins play in the organization of living beings, confirm in a clamorous way the principles just mentioned. They definitely discredit what could still remain of the old «linear» model of conceiving biological information. This is the model which asserted that all the information of the organism is in the genes, in the initial conditions of the dynamic process of ontogenesis, as if genes were similar to the «programs» of our computers.

On the other hand, these non-linear models will be the more scientifically meaningful the more a *new mathematical formalism* can be developed, which could be capable of dealing with non-linear systems as such. But we shall briefly return to these subjects, from the point of view of physical sciences, in § 2.6, where we shall refer to *complex systems*, and return to them more extensively in Part Four, from the point of view of biological sciences and, in Part Five, from the point of view of cognitive sciences and of the neural basis of mental activities.

2.4 Quantum mechanics

Study of the microscopic level of the structures of matter

A further reconsideration of classical mechanics has occurred at the level of the study of *microscopic* structures of matter of which all physical beings are made. Such structures can be found when one goes beyond the mesoscopic level, «inside» the structure of atoms and molecules, for magnitudes that are in the order of 10^{-8} cm (a hundred-millionth of a centimeter: the diameter of an hydrogen atom) and smaller.

2.4.1 The principle of quantization

The revolutionary discovery of Planck's principle of quantization and its first confirmations

This conceptual revolution has a date that coincides with that of the beginning of the XX century — or, more exactly, with the end of the XIX century: 1900. In that year, more exactly on December the 14th, speaking at a meeting of the German Society of Physics, Max Planck (1858-1947) claimed that it was possible to free ourselves from the paradoxes of the classical theory of the emission-absorption of light by matter, if one accepted that *radiant energy could exist only under the form of discrete packets* that he defined as *quanta of light*.

Einstein's discovery of the photoelectric effect

Such an hypothesis was confirmed in 1905 by the discovery of the *photoelectric effect* by Albert Einstein (1879-1955). Such an effect consists in the emission of electrons by metallic surfaces that are bombarded with violet and ultraviolet light. The effect in question could be explained only by admitting the quantum nature of electromagnetic radiation, that is, the existence of *photons* or elementary quanta of electromagnetic energy. On the other hand, the existence of photons could be directly derived, in the special theory of relativity, from the principle of the finite velocity of the propagation of electromagnetic waves (light). In any case, Einstein received the Nobel Prize for physics in 1921 for *this* discovery (which confirmed Planck's theory) and *not* for the theory of relativity!

The discovery of Compton's effect

Another success that determined the definitive confirmation of Planck's hypothesis was the discovery of the so-called *Compton effect*,

from the name of its discoverer, the American physicist Arthur Holly Compton (1892-1962), Nobel Prize winner for physics in 1927. Such an effect consists in the diffusion of X-rays by the electrons of some specific matter (paraffin). The revolutionary discovery is that such phenomenon can be explained only if one interprets the collision of electromagnetic radiation at high frequency (x-rays), as an elastic collision of two particles. That is, as if light were composed of particles without mass, precisely Planck's «quanta of light», to which Compton, for the first time, gave the name of *photons*.

2.4.2 Bohr's atom

The semi-classical
model of atom
created by Bohr

The picture of confirmations of Planck's discoveries was completed when, in 1913, the Danish physicist Niels Bohr (1885-1962) applied this quantum hypothesis to the model of the atom endowed with an internal structure, discovered by the New Zealand physicist Ernest Rutherford (1871-1937). That is, an atom which is no more the simple, indivisible, little sphere of homogeneous matter of the ancient and modern atomists, but which is endowed with a *structure*, it is itself a compound. That is, it is constituted:

- ◆ By a center of mass, the nucleus, where almost the whole mass of the atom is concentrated, and which is endowed with a positive electric charge;
- ◆ By a group of particles, the electrons, which rotating around the nucleus, as the planets around the sun, have a much smaller global mass than the nucleus, and are endowed with a negative charge.

Bohr's fundamental contribution was that of supposing that these «orbitals» were *discrete*. That is, unlike the planetary system, only certain orbits were admitted, so that the electrons could receive energy from the outside only at discrete packets, thus, for well defined values, and not in an uninterrupted way.

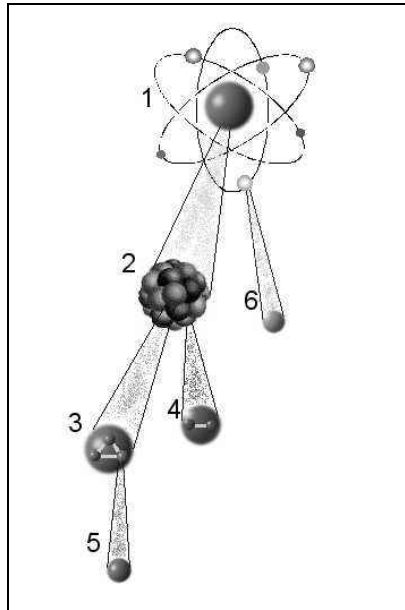


Figure 2-1. This image synthesizes the history of the modern notion of the structure of the «atom», beginning with the initial Bohr-Rutherford planetary model (1). In this notion, the nucleus was conceived as an elementary particle. Only later it was discovered that it had, in its turn, a structure composed of *protons and neutrons* (2). Each of them was composed, in their turn, by three *quarks* (3), while other inhabitants of the nucleus, the *gluons*, were composed of two *quarks* (4). Vice versa, *quarks* (5) and *electrons* (6) are today considered *elementary particles*, i.e., not composed. We shall deal with other parts of the history of the structure of the atom in the Third Part.

«Quantum»
character of the
orbitals of Bohr's
atom and first
successes:

In this way, each of them could «jump» to the higher level orbital, and could then return to their own place or «fundamental state» putting back (in a quantum way) the energy received, according to the typical *spectrum*, different for each type of atom and, hence, different for each type of material. Through the idea of the quantum orbitals, Bohr obtained three extraordinary results:

1. To give
coherence to
Rutherford's model

- ◆ *To give coherence to Rutherford's model.* Indeed, without the quantization of orbitals, one could not understand the reason why, when the electrons, which have an electric charge opposed to that of the nucleus, diminish their velocity, they do not «fall into the nucleus», attracted by the nucleus itself. Exactly like a satellite in orbit around the Earth which diminishes its velocity: it would begin to make increasingly narrow spiral-orbits around the Earth, until, captured by its gravitational force, it would fall down on it.

2. To give an explanation of the spectrum emitted by each material

- ◆ To give an at least preliminary explanation of why each type of atom is characterized by a specific *discrete spectrum of electromagnetic emission*, when the atom itself is «bombarded» from the outside by «disordered» energy (heat)⁴⁷. One could say that, in this way, the atom adds «order» to the energy that it has received, and re-emits it under the form of electromagnetic energy endowed with characteristic frequencies (colored light, in the spectrum of the visible), an order that provides to the physicist essential information about the atom's ordered internal structure.

3. To give an explanation of the Periodic Table of elements in chemistry

- ◆ To give an at least initial explanation of the *periodicity of the properties* of chemical elements, already discovered and described, but not yet explained, by the Russian chemist Dmitrii Mendelejev (1834-1907) with his famous *periodic table of the elements*.

With Bohr's discovery, which was only the first of a much more important series of other discoveries in nuclear and sub-nuclear physics, during the first thirty years of the XX century, discovery for which he obtained the Nobel Prize in physics in 1922, Planck's hypothesis obtained its most clamorous initial confirmation, after those of the photoelectric effect and those of the Compton-effect. What is more important is that Planck's discovery produced certain consequences which go far beyond what its own author suspected.

The revolutionary character of Planck's discovery: end of the myth of the infinite divisibility of matter.

It is important to have the revolutionary character of this new discovery clearly in mind. Through the systematic application of the notion of a «quantum» of action to the physics that studies the fundamental structures of atomic and sub-atomic matter, another principle of modern philosophy of nature was dismantled: that of the *infinite divisibility* of physical matter⁴⁸. A property that, instead, had led Descartes to establish

⁴⁷ An effect of this phenomenon, known to everybody, can be found in the lamps that contain within them, for example, neon or sodium. The gas of molecules is bombarded in these lamps by discharges of disordered electromagnetic energy (heat) —the alternating current of the line, even if its frequency is increased with respect to the usual 50Hz of the electric web, in order to avoid the visual effect of fluttering during the positive phase in which the atom receives the electric discharge under the form of discrete wave-packets, according to the quantum principle. So, the electrons «jump» to the superior orbital («level of energy» in Schrödinger's ondulatory model: § 2.4.6). During the negative phase in which the atom does not receive the discharge, it again emits the packet of energy that it has received during the positive phase. And it emits it again in an ordered way, with a determined wave-length according to the kind of atom, so that the emission acquires the typical «bluish» hue if it is a neon-lamp; «yellow», if it is a sodium-lamp. In the meantime, the electrons return to the fundamental state, to be ready to «jump again» to the subsequent emission of energy, and so on, at a quantity of one hundred times per second.

⁴⁸ It is clear that the quantum principle is applied above all to matter in its manifestations as *energy*. But, after the quantum mechanics application of the basic principle of restricted

an identification between physical matter and geometrical extension, the so-called *res extensa*, as the second of his «clear and distinct ideas» — the first being the idea of the *res cogitans*. The identification of physical matter with geometrical extension — that is, the anti-Aristotelian assertion that physical matter was infinitely divisible as geometrical extension is — was, in conclusion, the second pillar upon which Descartes intended to ground the new modern philosophy of nature, after the birth of the new Galilean science of nature.



As in the case of irreversibility, a fundamental ingredient of the explanation of the mesoscopic structure of the matter of physical bodies (Cf. footnote 42), also in the case that regarded the microscopical structure of the matter of physical bodies, it was another axiom of the Aristotelian philosophy of nature which had to be re-evaluated. The axiom was, precisely, the one that asserted that physical matter and geometrical extension were not identifiable, mainly — but not only — because the first, unlike the second, is not infinitely divisible.

Scientific basis of the theory of *minimal naturals* and thus of the notion of *species* in physics

For each type of material there exist, indeed, according to the Aristotelian vision, certain *minimum natural magnitudes* (*minima naturalia*) that characterize the dynamical properties of the material itself, that is, its way of interacting with the rest of the physical universe, and hence, also its way of manifesting itself to human knowledge. Of this metaphysical notion of the Aristotelian philosophy of nature, crucial in order to give an empirical basis to the distinction of *various species* of elements, and thus, of bodies, against the geometrical atomism in which all the elements existed without a qualitative distinction, Planck's quantum axiom, and the consequent discrete scheme of electromagnetic emission of atoms, provides the basis for its *operational**, quantified and, then, calculable version. This empirical datum will be confirmed, on a foundational level, when we examine the common causal structure of foundation of the distinction between several species of elements and of bodies (non-living and living) in quantum physics and in Aristotelian physics (Cf. § 5.4.4).

Planck's constant as a fundamental constant of nature

All the fundamental magnitudes of matter at the microscopical level are multiples of Planck's *h*. In other terms, Planck has introduced in modern physics a criterion of quantization which has given place to the birth of a new *fundamental constant of nature*, the most measured and reliable constant to date: Planck's constant:

$$h = 6.626176 \times 10^{-34} \text{ J/sec}$$

relativity concerning the convertability *mass-energy* $\times$ *velocities close to those of light* — that is, precisely, those towards which sub-atomic particles generally move —, the quantum principle could be extended also to the manifestations of matter as *mass*.

It can be defined as the relation between the energy of a photon and its frequency, or, more exactly, as an *elementary quantum of action*. Indeed, as can be observed from its mathematical definition, h has the dimension of an action, of an energy across time. We can then define the following:

Principle of quantization. Every physical magnitude, in particular every dynamical magnitude or intensity of an energy E , is an entire multiple n of h , according to the relation: $E = \hbar \nu \times n$,

where $\hbar$ (to be read «cut h ») is $h/2\pi$, that is, h renormalized upon the circumference, and n is the wave-frequency associated with the intensity of energy E .

2.4.3 The principle of indeterminacy

End of the myth of absolute precision in measurements and/or in observations.

The fact that in *microscopic* physics a principle of quantization or of discretization of matter was valid introduced another change of perspective in the philosophy of nature associated with modern science. The ideal of «Laplace's demon» was immediately put into question. That is, the ideal of a determinist vision of nature, based on mechanics, and ultimately related with the supposition that *the precision of measurements* — in particular, the precision with which the *initial conditions* of the motion of a particle are defined, its «position», q , and its «quantity of motion», p — could *be always and in any case increased at pleasure*.

And, indeed, the second conceptual revolution, which follows the first one of the introduction of the quantum criterion in microscopic physics, was the definition in 1927, by a young German physicist, Werner Heisenberg (1901-1976), of a subsequent new axiom, which holds in quantum mechanics, and which distinguishes it from classical mechanics. It is the so-called *principle of indeterminacy*.

In order to illustrate it, let us begin with a classical «thought experiment» that Heisenberg himself proposed. Let us imagine that we must follow the trajectory that an electron covers in its orbits around the nucleus, according to the Rutherford-Bohr semi-classical model of the atom just illustrated. When we work with distances in the quantum order (in the order of 10^{-8} cm, the diameter of the orbit of the electron around the nucleus of the hydrogen atom), the «light» that I had to use to «illuminate» the position of an electron in an atom — e.g., by illuminating with increasingly closer flashes of light different points of its trajectory — will necessarily modify the quantity of motion (velocity) of the electron itself, which will begin to proceed by «jumps». Indeed, the smallest quantity of energy with which the above mentioned trajectory can be illuminated

will be equal to $h\nu$, which will correspond to an energy communicated to the particle equal to $h\nu/c$, where c is the speed of light. So, the resulting imprecision in the determination of the quantity of motion p of the electron will be of the same order, that is: $\Delta p \approx h\nu/c$. Vice versa, if one wants to avoid this distortion, one should content oneself with an indetermination in the position: that is, one should give a less frequent «illumination». Such imprecision, in one case or in the other, could not, however, be inferior to h .

Heisenberg's
indeterminacy
principle

Principle of indeterminacy. The product of the uncertainties by which a magnitude and its conjugate are known (e.g., position and quantity of motion) will never be inferior to h : $\Delta p \Delta q \geq h$.

To persuade us of all this, let us follow the Nobel Prize George Gamow (Gamow 1980, 112ss.) in a simple demonstration about how the principle of indeterminacy produces remarkable effects at the level of *microscopic physics* and not at the level of *macroscopic physics*. This is the reason why it should be included, together with the quantum principle from which it derives, among the principles of quantum mechanics and not among those of classical (Newtonian) mechanics. Let us immediately introduce in the preceding mathematical formula, instead of the quantity of motion $m \times v$, the velocities, in order to show the mass m , and, thus, the fact that the principle is valid only for extremely small masses. In that way, the above mentioned Heisenberg's indeterminacy relation becomes:

$$\Delta v \Delta q \cong \frac{h}{m}$$

Its relevance in the
world of
microscopic events

So, for example, if we take a mass of 1 mg. (10^{-3} g), approximately that of a bullet of a hunting rifle — an enormous mass with respect to that of an electron which is nearly 10^{-27} g — we obtain the relation⁴⁹:

$$\Delta v \Delta q \cong \frac{h}{m} = \frac{10^{-27}}{10^{-3}} = 10^{-24}$$

⁴⁹ Let us notice that in the formula below the order of magnitude of h is of 10^{-27} , because, following Gamow, we prefer here to express the magnitude of h in terms of erg/sec, and not of J/sec (1 erg = 10^{-7} J) as in the corrected formula of page 116, where the order of magnitude of h , was, precisely, of 10^{-24} . The motive of such transformation is only intuitive. Given that here, in order to facilitate the comprehension, we are speaking in terms of centimeters and milligrams, it is better to use «erg», as the unit of measurement of energy (or work), than to use the «Joule», even if the use of erg is prohibited by international conventions since 1979. Finally, it is of no little significance that, as it appears before in the text, 10^{-27} is also the order of magnitude of the mass of the electron expressed in grams.

that can be satisfied, for example, by taking:

$$\Delta v \approx 10^{-12} \text{ cm/sec}; \quad \Delta q \approx 10^{-12} \text{ cm}$$

This means that the degree of error under which we can measure the velocity of our bullet is inferior to 0,3 m in one century, while the indeterminacy of its position can be compared with the magnitude of an atomic nucleus. But if we take instead of our bullet, an electron whose mass is, let us say it again, 10^{-27} grams, then:

$$\Delta v \Delta q \approx \frac{10^{-27}}{10^{-27}} \approx 1$$

Now, since that to say that an electron moves inside the atom means that the variation of its displacement can be at most that of the radius of the atom itself, thus $\Delta q \approx 10^{-8}$ cm, so the indeterminacy of the velocity appears at the same level, that is:

$$\Delta v = \frac{1}{10^{-8}} = 10^8 \text{ cm/sec}$$

which is an enormous indeterminacy, along the lines of millions of meters per second.

2.4.4 The principle of exclusion

Explanation of the periodicity of the elements by means of the distribution of the electrons in the orbitals

Surprises, however, were not finished yet. On the basis of Bohr's discoveries, the periodic distribution of the chemical properties in the table of elements is related to how the electrons distribute themselves along the several levels of energy (orbitals) around the nucleus. One of the most interesting properties is that, while the external dimensions of the atom are more or less always the same, the number of the atomic orbitals changes a lot: for example, it is possible to go from the electron of the hydrogen atom, to 92 of Uranium, 238. Hence, with the addition of always new electrons, the volumes occupied by the several quantum states (the different orbitals) contract themselves, but the number of states occupied by the several electrons increases, so that the external diameter of the atom remains approximately the same.

Pauli's discovery of the intrinsic angular momentum of the electron (electronic spin)

The problem, then, is that of finding a mechanism through which the electrons do not accumulate at the minimum level, that is, at the level in which they do not occupy the most internal «orbital». Each quantum state or «energetic level», corresponding to a Bohr's «orbital», were connoted by *three quantum numbers*. An hypothesis originally perfectly in accordance with the fact that the model of Bohr's atom develops itself

over three dimensions. On the basis of certain experiments (the so-called *Zeeman-effect*, namely, the splitting of the spectral lines of the atom due to very strong magnetic fields), Bohr suggested that a *fourth quantum number* was necessary and that such a number did not define a property of energetic level and/or of the orbit inhabited by an electron, but a property of the electron itself.

In particular, the Austrian physicist Wolfgang Pauli (1900-1958) suggested that a *fourth quantum number* must exist. It was thought that this number, in the case of the electron, should have the dimensions of a *magnetic moment*. That is, it should have the dimensions of a quantity of magnetic force, linked to a sort of spin motion of the electron upon itself⁵⁰. Pauli hypothesized that for each «orbital», characterized by three quantum numbers, only *two* electrons could exist, but each one with its magnetic moment. So, he hypothesized the existence of a *fourth quantum number* concerning the rotation of the electron, to the right or to the left, upon itself⁵¹. From here the formulation of the:

Pauli's principle of exclusion

Principle of exclusion. In a system of particles endowed with fractionary spin, which otherwise would be indistinguishable, two of such parti-

⁵⁰ We recall here that the magnetic force is generated by an electric charge in movement. This is Maxwell's great discovery which has unified the electric and magnetic force in only one force: the *electro-magnetic* force.

⁵¹ Later on it was discovered that the *spin* is a property that characterizes *all* the quantum particles. In this sense, the spin has been defined as the «intrinsic angular moment» of a sub-atomic particle, that is, a sort of *vector*, of «preference» of the species of particle for a determined sense in the direction of the interaction with other particles and that «weighs» in the determination of the final result of the interaction itself. In this sense, there are many types of those *spins*, characterized numerically, not only by fractional numbers as in the case of the electron (whose spin equals $\approx 1/2$), but also by whole numbers. The first case is about those particles which constitute the bricks of matter (e.g., electrons, protons, neutrons, etc.), the particles from which all atoms, and hence all bodies, are made. Because of this fractional spin property they will distribute themselves, at their fundamental state, along different energetical levels inside and outside the nucleus, and not only along the lowest level, as it should be if the laws of classical statistical mechanics were meant to apply here. In mathematical terms, it is said that these particles follow a particular distributive function, studied in statistical mechanics by Enrico Fermi (1901-1954) and Paul Adrien Maurice Dirac (1902-1984). From here the name *fermions* attributed to this type of quantum particles. The second case, that of the entire-*spin* particles, is about particles that are not the bricks of atoms (protons, neutrons, electrons), but «exchange particles». They are intermediaries of the several forces that act in the atom: electromagnetic force (*photons*); weak force (*intermediate bosons vectors*), strong force (*gluons*). Such particles, which mediate the quantum exchanges of force between phenomena, follow a more classical distributive function in statistical mechanics. According to this distributive function, at the fundamental state, all the particles have the same level of energy, the lowest level. Such distribution has been studied in particular by Satyendra Nath Bose (1894-1974) and Albert Einstein. For this reason, these particles are also called *bosons*.

cles cannot be in the same quantum state, characterized by a n -times (4, hence a *quadruple*, in our example) of quantum numbers.

Pauli also extended these considerations to the structure of the nucleus. Also here, as the electrons around the nucleus, the particles (nucleons) distribute themselves along different energetical levels. But, given that in this case there are two types of particles (protons and neutrons) and not only one (electrons), the presence of two particles in the same quantum state will be admitted.

2.4.5 The wave-particle duality

De Broglie's intuition: ondulatory and probabilistic character instead of line-form and deterministic character of the propagation of quantum events

But the surprises of the study of the ultimate structure of matter were not yet over. A further «scandal» was caused by the elegant mathematical hypothesis of the French physicist, Louis Victor Duke of De Broglie (1892-1975), who, in his Ph.D dissertation in physics in 1925, advanced an extremely original idea that, linked with Heisenberg's indeterminacy principle, constituted the real turning point of the just born quantum mechanics. Synthetically, in classical mechanics, the possibility of determining unambiguously, with an infinitely increasable precision, the position and quantity of motion of a particle, justifies the mathematical representation of motion in terms of the displacement, proportional to the quantity of motion, of a material point (a -dimensional) along a single trajectory (or along a one-dimensional line).

But once the possibility of an unambiguous determination is undermined due to the quantization and indeterminacy principles, such a type of representation becomes completely wrong. The quantum nature of the phenomena under examination does not make it possible to speak of an unambiguous and continuous determination of a position along an a -dimensional line, but it makes it possible to speak of a *probabilistic and segmented determination of the position, no more along a one-dimensional line, but, rather, along an n -dimensional surface with different thicknesses*, precisely a curved surface. This conceptual passage is intuitively represented in Figure 2-2. In other words, the representation of the motion of a particle as a continuous succession of positions along a one dimensional line, must be replaced with another kind of representation. That of the motion of the particle as the propagation in the space of an n -dimensional wave (with $n \geq 2$) of probable positions, the so-called *De Broglie's wave*, a wave whose length λ , in the *prototype-case* of the electron of the hydrogen atom studied by Bohr, is proportional to the quantity of motion mv , according to a relation discovered in 1922 by Compton, namely:

$$\lambda = \frac{h}{mv}$$

Equation 2-1

The ondulatory
model of the atom
as harmonic
oscillator

Through such a simple and disturbing idea, the phenomenon of «orbital jumps» of the hydrogen atom, when it is excited from the outside by quantized impulses of energy — the phenomenon by which the electron passes from the most internal orbit of its fundamental state to increasingly more complex orbits around the nucleus, according to Bohr's model —, can be explained through an ondulatory model. It could be explained, in a more coherent way, by considering the atom not as a planetary system, but as an harmonic oscillator (as, for example, in the case of acoustic vibrations: a bell) that «vibrates» when it is struck by a certain amount of energy. It produces a wave (a sound wave, in the case of the bell, an «electronic» wave in the case of the atom) which propagates itself in space.

De Broglie's hypothesis, when it was formulated, was very disturbing since it was formulated two years before the discovery of Heisenberg's principle of indeterminacy. For pedagogical reasons, we put it after Heisenberg's discovery, in order to make it immediately evident that De Broglie's waves *are not energy-waves*. They do not represent a state of matter, they do not represent the vibration of a field of forces, as, for example, electromagnetic waves. They are anything else but a change in the mathematical representation of physical phenomena that, in the classical representation of trajectories of particles unambiguously and not only probabilistically localizable, would be inexplicable.

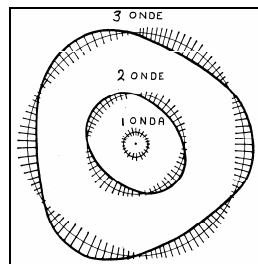


Figure 2-2. De Broglie's waves adapted to the quantum orbits of Bohr's atom (from Gamow 1980, 85). What can be seen here is the passage of representation from Bohr's one dimensional trajectories to De Broglie's probabilistic waves — in this bi-dimensional representation. Within the thick lines which define the profile of our bi-dimensional curve, the particle can be localized anywhere. For this reason it is said that with De Broglie–Schrödinger–Dirac's ondulatory representation we have passed from the «planetary» model of Bohr's atom (electrons likened to planets) to the «cloud» model of the atom.

In the same way that the volume of a cloud is made of drops of water that move whirling inside that volume without a precise localization, but only a statistical one, so also in the «electronic cloud» of the atom ondulatory model. The difference with the cloud of water is that, if one would want to, one could localize the position of the drop with an increasing degree of precision, while in the electronic cloud this is not possible.

The first
experimental
evidence of the
ondulatory model

For these reasons, the *experimental verifications* of the ondulatory model were so disturbing. First of all, if elementary particles propagate themselves as waves and not as bodies along trajectories, when they pass through matter they should produce diffractive phenomena, exactly as beams of light. For example, according to the above mentioned formula (Equation 2-1), by appropriately accelerating some beams of electrons, a De Broglie's wave length could have been obtained, $\lambda \approx 10^{-8} \text{ cm}$, comparable to that of x-rays with electromagnetic waves — even though the quanta of the electromagnetic waves are photons, particles without mass, while the electrons are endowed with mass. If, then, De Broglie's hypothesis were correct, «the electronic wave», passing through a network of atoms, should have produced a diffractive phenomena⁵² as if they were rays of light at high frequency, that is, x-rays. The experiment was accomplished independently by Thomson in England and by Davison and Germer in the United States. In both cases, the diffractive phenomena predicted by De Broglie's theory (cf. footnote 52), can be observed in the photographic plate. In the case of a group of electrons, these diffractive phenomena correspond to the wave-length predicted by De Broglie's formula. Moreover, even the same phenomenon was obtained by the German physicist Otto Stern by accelerating *atoms* — thousands of times heavier than an electron! — until obtaining a De Broglie's wave length of $\lambda \approx 10^{-8} \text{ cm}$.

The birth of wave
mechanics

By means of its experimental control, essential, like its mathematical formulation, to be counted among the groups of «Galilean», modern scientific hypotheses, we can say that a new branch of physics was born:

⁵² Diffractive phenomena are characteristic of all ondulatory phenomena. Intuitively, let us suppose that we have a series of waves, for example, electromagnetic waves, that show interference among themselves. It is clear that where peaks correspond to peaks, when waves interfere (phase waves), the two peaks are added up, and form a peak whose height is proportional to their addition. When, instead, a low point (not in phase waves) corresponds to a peak, they cancel themselves. If we applied a photographic plate to the wave produced by the interference, we shall obtain a characteristic figure with light and obscure stripes, where the former should correspond to the low points, and the latter to the peaks of the light wave. Now, if we accelerate a bundle of particles, for example, electrons (but also atoms!), and we make them interfere appropriately, we obtain the diffractive phenomena! So, particles endowed with mass behave like waves.

wave mechanics. For this discovery, that changed the course of the history of physics, De Broglie would win the Nobel Prize in 1929.

Other phenomena, that could not be explained with the classical mechanics «of a trajectories-like» model — the well-known uranium and radium radioactive phenomena, the possibility of bombarding atoms with high-speed atomic particles until they split (nuclear fission) or fuse together (nuclear fusion), as it continually occurs in nature, in the stars and in the sun in particular —, could now be explained from De Broglie's⁵³ ondulatory hypothesis.

2.4.6 Schrödinger's equation

The passage to
Schrödinger's
ondulatory equation

The definitive consecration of the new wave mechanics takes place after the publication of De Broglie's extraordinary idea, in 1926, when the Austrian physicist Erwin Schrödinger (1887-1961) formulated a new and elegant mathematical theory of the hydrogen atom in terms of De Broglie's waves mechanics. For this, he received the Nobel Prize in 1933. Bohr's semi-classical «planetary» model of the atom was replaced definitely by the «wave» model originated from De Broglie's intuition. We thus will no longer speak of «electrons» rotating in «orbits» around the nucleus, but of levels of energy of De Broglie's «electronic» waves, of an atom that «vibrates» as a quantum harmonic oscillator.

The model of the
hydrogen atom at
circumscribed
stationary waves

According to Schrödinger's equation, the different levels of energy to which a single electron of the hydrogen atom «jumps» when it is «excited» by the injection in the form of discrete energy from the outside, are calculated with incredible precision as «vibrations» of a quantum harmonic oscillator. They are calculated according to the «stationary-circumscribed-waves» model. This model describes the vibrations of an elastic rope that is fixed at both extremes, as the strings of a violin or of a guitar, when it is played. By striking it, with an intensity that is two, three, or four times that of the original one, the string will vibrate proportionally with a frequency that is two, three, or four times the original. So, we have a linear equation: there is proportionality between what goes in and what comes out.

⁵³ In the classical-material-points-trajectory-model it was impossible to explain how particles with inferior energy could jump higher «barriers of potential», determining, for example, the classical radioactivity phenomena, where alpha particles overcome the attraction of the uranium atom's nucleus. But if phenomena have an ondulatory nature it becomes possible, with some probability, that particles with an energy that is lower than that of the barrier, sometimes, «pass through» the barrier itself.

Determination of the scheme of the terms of the various energetic levels allowed to the atom (Bohr's old «orbitals»), as values of the respective wave functions (auto functions)

More exactly, the wave function $\psi(x, y, z)$ that appears in Schrödinger's equation is a function of the spatial coordinates of the particle. If it is possible to find the solution to that equation for a given system (for example: an electron in an atom), then the solution, which depends on the surrounding conditions (for example, the energy introduced into the system), is a set of the permitted wave-functions (*eigen-functions*) of the particle, each of them will correspond to a permitted energetic level (*eigen-value*). Physically, in every point, the square of the wave function is proportional to the probability that the particle will be found in an infinitesimal element of volume, $dx\,dy\,dz$, centered on that point. In this sense, an atomic or molecular *orbital* will not correspond anymore, as in Bohr's model, to a definite orbit or trajectory. The electron has only a certain probability of occupying a given position of space. Such probability is given by the solution of Schrödinger's equation, to obtain the wave function ψ : indeed the probability of finding the electron in a certain position is proportional to $|\psi|^2$. An *atomic orbital* instead of being an orbit, a trajectory in the classical sense, corresponds to a distribution of probabilities of a spatio-temporal localization around the nucleus or, which is the same, to a distribution of electric charge conceived as an average with respect to time. As we shall see, this equation changes not only the history of physics but also the history of the modern philosophy of nature (Cf. § 5.4.4).

Final stage of this first phase in the history of quantum theory is Dirac's equation about the discovery of anti-matter

As already mentioned, the last chapter in the history of the birth of quantum physics until now is the one which has been written beginning with Paul A. M. Dirac's equation (1902-1984; Nobel Prize in 1933). Aside from the disturbing hypothesis that such an equation advanced in a theoretical form, i.e., the existence of anti-matter, an hypothesis that has its first experimental confirmation some years later, after M. Anderson's discovery of the *anti-electron* or *positron*, it finally added to the equation of quantum mechanics the principles of the theory of special relativity, that is, the mass-energy transformability for particles that move with a velocity close to the speed of light (= limit-velocity). It is indeed from Dirac's equation that the successive development of quantum electro-dynamics (and of chromo dynamics) springs.

2.4.7 Principle of complementarity

To understand where quantum physics' actual research lies, at a distance of one hundred years of Planck's discovery, preparing ourselves — probably in a not too distant future — for surprises greater than the incredible ones of the first thirty years of the XX century, we need to mention the final revolutionary principle that distinguishes quantum

mechanics from classical mechanics. The new exceptional situation — the same physical reality represented according to two formalisms, the semi-classical formalism, according to the trajectory-function of Bohr's atom, and the ondulatory formalism of De Broglie–Schrödinger's atom — was photographed in the formulation by Niels Bohr himself, of a further principle that characterizes, with respect to Newton's classical mechanics, the new quantum mechanics, waiting for its future definitive formalization, which is on the other hand still to come⁵⁴. It is the so called:

Bohr's principle of complementarity

Principle of complementarity. All the microscopic phenomena are characterized by the presence of a double representation, in terms of particles and in terms of waves. Both aspects are related by the twofold relation $p = h\lambda$ and $E = h\nu$, where h is Planck's constant, p and E represent, respectively, the quantity of motion and the energy associated with the quantum entity under the form of particles; while λ and ν represent the wave-length and the frequency that can be associated with the quantum entity under an ondulatory form.

Now, if we carefully observe these two formulas of the principle of complementarity one thing can be clearly seen which is of great interest for a metaphysics of the physical quantum entity.

Quantum indeterminacy linked to the reduction of the wave function

All the indeterminacy — linked to h — is on the right side of the two equations that synthesize the statement of the principle of complementarity. The left side contains only variables which are typical of the «particle-like» aspect of the quantum event. In other terms, the indeterminacy manifests itself when the so called *wave-function reduction* is operated. That is, when the quantum events that have their mathematical representation in ondulatory terms are constrained to a representation in terms of the trajectory of particles. That is to say, events that mathematically speaking have their correct representation in terms of *wave-functions* and not in terms of *trajectories*. In terms of wave functions quantum events are perfectly *deterministic* and, above all, *predictable*, because of their *dynamic stability* (Cf. § 2.6.1). In any case, there exist several promising attempts to provide a *realist* interpretation of the wave-function reduction rendering the observer-based paradoxical, physical phenomena linked with it independent of the representation and/or of the act of

⁵⁴ Such formalization will be partially obtained in the following thirty years, after the integration with the theory of special relativity, by means of Dirac's equation, in terms of *quantum electrodynamics*. And this due to the work of eminent mathematicians and physicists as Hilbert, Von Neumann e Feynman, but that still waits for a satisfactory definitive formulation. We shall turn to this issue in the Third Part of our work..

measurement of the observing subject, linking them instead to the so called *decoherentization* phenomenon (Tegmark & Wheeler 2001).

Weakness of the current interpretation of the indeterminacy linked to the interference of the instrument of measurement

But this also means something else. Generally, in the usual interpretation of quantum theory, it is said that the indeterminacy, which Heisenberg first told us about, depends on the irreducible interaction of our instruments of measurements with the observed event. Often this has been used to speculate about a kind of subjectivist epistemology concerning the unavoidable influence of the subject upon the nature of the observed object, in heart of the physical science. But in the other interpretation that appeals to the so called «Copenhagen's School» of the «hidden variables» — an interpretation that, among the theorists of quantum physics, is gaining more and more ground (Hawking 1988; Hawking & Penrose 1996; Ghirardi 1997; Penrose 1999; Tegmark & Wheeler 2001) —, the indeterminacy has a much deeper reason, completely objectivist and realist. If a representation — that of the wave-function — seems more adequate, «truer» than another — that of classical trajectories —, is because it seems intrinsically related to the *spatio-temporal non-locality* of the quantum phenomena, understood as one of their specific properties. This seems like a scandalous property to the 'modern' philosopher of nature, but not at all to someone who adheres to an Aristotelian philosophy of nature or, among contemporary philosophers, to Whitehead's Process Philosophy.

Towards a realistic interpretation of the relations of indeterminacy

For the same reason, the indeterminacy phenomena that result from the interference of the measurement instruments, would not be linked to an interference of «knowledge» on the «being», but much more trivially and realistically to the fact that another physical system (the measurement device) interferes with the quantum event by «decoherentizing it», breaking the magic of the *entanglement* among quantum states and transforming the fascinating non-locality property of these events into the much more prosaic impossibility of unambiguous localization of the quantum particle, typical of the indeterminacy relation. At the same time — and here resides the great theoretical interest of the whole question — it seems to be evident how the properties of physical space-time appear here intrinsically linked to the dynamical interaction among bodies in a way even more intrinsic at the microscopic level than the general relativity itself has taught us at the megaloscopic level (Basti & Perrone 1992). From this point of view, we can understand the interest of a great theorist of general relativity, such as professor Wheeler for the theory of decoherentization. Let us deeply investigate the notion of «quantum non-locality» as an intrinsic property of a quantum system isolated from external interferences and thus «non-decoherentized».

EPR's paradox and its meaning

The non-locality phenomenon is considered a famous paradox, stated by A. Einstein, B. Podolski e N. Rosen in an article of 1935 (the EPR paradox). According to these authors this paradox aims to show the incompleteness of quantum theory.

From «thought experiment» to physical experiment

The article proposed what at the time in which it was conceived by their authors could only be a «thought experiment». If we imagine that two particles (e.g., two photons) have interacted quantistically, it is impossible for the observer to conduct measurements of the state of both particles with an imprecision less than $\hbar$, as Heisenberg's principle of indeterminacy shows. But nothing should prevent that, and here lies the intuitive substance of EPR's idea, when both particles are far enough, it could be possible to conduct a measurement with all the desired precision and from here go back in space-time, until redefining precisely which was the state of the particles at the moment of the interaction, eliminating in this way the indeterminacy. Through a series of works, first, in the '60s, the English physicist J. S. Bell defined in a quantitatively rigorous way the phenomenon that should be measured empirically (the famous «Bell's inequalities») to see whether EPR were right or not. The experiment was performed during the '70s and '80s, either with particles endowed with mass or with particles without mass. The most convincing of all was carried out by the French physicist A. Aspect in 1982, and then repeated several times with similar results, showing in an irrefutable way that EPR were wrong. The experiment concerned the state of polarization of two photons that interact quantistically, that is, that find themselves in an *entangled* state, in which the state of polarization of one determines the state of the other according to the predictions of the theory.

Representation in terms of wave function linked to the intrinsic non-locality of quantum phenomena

The detectors of the state of polarization of each photon emitted by the source where placed at 13 meters one from the other, and their state was updated every 10 nanoseconds, every one hundred millionth of a second. This was done to make sure no message could be sent from one detector to the other, so that the state of one of them, at the time of passage of its own photon, would influence the state of the other. Indeed, at the speed of light, that is, at the maximum speed conceivable for whatever propagation of a physical event a message would have employed, it would have taken 40 nanoseconds to arrive! Well, also at that distance, «enormous» for any microphysical event, the state of polarization of every photon is influenced, *entangled*, by the state of the other, in accordance to the predictions of the quantum theory. Given that the way the experiment was conceived leaves no reason to think that, in the experiment itself, c , the speed of light (Ghirardi, 1997, 236-361), has been violated, we must give up to the hypothesis of the non-

localization of the microscopic phenomena. The representability of quantum phenomena in terms of wave-functions appears thus intrinsically related to the non-localization of space-time properties at the level of microscopic events and with the fact that the observed quantum system is perfectly «isolated» from interactions with the rest of the physical world (Tegmark & Wheeler 2001). As Ghirardi ironically says, it is as if quantum events had «telepathic» properties!

Non-locality and indeterminacy

To conclude this section dedicated to quantum domains, let us make an observation related to the importance of what we have said regarding the link between *non-locality* and *indeterminacy* in quantum mechanics that will be extremely useful in the Second Part of our work. If we could make a metaphysical kind of deeper consideration — but, on the other hand, Aspect's experiment has been defined as an extraordinary case of «experimental metaphysics», a case of extraordinary tangentiality between physics and metaphysics, between science and philosophy of nature — with the discovery of quantum non-locality the close relation between space and time that relativity theory bore out acquires a significantly greater value.

Intrinsicness of the space-time relation in quantum physics

The special theory of relativity teaches us that we are constrained to consider time as a fourth dimension of the space. Hence, once it is admitted that c is finite and that it is the upper limit of velocity of whatever physical entity, there no longer exists absolute simultaneity between physical events, and for this reason, the spatial localization of an event *as a measured one*, in the classical terms of points and trajectories, necessarily requires a consideration regarding the time with which the signals reach the measurer, and the subsequent correction of the measurements. Dirac's equation teaches us that similar considerations should be extended also to the microscopic world of quantum physics, precisely because the particles under examination move at a velocity comparable with c . The lesson given us by EPR's mistake and by the discovery of non-locality is that the space-time relationship is much more intrinsic — at least in microscopic phenomena, but not only in those phenomena — than the mere necessity of adding a fourth dimension, that of time, to a geometric three-dimensional space, in which events can be represented. Such intrinsicness is later on confirmed in the case of the cosmological interpretation of the phenomenon of quantum decoher-entization (Zeh 2001).

Non-locality of time and non-local character, discrete and indetermined of quantum events

Indeed, what has passed completely unnoticed to modern physics from Newton to Einstein, but not to the ancient philosophers, is that time has an intrinsic non-local nature: let us try to localize unambiguously the present instant, as if it were a point in space! It is systematically elusive as Gilbert Ryle has recently said in a famous essay (Ryle 1949). If I

would try to fix «the now» as a «here» in space, it is irremediably passed. This is the truth of the unambiguous non-localizability of the present. For this reason time is not representable as a geometric point unambiguously localizable. This truth was known to Plato and Aristotle and in the 1900s, was claimed by another philosopher and by his followers: Henri Bergson (1859-1941). But common to all these contemporary philosophers is a reference to the property of the non-locality of time in a way that is exclusively dependent on the subjectivity of consciousness, unlike what the ancients did, especially Aristotle. Vice-versa, the quantum non-locality as an intrinsic property of the space of microscopical events and its close relation with the principle of quantization⁵⁵, and hence with the principles of decoherentization and indeterminacy, make reference to the fact that spatio-temporal non-locality is an *objective* fact of the physical microscopic processes. Consciousness does not matter: we are not in front of measurable events, unless we want to attribute «telepathy» to microscopic physical events as Ghirardi ironically maintained!

The intrinsic character of the bodies of the space-time and its detachment from the modern Newtonian-Kantian conception of the origin

More generally, all this is linked to the absolutely disturbing fact for a modern person — but not at all for an Aristotelian — that space and time are not «*a priori* forms» of our knowledge (Cf. footnote 57). Their structure and properties (first of all, *locality* or *non-locality*) critically depend on the (*dynamical*) *causal interactions* among certain bodies placed in them, of which, then, as the Aristotelian table of categories proposed, space and time are *accidents*, typical attributes whose properties change according to the variation of the nature of the bodies which they referred to and according to their relations (Cf. § 5.4.2)⁵⁶. This was a novel item for the modern physicist, for which the theory of relativity and, especially, its explanation of gravity had in part prepared us (Cf. § 2.5.3), but that a unified, relativity/quantum theory or «theory of quantum gravity» based on such premises would go much beyond. Wheeler is thus quite right when he concludes in his often quoted central article on the first one hundred years of quantum physics with the following reflection:

⁵⁵ The quantum is «quantum of action», that is, of a force applied upon the space for an interval of time!

⁵⁶ A consequence of this approach is that for Aquinas only the bodies that exist in act are unambiguously localizable, while the elements «in continuous motion» that constituted them are absolutely non-local, for this reason they exist in them *virtually*. That is, in an intermediate state between potency and act, because they are *distinguishable* within the compound which can, indeed, be splitted, reobtaining the constitutive elements that are not perfectly «in potency» in it. At the same time, however, they are unambiguously non-localizable within the compound — and so, not perfectly in act — guaranteeing in this way the *unity* and *impenetrability* of the bodies. We shall return to these concepts in the Third Part (Second Volume) of this work.

The first one hundred years of quantum mechanics have provided us with powerful technologies and have answered many questions. But physics has generated many other new and equally important questions (...) — questions that concern both quantum gravity and the ultimate nature of reality. If history is something that moves on, the upcoming century should be full of exciting surprises (Tegmark & Wheeler 2001, 75).

2.5 Theory of relativity

2.5.1 Newtonian conception of space-time

In Aristotelian mechanics, absolute distinction between rest and motion because science is causal

To begin to complete our initial picture about the scientific revolutions of the 1900s we should now say something about the most well known one, exactly because to the common man, such a revolution was produced by the prototype of the modern scientist, Albert Einstein (1879-1955): the *theory of relativity*.

One of the conceptions that deeply differentiates the vision of physical *space and time* of Newtonian mechanics with respect to that of the Greek vision, for example, the Aristotelian vision is that for the laws of Newtonian mechanics no substantive difference between the state of *rest* and the state of *motion* exists. The Aristotelian philosophy of nature searched for causes and not for mathematical laws to represent and predict measurable phenomena. The Aristotelian mechanics searched for the *causes* of the motion of bodies at rest.

Lack of an absolute distinction in modern mechanics, between rest and motion because science is phenomenical

This perspective is radically modified by modern *mechanics** and *dynamics** beginning with Galileo. Given that they are interested essentially in the mathematical laws that determine the relation between measurable magnitudes, when they study the *forces*, they are not studying «causes» that determine the existence of events (accidents): for example, causes that determine the existence of the event of the passage from a body's (substance) state of rest to its state of motion or vice versa, but they study the relation between the variation of a determined dynamic magnitude (independent variable: the force) and the variation of another mechanical magnitude (dependent variable: the position). By applying this method Galileo reached the relevant result of demonstrating that bodies of different weight fall at the same velocity — so weight is not «the cause» of the fall. Newton, following and formally improving upon this method, defined the law that determines the intensity of the force of gravity that later determines the quantity of motion in question. That is, Newton discovered that the intensity of the force is directly proportional to the masses of the involved bodies, and inversely proportional

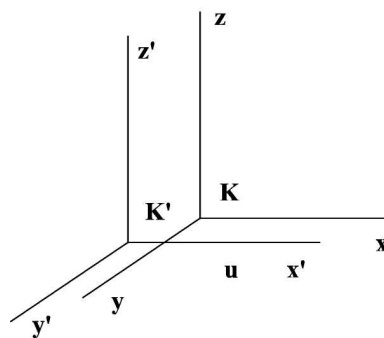
to the square of their distance. In sum, in modern dynamics the forces *do not cause* displacement of bodies at rest, but they *modify the velocity*. Rest and motion are not absolutely distinct physical states of the bodies, as the principle of inertia in its Newtonian formulation shows. They are just physical states characterized by two numerical magnitudes different from velocity: $= 0$, for rest, $\neq 0$ for the state of motion.

Galilean principle of relativity and lack of an absolute system of reference for bodies at rest.

This difference in the above mentioned approaches implies a deep change regarding the notion of *time* and *space* in modern mechanics with respect to the ancient science. While in the latter there existed an *absolute* system of reference to judge the state of rest of the bodies (in the cosmological vision, the sky of fixed stars with concentric spheres of the Aristotelian-Ptolemaic cosmology), this is completely absent in Galilean-Newtonian classical mechanics. That is, in this one, the so called *principle of Galilean relativity* holds, which asserts that there is no privileged system of reference with respect to which it could be said, when describing a motion, which is the body in movement and which the one at rest. Intuitively, this has an immediate correspondence in the experience that we have about a train which is at rest in the railway station, when, looking through the window, we cannot say which one is moving: whether us or the train that is parked just aside.



Principle of Galilean relativity is stated by the following: if in the description of a physical phenomenon the position of the bodies is referred to a system K of orthogonal Cartesian coordinates, so that the principle of inertia is valid (inertial system), this law is also valid for another system K' , with respect to which the former is in uniform motion. In the preceding example, let K be fixed to the train, K' to the railway (even though both of them are not perfectly inertial), the axes being x, y, z ,



which define the system K , should be parallel to the axes x', y', z' which define the system K' , and the motion be parallel to x ; the relationship among the coordinates x, y, z and x', y', z' , according

to Galileo's transformation, is the following:
 $x' = x - ut$, where u is the velocity and t the time
 $y' = y$
 $z' = z$

Nonetheless,
 presence of an idea
 of absolute
 measurement of
 space and time in
 classical mechanics

Despite this renunciation of an absolute system of reference for rest, in Newton and Galileo the idea of *absolute* space (more exactly, of a measurement of spatial distance) and time (more exactly, of a measure of temporal distance) remains, that is, of an absolute system of reference with respect to the observer. In such a way, space and time are considered by Galilean–Newtonian physics as a sort of «container» of the physical events that take place inside it, without being influenced by these events⁵⁷. In particular, it is typical of the Newtonian conception that *time* could be measured with absolute *precision*, whatever the position in *space*, provided that one had a good clock. This implied that we can consider:

- ◆ *Space and time* as two absolutely *independent* coordinates, and consequently,
- ◆ That the hypothesis of the *absolute simultaneity* among events was perfectly plausible, which physically supposes that we could send signals (for example, between two clocks in order to synchronize them) at an infinite velocity.

On the other hand, the infinity of the speed of light was also a common conception in ancient physics.

The problem of the
 finite velocity of light
 and Maxwell's
 ondulatory theory of
 light

However, that light had a finite velocity was a discovery available even eleven years before the publication of Newton's *Principia*, in 1675, by the Danish astronomer Olaf Christensen Römer. A discovery made by observing the eclipses of Jupiter's satellites during different periods of the year, when the Earth–Jupiter distance varies due to the elliptic orbit that these two planets of the solar system cover. Aside from the non-exactness of the measurement of this velocity that Römer had attempted, we shall need to wait until 1865, when the English physicist, James Clerk Maxwell (1831–1879), provides modern physics with a consistent theory of light radiation (electromagnetic), based on the idea of the finite velocity of the propagation of radiation itself.

⁵⁷ Furthermore, in the Newtonian metaphysic interpretation, absolute space and time were defined *sensoria Dei* (the way in which the Newtonian «God–clock maker» enters in relation with the world) and the absoluteness of the laws of mechanics were thus guaranteed. Kant will be given the task of «demystifying» such an idea by making absolute space-time *sensoria hominis*, namely, «a priori forms» of human sensibility in the empirical knowledge of physical events.

The ether hypothesis and its dismissal

The problem was that, if both Maxwell's equations and the finite velocity of light are accepted, the Galilean principle of relativity is not valid anymore. If, indeed, such a principle were valid, with respect to a system at rest (for example, the «ether» understood as the hypothetical medium at rest that light radiation would stimulate, a kind of electromagnetism, corresponding to that which air does for sound waves), Maxwell's equations were verified just as they were, while with respect to other inertial systems in motion regarding ether, they should have provided different results. A famous experiment conducted by Albert Michelson (Nobel Prize in 1907) and Edward Morley in 1887 demonstrated the baselessness of the above mentioned hypothesis. They compared the speed of light in the direction of the motion of the Earth and perpendicularly with respect to it. If the hypothesis of the ether had been true, the speed of light and the speed of the Earth should have had to be added together and, thus, light would have to be faster measured in the first case with respect to the second. Vice-versa the velocity resulted to be basically identical — although, later, objections to this measurement forced them to make even more precise measurements, and the controversies are yet not completely over.

2.5.2 Special Relativity

Proposal of solution in terms of A. Einstein's Special Relativity

In 1905, the young A. Einstein, who was at that time employed at the Patent Office of Vienna in the attempt to make a living after his degree in physics, published a famous essay *On the Electrodynamics of Bodies in Motion*. Einstein noticed that certain dissymmetries in Maxwell's electrodynamics, applied to bodies in motion, and the failed attempts to emphasize the motion of the Earth with respect to the hypothetical medium of light propagation («ether») led one to maintain that:



Principle of special relativity. The laws ruling all physical phenomena are the same for two observers who move in uniform rectilinear motion, one with respect to the other. [In other words, no experiment, either mechanical or electromagnetical, can prove such a kind of motion: the Galilean classical principle of relativity claimed the very same thing, but was limited to mechanical phenomena].

This principle is immediately linked to the other one concerning *the universal constancy of the speed of light*:

Light propagates in a vacuum in every direction at constant velocity, independently of conditions of motion of the source and the observer.

Absolute character
of the speed of light

This second principle contrasts with the law of composition of velocities derived from Galileo's transformations: indeed, in classic physics one finds that the velocity of a material point varies in accordance with the variation of the inertial system where the measurement is made. On such a basis, the ether hypothesis was formulated. More simply: if I throw a stone at 5 Km/h to a car that is traveling at 90 Km/h, the stone will hit the car at a velocity of 95 Km/h. Vice-versa, admitting the constancy of the speed of light, if I illuminate it with a ray of light, which is traveling at 1,080,000,000 Km/h (300,000 Km/sec), it will not hit our car at the velocity of 1,080,000,090 Km/h, but always at 1,080,000,000 Km/h! One thus needs to replace Galileo's transformations between inertial referential systems (e.g. between a moving observer and a body in motion), with other transformations that satisfy the postulate of the constancy of the speed of light c . Einstein discovered that these transformations were actually the same ones already discovered by the Dutch physicist Heinrich A. Lorentz (1885-1928) to explain the results of Michelson and Morley, but which were still waiting to be settled in a coherent physical theory, as was Einstein's theory. It follows that value c is a universal constant, that is, it has the same value for all the inertial referential systems.

The incredible
consequences of
relativity:

However, the whole structure of the theory of relativity led to paradoxical consequences, ultimately linked to the demise of the idea, which was typical in classical mechanics, of an *absolute space and time*, «space» and «time», of course, not considered in themselves — this is not a task of a modern physical theory —, but conceived as measured, that is, as «measured spatial distance» and as «measured temporal interval». Synthetically, the paradoxical consequences are the following:

1. Dilation of times

- ◆ *Dilation of times (= relativity of time)*. To understand the critique of the principle of the absolute character of time, let us return to the motion of the train of L length. The two extremities of the train correspond to two points A, B, close to the platform where, in point M between A and B, an observer O_b is at rest.

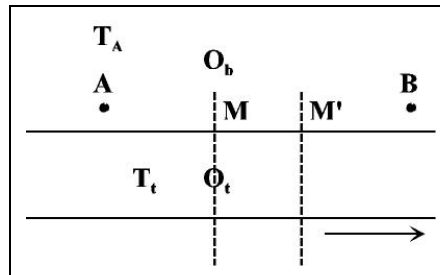


Figure 2-3. Intuitive illustration of the principle of temporal relativity

If two light impulses are emitted at the extremities of the train so that O_b could see them simultaneously by means of an optic system, he will claim that the two emissions have occurred simultaneously. But another observer O_t , placed in the middle point of the moving train, will not see them arrive simultaneously because in the meantime the train has moved to M' , and if the train is moving in the direction AB , he will claim that the emission A has occurred after the emission B . So, the concept of simultaneity is relative to the referential system. From the *relativity of simultaneity* one moves on to the *relativity of time* by considering a clock T_A which has been fixed in A and another clock T_t in the moving train, a clock that by hypothesis should be synchronic to the preceding one, if the train were at rest. As clock T_t moves from A towards B , its hands, which we imagine could be seen by A , will seem to rotate more slowly than those of T_A depending on the velocity of the train, given that as the distance increases, the time of propagation of light will accordingly increase (*dilation of times*).

2. Contraction of lengths



- ◆ *Contraction of lengths (= relativity of space)*. The inexactness of the absolute character of the distance derives straightly from the previous considerations. Indeed, a distance l measured on a body at rest is the difference $x_2 - x_1$ of two spatial coordinates with respect to a given system K ; when this is in motion with respect to another system K' , x_1 and x_2 and thus also their difference, become, with respect to K' , functions of the velocity u and of the velocity of light c . The outcome is a *contraction of length*.

3. Time as fourth dimension of space



- ◆ *Spatio-temporal referential system (=chronotopa)*. The constancy of the speed of light in inertial systems, the relativity of time and the relativity of distance *make it necessary* to always *introduce time in the description of physical phenomena*, also in their purely *geometrical* aspect, namely in the *spatial* coordinates. E.g., in the figure that illustrated the Galilean principle of relativity it was sufficient to point to the three spatial

coordinates (Cf. p. 131). But to illustrate the Einsteinian one it is necessary also to point to the temporal coordinate. It said, then, that the referential system becomes *spatio-temporal*. Or, as one could find in certain superficial popularizations of the theory, time becomes the *fourth dimension of space*. In any case, space and time *cease to be independent from the dynamic phenomena* that they describe, as they were in classical mechanics. That is, they cease to be represented mathematically in the equations of mechanics as if they were absolute «containers» of dynamic phenomena.

Lorentz's transformations and their relations with Galileo's transformations

At this point, one also better understands the sense of Lorentz's transformations between systems of coordinates that replace those of Galileo (Cf. above, p. 131), typical of classical mechanics. Galileo's transformations are replaced by the following, due to H. A. Lorentz, where, as one could see, it becomes essential to define the transformation also with respect to a t , and not only with respect to the three spatial dimensions x, y, z :

$$\begin{aligned}x' &= \frac{x - ut}{\sqrt{1 - \beta^2}} = \gamma(x - ut) \\y' &= y \\z' &= z \\t' &= \frac{t - \beta \frac{x}{c}}{\sqrt{1 - \beta^2}} = \gamma\left(t - \beta \frac{x}{c}\right)\end{aligned}$$

where c is the speed of light in the vacuum and $\beta = \frac{u}{c}$ $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$.

Lorentz's transformations have, in the end, the further property of leaving Maxwell's equations unchanged.

What is new about relativity: unifying character of many solutions to different problems

As one sees, with a single idea, the relativity of the measurements of time and space, Einstein not only had given coherency to Maxwell's electrodynamics by guaranteeing its invariance among inertial systems —just as the same invariance was guaranteed in the dynamical systems in classical mechanics—, but he also had found that the transformations that guaranteed such invariance were those of Lorentz's transformations that already explained Michelson–Morley's result. In such a way, finally, both the experimental result and the transformation that gave a theoretical account of it, were part of a general and coherent physico-theoretical framework for all electro-dynamical phenomena.

Galileo's transformations as bound-transformations of those of Lorentz

Furthermore, we should have clearly in mind the following considerations. Both the contraction of lengths and the dilation of times are the more sensitive the greater the velocity u in question. When such velocity is much smaller than that of light in the vacuum, as happens in ordinary experience, such effects are completely negligible. Indeed, it is easy to see that *Lorentz's transformations reduce to those of Galileo's* when u/c tends towards 0; therefore, in such a limit, the laws of classical mechanics hold with excellent approximation.

Physical causality in the theory of relativity and concept of «cones of light»

Moreover, the theory of relativity introduces a deep modification of the concept of temporal ordering, by claiming in substance that a clock that measures time everywhere the same in the universe does not exist, but rather there are as many clocks as the employed referential systems. Given two events, a and b , that occur in two different points in space, one can determine in a given inertial system the chronological order of such two events; however, it could sometimes happen that, by moving to another inertial system, such order is reversed. If, for example, for a given observer, a precedes b , it could happen that another observer, in motion with respect to the former, could see instead b preceding a . It is clear, then, that the chronological order of two events does not always have an intrinsic physical meaning. By means of Lorentz's transformations, one could determine which conditions the two events should satisfy so that a given chronological order be the same for all the inertial systems. One finds that a precedes b in every inertial system if the event b is reachable by a signal that starts from a and travels with a lesser or equal speed than that of light, that is to say, only if the event a could influence in some way b . From this principle, it particularly follows that, not only no physical body could travel at a velocity higher than that of light in a vacuum, but that some signal or physical medium that could allow the transmission of information at a velocity higher than c does not exist.

The principle of light-cones and its meaning in physics

This idea could be summarized in the famous principle of *light-cones*. Every event propagates in the future its causal effects more or less as the concentric circles of a stone thrown into water. By adding the third dimension of the height to represent the temporal development, the unfolding in time of the causal effects of an event looks like an inverted cone, which has as its vertex the very same event. All the events, which are causally reachable by the event in question, are comprised in such a cone, taking into account that the velocity by which these effects propagate cannot overcome c , the speed of light. This is the reason why one speaks of *light-cone*.

Vice-versa and complementarily, the considered event is the effect of another whole series of causal events, which are comprised in a second

light-cone, which has always the considered event as its vertex, and which contains all and only the events that could causally get in contact with the given event. That is, that they could reach it with a velocity lower than, or equal to, c .

All this is summarized in the following figures:

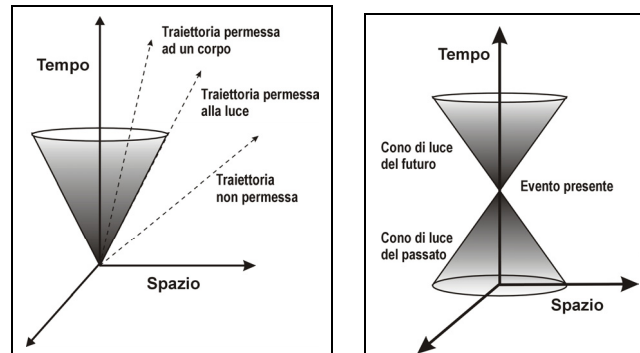


Figure 2-4. The principle of light-cones that determines the admissible causal events on the basis of the theory of relativity.

But the explanatory force of the theory did not end here, nor did their paradoxical consequences. Indeed, what our description is lacking is the most paradoxical and at the same time most fascinating of these consequences.

Two further consequences of relativity, linked to the bound-nature of the speed of light

In the end, the paradoxes that we have examined thus far modify the representative frame and the mathematical formalism of classical mechanics, but still cannot give the sense of how much the principle of spatio-temporal special relativity also influences the *physical content* of both this frame and formalism. The other two paradoxical consequences of the theory have instead these very same effects, which make them, in turn, explanatory of those terrible and marvelous phenomena – radioactive decay and nuclear fission and fusion – which the other conceptual revolution of the XX century, quantum mechanics, begun to discover and study exactly in those years.

The last two main paradoxical consequences of the theory are strictly linked to the fact that the *speed of light*, on the basis of the previous relations, constitutes a *maximal unbreakable limit* for bodies in motion. Hence, when a body gets accelerated, its quantity of motion mv increases. But if v , when it surpasses a certain limit, and gets close to c , cannot increase more than a given amount, then mass m will begin to increase! And such an increase of m will be the stronger, the more v will approximate its up-

per limit c . There are two fundamental consequences that follow from here:

4. Relativistic
increase of mass

- ◆ *The relativistic increase of the mass of accelerated bodies* for velocities close to the speed of light, on the basis of the relation:

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

In this sense, it is necessary to distinguish between the *mass at rest*, m_0 , which the body possessed when it was at rest and the *kinetic mass*, m , which the body possesses when it is accelerated to velocities that approximate that of light. The sense of the above relation is clear: when the velocity of body v tends to be the same of c , the value of the denominator of the right side of the equation tends towards 0, and thus the value of m tends towards infinity.

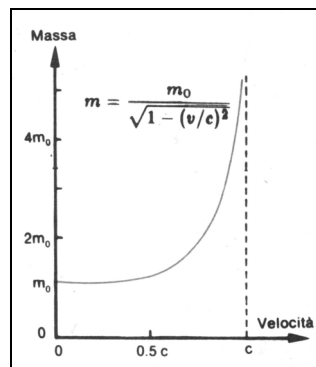


Figure 2-5. Course of the increase of kinetic mass for velocities close to that of light. One should take notice of the exponential increase of mass for values of $v > 0.5c$.

5. Equivalence*
mass-energy

- ◆ *The equivalence and reciprocal transformability between mass and energy* for bodies that move at velocities close to that of light, according to the famous equation:

$$E = mc^2$$

These latter relations have allowed us to explain the exceptional production of energy that arises regarding phenomena such as radioactive decay and nuclear fission and fusion. More generally: these two relations, especially the former, capture what happens every moment in particle

The value for
quantum
mechanics of such
discoveries

accelerators in physics research centers at high and very high energies, like, for instance, at CERN, in Geneva.

In particular, in the case of accelerated particles at velocities close to that of light, particles could acquire a new mass (kinetic mass) with respect to the mass that they had when they were at rest (mass at rest). Or, all of their mass «at rest» and «kinetic» could be transformed into another form of energy, both in the form of radiation and in the form of particles endowed with mass that were certainly not «inside» the original particles, provided that the total sum of mass-energy before and after the event remain the same. This is a phenomenon called the *annihilation-creation* of particles that one often observes in elementary particle physics, and which from a metaphysical point of view violates the principles of naive or metaphysical atomism (cf. *infra* § 5.2). For example, an electron and a positron (anti-electron), which are endowed with a given mass, although very small, can interact among themselves and become «annihilated». The product of this process is not a particle with mass, but pure, electromagnetic radiation at a very high frequency (γ ray). If we accelerate these particles to velocities close to c , *by acquiring kinetic mass*, their annihilation will also produce particles endowed with mass: neutrinos and neutrons that will quickly decay as protons, which are much more stable. These are particles which are very heavy and that certainly were not «inside» the original particles, as metaphysical atomism could make one suppose⁵⁸. As already said at the end of § 2.4, it is due to Dirac's equation that the equations of special relativity have been able to be incorporated into the theoretical framework of quantum mechanics, giving rise to the exceptional developments of the research in physics of the high and very high energies that are in front of us. But we shall return in the Third Part (Second Volume) of this work to discuss Dirac's equation, its development in Feynman's formalism and the role of *quantum electrodynamics* — and *quantum chromo-dynamics* —, both of the material constitution of all physical bodies, and the structure and evolution of cosmos itself.

⁵⁸ As we shall see later, the concept of relativistic mass-energy is a sort of quantifiable corresponding Aristotelian notion of *prime matter*, «in potency», with respect to the elementary particles that constitute the material substratum of all physical bodies. By means of such a notion, Aristotle corrected Democritean atomism, by maintaining (on the basis of the experimental evidence available at that time) the transformability of elements into each other. E.g. by following Empedocle, who identified the elements in terms of water, air, earth, fire, Aristotle had empirical evidence concerning the «volatilization» of water into air (evaporation), and of the transformation of earth into fire (probably due to dirt full of hydrocarbides), etc. So, the very same elements, considered by Democritus as eternal and unchanging, or just «atoms», do not constitute for Aristotle the ultimate basis of the material substratum of the bodies, but such ultimate substratum was prime matter, understood as that which could be transformed within each of the elements.

2.5.3 General Relativity

General theory and the problem of reconciliation between special relativity and gravity

Special relativity, and classical mechanics, assigns to the systems in uniform rectilinear motion a privileged situation, because only with respect to them the physical laws are invariant. The difficult task that Einstein faced in the elaboration of general relativity is that of making sure that the laws of physics conserve their structure in every referential system no matter how they would be accelerated: in other words, the laws of physics must be such that their form remains unaltered with respect to every observer; hence the equation of physics must be invariant not only with respect to Lorentz's transformations, but also *invariant with respect to every transformation*.

Einsteinian brilliant solution: gravitation is a force that modifies the structure of space-time

Let us say the same thing in more simple terms. The theory of special relativity with the importance that it gives to the finite speed of light, leaves a difficult problem unsolved: it runs into conflict with *gravitational force* and its capacity *to act instantaneously* at a distance. Between 1908 and 1914, Einstein tries in different ways to make special relativity consistent with gravity. From here we have the revolutionary hypothesis that gravity is not a force (like other forces), but rather the consequence of the fact that space-time is not flat, but *curved, twisted* by the distribution of mass and energy of the bodies that inhabit it.



In other terms, the celestial bodies follow a curved trajectory not because they move along orbits curved by gravity as in the usual intuitive representation, but rather because they follow that which most closely looks like a rectilinear trajectory in a curved space: *geodetics**, that on a sphere corresponds to a circular arch of greatest distance. In other words: in general relativity, *bodies follow straight lines in the four-dimensional space-time, but in our three-dimensional space, they appear as curved*. As when we project a straight line in three-dimensional space (e.g., the rectilinear route in three-dimensional sky) on a bi-dimensional curved surface (e.g. on the earth), it will appear to us as a curved line, at most a geodetic that connects on the curved surface the projections of the points that the straight line unites, as the shortest line between the same two points in three-dimensional space.

Employment of Riemannian geometry of curved space

General relativity, thus, uses for its mathematical representations, not Euclidean flat space (as in classical mechanics), but *the curved space of Riemannian geometry* (Cf. above, Figure 1-5). And, as plain Euclidean geometry can be considered a particular case of the curved one, for spaces with a curving equal to zero, e.g., valid for infinitesimal portions of curved space, so general relativity incorporates classical mechanics, by making physics capable of explaining phenomena which Newtonian classical mechanics could not.

The initial and extraordinary experimental confirmations of the theory

For example, one of the first confirmations of the theory of general relativity was the correction of Mercury's orbit, the planet closest to the sun, which is strongly influenced by the action of gravity exerted upon it: an experimentally proven correction, but which Newtonian theory could not explain. Another effect, which general relativity is able to explain, is the so-called «Doppler's effect» concerning the light of the stars.

It is well-known that the light of the stars appears to our eyes primarily as reddish, though its chemical composition does not justify at all such an appearance. The explanation is found as soon as one thinks that such an appearance is the effect of the curving imposed on the light rays by the surface of the Earth. The curving exerted upon a wave has, indeed, the consequence of «stretching it», by extending the wave-length, which means, in our case, to deviate the luminous radiation towards the lower part of the visible spectrum, precisely towards the red.

The most clamorous initial confirmation in the eclipse of 1919

But the most clamorous confirmation of the theory since becoming public occurred in 1919, when it was possible to verify that, during a total eclipse of sun, the light coming from the stars was distorted by the fact that it was moving close to the sun. At night, when the sun is not interposed between the star and us, the star appears in a certain position in the sky. At day, if general relativity were true, the position of the star in the sky should be seen as displaced to a certain degree, proportional to the curving imposed by the mass of the sun on the light ray coming from it. During normal days, it is however impossible to observe this, for the light of the sun cancels that of the stars. Vice-versa, during the eclipse of the sun of 1919, in a particularly clear sky (like that of Africa), a scientific expedition was able to verify and measure this phenomenon. In essence, a star which was observed in a certain position in the preceding night now appeared to be displaced in the morning, due to the curving of its light rays imposed by the interposition of the sun.

Theoretical consequence: the principle of equivalence

The unification of inertial phenomena with gravitational ones, which is one of the most important theoretical consequences of the relativistic theory of gravitation, was obtained by Einstein by formulating the famous:



Principle of equivalence. In a gravitational field (of small spatial extension) everything occurs as in space free from gravitation, provided that one introduces, instead of an inertial system, a referential system which is accelerated with respect to the inertial system.

This principle has also been confirmed to the general public by observing capsules returning from space flight. Inside of them, although they are in the gravitational field of the Earth and not in space any longer, there is an absence of gravity. The fact that the space craft, in a

General relativity
and cosmological
theories

free fall accelerated motion towards Earth, constitutes an infinitesimal part of the space upon which terrestrial gravitational force acts, makes it possible that the space inside the space craft be practically of a zero curving, and as if, inside it, earthly gravitation were not acting.

To conclude, the theory of general relativity, aside from providing a coherent physical theory for bodies which move at cosmic distances (the *megaloscopic* level of physical observation), has made *scientific cosmology* once again possible, i.e., the study of the origins and the evolution of the universe. But unlike ancient philosophical cosmology, the new cosmology uses the mathematical and experimental, Galilean method. The principle, which made it possible to extend the mathematical-experimental study to the inquiry concerning the origin and development of the universe, is linked to the conceptual revolution that general relativity has introduced in the physico-mathematical conception of space-time. In classical mechanics, but also in restricted relativity, space and time were thought as containers, not influenced by the dynamic events occurring inside them. Vice-versa, with general relativity, the structure of physical space-time is modified by the events that occur inside it. Moreover, given the expansive character of the universe, space and time, which are internal to the physical universe, are «created» by the reciprocal moving apart of the bodies that compound the universe itself. Even from this point of view, the monolithic framework of ancient and modern science concerning the beginning of the universe has been deeply modified.

2.6 Science of complexity

2.6.1 Dynamic Instability

The final state is
defined uniquely by
the initial conditions
in the systems
studied in classical
mechanics

As we have seen, the quantum revolution weakens the «classical» Newtonian ideal of an explanation of physical phenomena. The discovery of the law of phenomena that concern elementary particles no longer follows the principles of classical mechanics and electrodynamics. Such phenomena allowed physicists to conceive of the motion of these particles as events in which a body exactly identifiable follows a well-defined trajectory in space and time. In this sense, the objects of classical physics look like planets and are schematized as material points that move under the action of fields of forces. According to this scheme, besides the elementary particles (electrons, quarks, etc.) there are radiations, such as light, that propagate under the form of waves. In any case, the motion of these elementary «objects» is exactly determined by their initial conditions. If as well as these, one also knows the law of dynamic evolution of the system (e.g. the laws of motion), we can predict with exactitude

the future trajectory of the body and can, in a sense, «imagine it» as a displacement of something in space and in time.

In statistical mechanics one still moves in the ideal classical context: uncertainty due to ignorance

If probabilistic considerations intervene, as in the case of statistical mechanics, one could think that they are a short cut for dealing with overly complex systems or that they are the effect of the ignorance of the detailed circumstances in which the phenomenon is generated. Just as Aristotelian and Cartesian physics detested vacuums, so classical physics, in its integrity, detested chance: one cannot admit the existence of phenomena that escape every law and are intrinsically unpredictable.

Quantum mechanics brings about a first revolution, by highlighting the intrinsic limits of our representations

We have seen that quantum mechanics placed these traditional certainties in doubt. The necessity to represent mathematically quantum phenomena by means of the formalism of wave-functions destroys the ideal of the *unambiguous representability* of their motions as trajectories of material points. The necessity of probabilistic considerations manifests itself as expression of an intrinsic 'chance' in the mathematical representation of these phenomena, which makes the ideal of «absolute precision» of the observations and predictions — albeit an increasing precision — of the Newtonian–Laplacian paradigm of physical science, fade away forever.

But quantum mechanics still moves in a reductionist domain

However, quantum mechanics was not able yet to undermine another key-point of the classical paradigm, that of physical reductionism: what is simplest and most original explains what is most complex and recent. Rather, the intensive study of fundamental particles to solve the most intimate secrets of matter, and also clarify the problems concerning the origin of the universe, was driven by a reductionist ideal. The reductionist ideal, typical of classical physics but also accepted by quantum mechanics, has two directions, *synchronic* and *diachronic*.

Double direction of the reductionist principle, synchronic and diachronic

- ◆ *Synchronically*, to reduce the complex to the simple. To reduce the behavior of complex systems, that is, of systems constituted by a great number of particles — all the bodies which are the object of our ordinary experience studied by macroscopical physics, from minerals to living organism — to the laws that regulate the behavior of these elementary constituents.
- ◆ *Diachronically*, to reduce the final state to the initial conditions. All the many particle dynamic systems, studied by classical mechanics, statistical mechanics and quantum mechanics are stable systems, that is, the final state of the system can be unambiguously determined by its initial conditions (initial states + laws of dynamic evolution).

The discovery of dynamic instability in particular systems of classical mechanics in Poincaré's «three-bodies» gravitational problem



However, this framework had already been questioned at the end of the 19th century, by the discovery, which occurred in 1890 by the great French mathematical-physicist Henri Poincaré's (1854-1912), of the phenomenon of *dynamic instability* in the study of classical mechanical systems, not the macroscopical ones, apparently simple. For example, this would be applicable to the study of the dynamic evolution of a three-bodies system (e.g. sun, earth, moon) that interacted by means of gravitational forces. And so, although the system were governed by *differential equations**, the non-linear character of these equations, due to the fact that the object of the study was the interaction between three bodies — and not «two by two» (e.g., earth-sun, sun-moon, and earth-moon), as in the classical Newtonian approach to the «many bodies» gravitational problem — made it possible that the system of equations itself resulted not only *non-integrable* — in the sense of the impossibility to furnish an exact solution of the equations of motion —, but also *unstable*.

By varying the initial conditions to an ε degree, the subsequent trajectories in the *space of the phases** by means of which we can represent the dynamic evolution of the system, rather than remaining separated by the value ε (= condition of *dynamic stability*), they actually *diverge exponentially* among them (= condition of *dynamic instability*).

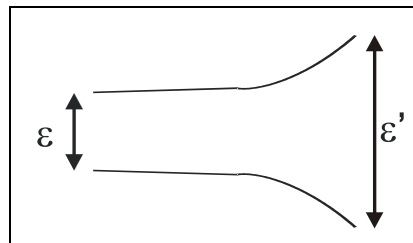


Figure 2-6. Exponential divergence of the trajectories in the *space of the phases** in phenomena of dynamic instability. The initial ε of imprecision in the determination of the initial conditions of the system is amplified by the dynamic itself, such that any prediction in the long run is impossible.

Uncertainty does not remain within the boundaries of a determined value as at the quantum level, but is amplified by the system

It is as if, the system itself, in the course of its evolution, *would amplify* the uncertainty ε — not eliminable in any finite physical system — in the determination of the initial conditions of the motion (position and quantity of motion) of the bodies that constitute the system. In an *unstable* dynamic system, a small difference in the determination of the initial conditions of motion produces great differences in the determination of the system's final state (Cf. Figure 2-6). This does not occur in the *stable* dynamic systems: the imprecision in the determination of the initial

conditions is the same that one finds in the final state. In such way, if one would like to establish with more precision the final state in a stable system it would be sufficient to increase the precision in the definition of the initial state. In unstable systems, instead, independently of the increase of the however finite precision of the definition of the initial state, one always obtains a much greater uncertainty in the determination of the final state. An uncertainty that makes the dynamic evolution of it, in the long run, completely *unpredictable*.

End of the
Laplacian
reductionist and
deterministic ideal
of classical science

To provide a sort of intuitive illustration of the principle: in stable systems it is as if I were to drop a ball down the walls of a closed vase. Even by varying significantly the initial condition of the position of the ball along the walls of the vase, with absolute certainty, I know that, after a more or less long drop, the final stable position of the ball on the bottom of the vase will be unique. Vice-versa, if I would put the same ball on the top of a mountain, a very little difference in the initial conditions of motion of the ball — a little variation in the position or in the direction of the initial impulse — would determine a great variation in the final state. For example, it will determine whether the ball will fall along the south or north side of the mountain in question. This is how Poincaré himself explained his disturbing discovery, which immediately cancelled the ideal of classical mechanics as the kingdom of the most absolute reductionist determinism, according to what «the principle of Laplace's demon», formulated just a century before and above mentioned, exemplified (Cf. § 1.4.2).



A very small cause that could escape our attention determines a considerable effect that we cannot fail to see, and so we say that the effect is the product of chance. If we were able to know exactly the laws of nature and the situation of the very same universe at the initial moment, we could exactly predict the situation of the same universe at a successive moment (Laplace's idealized demon, *N.o.A.*). However, even if it would happened that the natural laws would not hold any secret for us, *even in that case we could know the initial situation only approximately*. If this would allow us to predict the situation with the same approximation, we would not need anything more and should say that the phenomenon has been predicted and is governed by laws (classical mechanics as the partial realization of the Laplacian demon, *N.o.A.* Cf. § 1.4.2). But this is not always the case: it could happen that small differences in the initial conditions produce very great differences in the final phenomena. A little error in the former produces an enormous error in the latter. The prediction becomes impossible and one has a unexplained phenomenon.⁵⁹

⁵⁹ From *Science et Méthode*, quoted in (Ruelle 1984, 19).

The aleatority of an unstable system does not depend on an insufficiency in the number of variables regarding the representation, as in the systems studied in statistical mechanics



The complexity of the behavior in an unstable system does not depend, then, on the number of involved variables and hence on the insufficiency of the representation. A very small number of variables (*degrees of freedom**) of the system (three) can already determine an enormous and irreducible variability in the behavior itself. An unstable dynamic system is thus *qualitatively different* from the systems studied by classical statistical mechanics that satisfied the so-called *Boltzmann's hypothesis*. That is, the hypothesis that the aleatority were linked to the increase of the number of the degrees of freedom of the system, given that the dynamic evolution of it would have certainly brought it — as in the case of the thermodynamics of gases: Cf. above § 2.3.2, — to assume all the configurations that are consistent with the principle of conservation of energy. An hypothesis which made it possible, even in the case of these many-particle systems, to univocally define the final state. In classical statistical thermodynamics, the system will necessarily tend towards a unique possible final state, the most probable state, the state of equilibrium of minimal energy or maximal entropy: an ingenious way, this one by Boltzmann, to combine chance and necessity, so as to maintain the deterministic paradigm as unaltered, even within statistical mechanics. Indeed, due to the *ergodic hypothesis*, the irreversibility of thermodynamical phenomena ended up perfectly consistent with the deterministic, perfectly reversible, laws of classical mechanics. Other more ordered states of the system remained equally possible, except that, being particularly improbable for them spontaneously to reappear, one would need such a long time (the so-called *Poincaré's time of recurrence*) to make it practically unobservable.

We shall return to this essential point later, in order to comprehend metaphysically the relationship between *contingency* and *determinism* and thus the center of the theory of complexity (Cf. § 2.7.2).

In such systems the aleatority is extrinsic to the dynamic system



In sum, the uncertainty in the macroscopical determination of a many-particle system studied by statistical mechanics is only linked to an *extrinsic* limit of representation of the system. It is only linked to the fact that one is observing the system with an insufficient number of variables as soon as one would adopt an adequate resolution, that is to say, the number of degrees of freedom of the system would be increased — e.g., in a gas being heated one considers the motion of the single particles —, the aleatority would disappear, and the system would become perfectly deterministic.

In unstable systems aleatority is intrinsic; it is linked to the irreducible non-linearity of the system

Instead, the degrees of freedom of the dynamic system studied by Poincaré are always few in number, in fact, very few (three). Nevertheless, the system is characterized by an irreducible aleatority in the long run prediction. In sum, what Poincaré demonstrated is that mechanics, both classical and statistical, was until then applied to very simple problems, or at least reducible to very simple problems. As soon as things get complicated, as in irreducibly non-linear systems of the sort studied by Poincaré, the dynamic evolution resulted completely unpredictable, at least for the formal means of analytical calculus.

Difference with respect to quantum mechanics

But the unpredictability of a system characterized by dynamic instability is not even linked to a limit *intrinsic to the representation* of the system, and nonetheless *extrinsic to the dynamic* as the aleatority of quantum mechanics discovered some decades after Poincaré's work. In it, as it is made explicit by the principle of indeterminacy (Cf. § 2.4.3), the unpredictability seem to be linked to the conjunction of the variables, due to the non-eliminable interference of the instrument of measurement upon the measured quantity.

In quantum mechanics aleatority is linked to an intrinsic, irreducible limit, but always and only in the representation

Instead, as recently pointed out by the Italian physicist, Giulio Casati, the discovery of the dynamic instabilities of classical mechanics has created a *paradoxical situation*. Quantum mechanics is an *intrinsically probabilistic* theory indeed. It has an intrinsic nature, however, linked — at least in the classical model without decoherentization — to the representation and change of representation (from representation in terms of wave-functions to representation in terms of trajectories), not to the dynamics itself. In other terms, given a function of state $\psi(t)$ that represents the state of the system at the instant t , it is only possible to make probabilistic claims about the fact that, by performing a certain measurement upon a determined magnitude (e.g., the position, once the quantity of motion is known), we shall obtain a certain value that defines the final state. Nonetheless, with respect to the future evolution of the system, it is possible to make predictions, given that in quantum mechanics we are generally in the presence of dynamically stable systems. Given the laws of motion and the initial state $\psi(0)$, it is possible to *unambiguously predict* the state $\psi(t)$: the future state of the system depends unambiguously on the initial state, and the initial uncertainty linked to the principle of indeterminacy is not amplified by the dynamic evolution of the system.

Quantum systems are dynamically stable

Instead, it is not possible to say the same thing in those systems of classical mechanics in which one finds the phenomenon of dynamic instability. Here the uncertain nature of the prediction is not only irreducible, as in the case of quantum mechanics, but it is also *intrinsic* to the *dynamic*,

and not only to its mere formal representation, as in the case of the uncertainty of the prediction in stochastic systems and quantum systems. In sum,



In unstable systems aleatority becomes an unavoidable and intrinsic ingredient of the physical-mathematical study of nature

even though quantum mechanics is intrinsically probabilistic (but does have an intrinsic nature linked to representation, *N.o.A.*, Cf. § 2.4.7), given its character of instability, it turns out to be more predictable than classical mechanics (aleatority is not intrinsic to dynamics, *N.o.A.*) (Casati 1991, 9).

What makes the phenomenon of dynamic instability absolutely unique with respect to other forms of aleatority studied in modern physics and mathematics, is that for the first time the casuality, the aleatority — and an irreducible and catastrophic casuality — appeared as an *intrinsic and non eliminable ingredient* of the study of dynamic systems, independent of empirical observation and/or of formal representation. Thus it happened that non-linearity, the lack of proportionality between modification of the initial conditions and modification of the final state, or, if we want, a lack of proportion between cause and effect — a realistic interpretation of it⁶⁰ —, entered with force in the arena of modern science. And it entered with all its force of *confutation of the reductionist principle*, clearly based on an excessive confidence in the ultimately linear character of all physical laws, beginning from the paradigmatic one of the second law of dynamics.

The new situation introduced in the frame of modern physical sciences by the discovery of dynamical instability, is summarized in the following

Table I.

⁶⁰ For example, the one given by Aristotle, concerning the physical processes that today we model in terms of dynamic instability, of «deterministic chaos» or «of processes of self-organization». In his metaphysical realism, he instead talked about «processes of generation/corruption of natural forms» and, exactly because of this, he spoke of the fallacy of the atomistic-geometric hypothesis in the philosophy of nature. And this is the root of the famous Aristotelian theory of the «four causes», which we shall be dealing with at the beginning of the Second Part.

ALEATORITY				
Stable Systems		Classical Mechanics	Statistical mechanics	Quantum Mechanics
	None	Linear System		
	Extrinsic		Stochastic Systems	
	Intrinsic to Representation			Quantum Systems
Unstable Systems	Intrinsic to Dynamics	Non-linear Systems		Quantum chaos?

Table I. Summary of the distinction among stable and unstable systems in contemporary physics.

2.6.2 Deterministic Chaos⁶¹

The discovery of dynamic instability alone could not be enough to justify the birth of that new paradigm in contemporary science, which is labeled as the *theory of complexity*⁶². For this to come about, a further step was needed: a step linked to a «surprise» that the study of dynamic instability brought about more than seventy years after Poincaré's discovery.

⁶¹ For an introduction to the concept of «deterministic chaos» one can profitably read a now classic in popular science, the book by James Gleick, which has seen various reprintings in Italian (Gleick 2000). For an introductory anthology of more scientifically oriented essays, which are now classics as well, cf. (Casati 1991).

⁶² The scientific notion of «complexity» is particularly difficult to define. On this problem see the recent work by F. T. Arecchi, one of the first Italian scholars of deterministic chaos, and by A. Farini on the *lexicon of complexity* (Arecchi & Farini 1997).

Chaotic systems
which are found at
each level of the
physical inquiry

The «surprise» consisted in the fact that that which simply seemed a limit, in some way unsurpassable, to the predictive, and thus explanatory, possibilities of classical mechanics, it instead appears today as one of the most fruitful fields of physical research for the study of the emergence of order out of chaos under the action of determined causes — thus according to processes traceable back to laws —, in such a way to create a new branch of physical research itself, which goes under the name of the theory or *physics of complexity*. Still nowadays it is able to apply its *descriptive* models — even if still not explanatory *in the mathematical sense*, due to the lack of a formal adequate theory of complex systems — in fields thus far inaccessible to physical research. And it is able to do this in a way which is «transversal» with respect to the subdivision that we have adopted in this chapter among the *microscopical* (quantum mechanics), *mesoscopical* (thermodynamics), *macroscopical* (classical mechanics), *megaloscopical* level (general relativity) of physical inquiry. Instances of unstable systems can be found at each of these four levels.

Presence of chaotic
phenomena also at
the level of quantum
systems

In particular, a class of unstable systems, the chaotic systems, seems to be extremely promising (from the point of view of theoretical and experimental studies) for producing mathematical models applicable in many fields of physical study: from the study of turbulence to those, extremely complex, of oscillatory chemical systems, of various metabolic functions in biological systems (Ruelle 1992; Serra & Zanarini 1986), of physiological functions (e.g., heart-beating; dynamics of arterial flux) and of neurophysiology (e.g., dynamics of single neurons and of networks of neurons in the nervous systems of various animals) (Belardinelli & Pizzi 1993; Freeman 2000). Apart from the field of physical macroscopic and mesoscopic systems referred to in the quoted examples, other known examples of chaotic systems concern megaloscopic systems such as the dynamic of galaxies, but also microscopic systems, given the existence of *quantum chaos*, that is, given the discovery of unstable systems in quantum mechanics to which it is not possible to apply, with predictive success, the classical formalism of Schrödinger's linear wave-function (Prigogine 1999).

Possibility of
applying chaotic
models also in
human sciences

But the fields of applicability of chaotic models are not limited only to the study of natural, physico-mathematical sciences. It also extends to the human sciences, with models applied to the description of the macro- and micro-economical dynamics of markets (Anderson *et al.* 1988; Gatto & Marino 1998); to the study of the dynamics of popula-

tions in sociology; to the study of perceptual dynamics in psychology, even including the study of creative strategies concerning business management (Stacey 1996)... Practically, there is no domain in modern applied mathematical sciences, both regarding nature and human beings, where kinds of chaotic models could not be applied with different degrees of success, to an at least descriptive characterization of the dynamic behavior (evolution in time) of the most diverse, physical, chemical, biological and human systems, studied by these sciences.

We still lack an explanatory theory of deterministic chaos

Though we still do not have a mathematical theory which satisfactorily explains chaotic systems — and this is the reason why the chaotic models applied to the study of both natural and human phenomena cannot, in their turn, be explanatory of these phenomena, i.e., able to define *dynamic laws of evolution* of the phenomena themselves, in order somehow to be able to *control* them — we do, however, have the possibility of providing a *descriptive mathematical characterization* of such systems. There are, in other words, different measurements that can be used as definiens of other equally necessary conditions to assert whether a given dynamic system is «chaotic» or not⁶⁴. No one has yet come up with a condition or set of conditions that could be defined not only as necessary, but also as sufficient to the existence of a chaotic system. If we knew this set of conditions, it would mean that we would have an explanatory theory of chaotic systems.

Chaotic systems cannot be assimilated to stochastic systems

That which can be said, in the negative, is that chaotic systems are not reducible to stochastic systems traditionally studied in statistical mechanics, according to the paradigm given by *Langevin's equation* (Paul Langevin, 1872-1946), originally developed for the characterization of *Brownian motion*, that one, for example, which is typical of a casual, not balanced motion of particles in thermal agitation. According to this equation, the function $v(x)$, which describes the macroscopical evolution of an observable quantity x of the system, is not sufficient to describe the instantaneous state of x . To it, one should add a disturbance limit (noise) according to the following equation:

$$\frac{dx}{dt} = v(x) + \sigma(x)\xi$$

where $\sigma(x)$ is the amplitude of the disturbance and $\xi = dw/dt$ is known as «white noise», a disturbance limit that could be considered as

⁶⁴ One of these measures is that of *Lyapunov's exponent*, which measures the distance between successive points along the trajectory in the space of the phases, so that a positive exponent expresses divergence and thus a possible chaotic nature of the system (Cf. Figure 2-6, p. 145).

derivative of a Wiener's process. As one could see from the form of the equation, the noise, the aleatority, appears as an *extrinsic* limit, which is added to the dynamics $v(x)$. And indeed, in Langevin's formalization, such a limit exactly expresses the effect of a casual disturbance, constantly applied to the system «from the outside». In the case of dynamic instability, we are confronted with an *intrinsic limit of aleatority*, linked to the dynamic evolution of the system and not «added from outside».

Unpredictability in the long run of the behavior of chaotic systems

What is thus meant by the term *chaotic system* and by the apparently paradoxical expression of «deterministic chaos»? The expression aims to connote a physical *deterministic* process, in which the next step causally depends on the preceding state of the system as it is formally expressed by the fact that, for example, the dynamic can be represented by means of a differential equation⁶⁵. The irreducibly non-linear nature of the equation, however, is, due to the phenomenon of the exponential divergence of the trajectories, at the origin of the intrinsic unpredictability of the system concerning *long term* behavior. The initial conditions, which could be known with a precision that could be increased at will, though always in a finite way, could never predict with certainty the final state, or — experimentally the most realistic case — one could never, from the final state of the system, return to a unique set of initial conditions.

Non-ergodic character of chaotic systems unlike other kinds of unstable systems

What makes chaotic systems theoretically very interesting is that they are very simple, unstable, dynamic systems, the behavior of which does not follow the *ergodic hypothesis*: unlike, for example, the classical, unstable dynamic system studied by the Russian mathematician, Yasha Sinai. He has demonstrated the ergodicity of a very simple dynamic unstable system: that of a billiard table with circular, convex obstacles. After a few hits of a ball with a positively curved surface, the amplification of the initial differences increases so quickly as to make the final state absolutely unpredictable (Ruelle 1984, 15). This is the reason why the sides of traditional billiard tables are rigorously straight! Vice-versa, chaotic systems do not satisfy the ergodic hypothesis; nor, in truth, do many other dynamic systems, as the so-called KAM theorem (Kolmogorov-Arnold-Moser) has demonstrated in the '50s (Ruelle 1984; Vulpiani 1994).

Chaotic system studied by Lorenz

The first example of chaotic behavior of a dynamic system that is usually found in the literature is that studied by the American physicist Edward N. Lorenz of MIT, Boston, in 1963 (Lorenz 1963). He devised such a system as a mathematical model for certain phenomena of atmospheric turbulence, aiming at the improvement of meteorological

⁶⁵ There are of course also examples of non-differentiable chaotic systems, but here we do not want to enter into such questions.

predictions. Recently, in 1999, precisely regarding the chaotic system studied by Lorentz, Warwin Tucker, from the University of Upsala, has provided the mathematical demonstration of the chaotic behavior of the system of equation studied by Lorentz. In practice, he has demonstrated *the existence*, in mathematical terms, of the «strange» or chaotic Lorentz's *attractor** (Tucker 1999; Viana 2000).

Current
mathematical
confirmations of
chaotic systems

This demonstration does not leave room for any doubts — if any of them could have remained at all — that chaos is not a sort of invention of the computer, derived from the fact that it numerically calculates the dynamic evolution of a system, thus with a finite precision. On the contrary, the mathematical models of the chaotic systems constitute a class of mathematical entities with the same dignity of the others, and, certainly, much more fascinating and mysterious than perhaps all of those studied thus far (Stuart 2000).

Kinematic
(geometric)
characterization of
stable dynamic
systems. Notion of
attractor

To introduce ourselves to a *kinematic**, geometrico-mechanical, characterization of the chaotic behavior of a dynamic system like Lorentz's, one should carefully examine the notion of the *space of the phases** as a powerful instrument for the geometrical representation of the behavior of dynamic systems in the short and long run.

For example, if we represent in a bi-dimensional space of the phases, with *position* on the x axis and *quantity of motion* (velocity) on the y axis, the behavior of a pendulum, which undergoes a friction which slows down its motion, the different states that the pendulum will assume in time will be represented by an equal amount of points in the above mentioned space of phases. These points will dispose themselves along a spiral-trajectory, with increasingly narrow spirals, which will tend in the end to a point that will represent the state of rest which the pendulum will necessarily reach. The final point does not move: it is a fixed point, and since it is as if it were pulling to itself orbits that depart from different (initial conditions) points (e.g., one could start the oscillation of the pendulum from different heights), it is defined as an *attractor**, to be precise, as an *fixed-point attractor** of the dynamics.

Thus, the fixed-point attractor represents very well the dynamic behavior of many dynamic systems, such as, for example, our pendulum, which is subject to friction.

Fixed point attractor

In all of these systems, the final state will be a state of rest, independently from those which were the initial conditions of motion of this system (Cf. Figure 2-7).



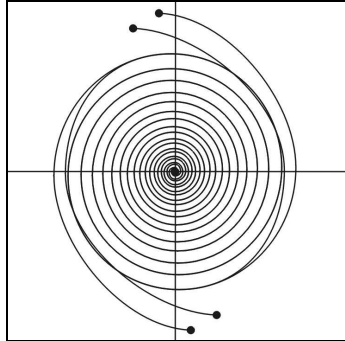


Figure 2-7. A fixed-point attractor. As one can see, even starting from very different initial conditions (the points of origin of the lines outside the spiral) the system will stabilize itself along a unique spiral trajectory and thus along a unique final state of rest.

Limit-cycle Attractor

By contrast, other stable systems do not tend, on the long run, to a state of rest, but rather to repeat periodically a certain behavior. An example is constituted by the above-mentioned pendulum, when the energy that is consumed by the frictions is reintegrated by the spring-mechanism of a clock. In such a case, the final state, the limit-behavior at which the system will stabilize, is not a state of rest, but a periodic oscillation. Geometrically, the attractor will not be represented by a fixed point (state of rest), but by a closed curve, by a limit-cycle (periodic motion). Many stable systems exhibit a similar behavior on the long run, for example, the beating of the heart after exerting effort.

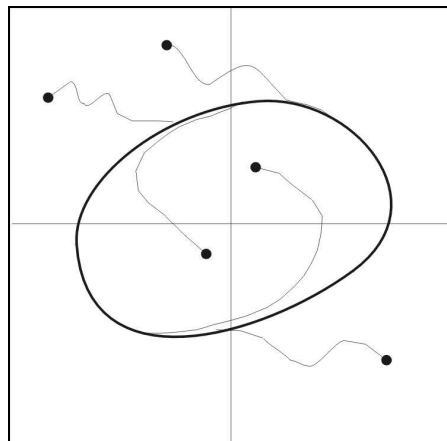


Figure 2-8. Limit-cycle attractor. Although it starts from very different initial conditions, the system will tend, in the long run, at most to stabilize in a peri-

odic behavior, a cycle, which is represented in a space of phase by a closed curve.

Toroid
attractor for more
complex stable
dynamic systems

Other systems stabilize themselves, instead, on quasi-periodic behaviors, that is to say, they stabilize themselves according to more complex cycles which are generated by the combination of two or more oscillatory (pseudo-cycles) behaviors. In this case, the attractor will not be constituted by a one-dimensional trajectory, a limit-cycle, but by a *bi-dimensional toroid* (a sort of ring-shaped donut) — but there could be *n*-dimensional trajectories in the space of phase —, constituted by the finishing of these pseudo-cycles. In spite of this complexity of structure, the system is, however, predictable. Even if the orbit never exactly repeats itself — otherwise we would be facing a limit-cycle —, as happens if the frequencies of a periodical motion do not have the same divisor, nevertheless orbits that start close to the toroid remain near to each other. We are thus always facing a *stable* system.

Traditionally, the
study of attractors
ended here.

Up to Lorenz's discovery, it was thought that the typology of attractors in the space of the phases had to be reduced to these cases: fixed points, limit-cycles, bulls, and thus to *stable* systems. They ended up as systems that satisfied the above mentioned Boltzmann's hypothesis. Their behavior with respect to limit-time, however, would have uniformly covered all the space of the phases, without giving rise to the existence of any attractor.

Lorenz's discovery
of «strange» or
chaotic attractors.
Anomalous
character of chaotic
systems: unstable,
although they show
a structure

To model mathematically the behavior of a very complex fluid (such as the atmosphere), beginning from the equations of the motion of a three-degrees-of-freedom fluid,

$$\begin{aligned}\dot{x} &= -\sigma \cdot (x - y) \\ \dot{y} &= R \cdot x - y - xz \\ \dot{z} &= -bz + xy\end{aligned}$$

Lorenz had discovered a behavior of this system that is not reducible to any of the typologies thus far known. The system clearly exhibited an aleatory behavior due to the phenomenon of the exponential divergence of trajectories. Orbits that begun close to each other did not remain in such a condition, but quickly moved far away from each other. Nonetheless, by simulating the system on a computer and reporting its long term behavior on a graph of the space of phase, the system did not cover all the space of the phase, but only a finite volume of it. That is, it gave rise to an attractor, but an attractor not reducible to any of the types regarding the stable systems studied thus far, and hence it was called a *strange* or *chaotic attractor* (Cf. Figure 2-9).

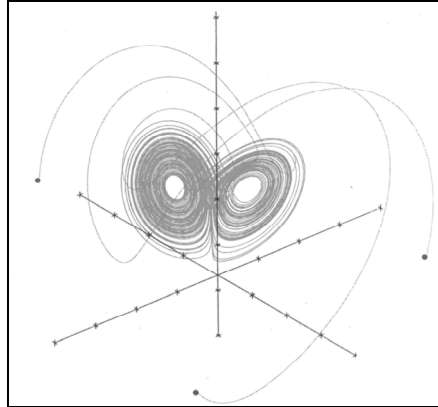


Figure 2-9. Lorentz chaotic attractor. One can notice how, despite the phenomenon of exponential divergence, the system tends in the long run to localize itself in a finite volume of the space of phase, defined «chaotic attractor».

After Lorenz,
discovery of many
other examples of
chaotic attractors

After Lorenz's discovery, many other strange or chaotic attractors were discovered both in the study of abstract mathematics and of mathematics applied to the attempt to model the most varied complex systems, both in the natural and human sciences.

Attempt to supply at
least an explanation
of the origin of a
chaotic behavior by
means of the
«baker's
transformation»

Returning to the dynamic systems of physics, an attempt to explain at least the origin of such an original behavior could be provided in the following way. Being the space of phase of a finite physical system, even in the case of exponential divergence, sooner or later they will have to end up close together. The most reasonable physical explanation of such a constraint is, for example, the presence of friction. This is the case of dissipative chaotic systems, among the most interesting to be studied in chemistry and biology, given that the majority of chemical systems and *all* biological systems are dissipative systems. Geometrically, one could represent all of this by claiming that the trajectories, which deviate and always go on to deviate in two dimensions, one with respect to the other, seem to get close in a third dimension.

Just like the baker when he kneads: when he stretches the pasta, two closed points move far way. When he folds it on itself, he reconnects these points, but in a third dimension. In the former two, the two points go on being distant.

Chaotic dynamics
characterized by
pseudo-cycles and
quasi-periodic
behavior

It is as if, in sum, the space of phase would fold upon itself, giving rise to a quasi-periodic behavior. The trajectories, though continuing to diverge in two dimensions, will pass again to be close to each other in the third dimension, within a given ε , without ever closing in on themselves, thus constituting certain pseudo-cycles very different from those of the

pseudo-cycle of a toroid. Indeed, trajectories that begin by being close to the attractor, do not remain close at all. Given its instability and non-stationarity, the system could unpredictably jump from one pseudo-cycle to another one, remaining on each cycle for absolutely irregular and non-predictable intervals of time. Hence we have the complex, «strange», structure of the attractor.

«Fractal» character
of chaotic attractors

All this takes us to *another property*, the so-called *fractal structure* of the strange or chaotic attractor. Given the phenomenon of exponential divergence and of folding in on themselves, it is as if the trajectories would go away (due to divergence) and get closer again (due to dissipation) continuously among them, thus giving rise to more and more complex and articulated pseudo-cycles. Returning to the metaphor of the «baker's transformation», when he stretches the pasta two close points get far away, when he folds them they get close again, but in an always different manner as they were before, through paths which are *irreversible* and thus absolutely unpredictable. Through this course of indefinite displacement-unfolding, of continuous mixing up of the trajectories, of creation of pseudo-cycles, more and more indefinitely complex, the chaotic attractor acquires the typical structure of a *fractal*, geometrical, self-resembling object. A structure that indefinitely repeats itself, in whatever way one is looking at it, without ever collapsing in a simple object, a point or a one-dimensional line (Cf. Figure 2-10).



Figure 2-10. Classical example of a fractal object, characterized by a property of *auto-similarity*. For example, if we would observe with a magnifying glass the first «A» of the figure, we would not see so many little «A's», but simply some little spots of ink. At different resolutions, the object does not maintain the same structure, is not auto-similar. The object has a structure only under a given resolution. Vice-versa, at whatever resolution one would observe a fractal object, it always manifests a structure that repeats itself indefinitely. In the figure, this is the case of the second, third and fourth image of the figure produced by the first «A» through a recursive, indefinitely repeatable process at greater and greater resolutions.

If these two characteristics — pseudo-cycles, or complex quasi-periodicity (Cf. Figure 2-11), and fractality — are the emerging geometrical properties of a chaotic attractor, one understands how, from a sys-

tematic point of view, the most promising attempt of providing a mathematical characterization of a chaotic system is that of characterizing it through a series of pseudo-cycles with increasingly longer periodicities⁶⁶ until the whole chaotic attractor is covered (Auerbach *et al.* 1987).

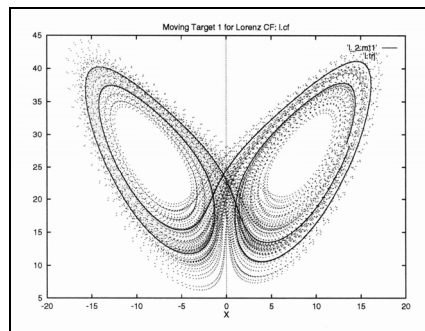


Figure 2-11 Typical pseudo-cycle of Lorenz's attractor. As one can see, we are not in front of a trajectory closed upon itself, but the system always passes only in the neighborhood of a determined point, without ever passing again exactly over it. This is the reason why one speaks of a pseudo-cycle, and correspondingly, of quasi-periodic behavior with fractal structure.

Typical
pseudo-cycle of
Lorenz's attractor

In this case, the aleatory behavior of the system would be interpretable in the space of phase as deriving from the stationarity of the same one, for a certain unpredictable temporal interval, on a pseudo-cycle, to jump suddenly, and without any predictability, over another one, and so on

⁶⁶ E.g., the symbolic sequence ABCDABCDABCD... has periodicity 4: i.e., the same subsequence will appear again at every four occurrences. So, all this constitutes a cycle of order 4. A similar case is that of the sequence ZBNPZBNPZBNP that will always be a cycle of order 4. The sequence XCGDXWQCOXUI XCGDXWQCOXUI XCGDXWQCOXUI has instead periodicity 12. The sequence:

XCGDXWQCOXUIYCGDXWQCOXUIWCGDXWQCOXUIZCGDXWQCOXUI is constituted instead by pseudo-cycles of order 12. Indeed, at the beginning of each regular sub-sequence expressed by the underlined letter, there is not the same letter X, but letters close to it in the international alphabet, e.g.: Y, Z, W. The cycle, every 12 steps, does not close upon itself again, but passes near its point of departure. Geometrically, in the space of the phases of this symbolic dynamic, it should be represented by an open curve with close extremities: it thus constitutes a pseudo-cycle.

XCGWRQTYZDFGDXWQLKJHYUFPOCOXUINGFDWCGDXCXDWQCB-VCFERTWPIUOXUI constitutes instead a typical example of a chaotic sequence where pieces of different length of a pseudo-cycle X(W)CGDXWQCOXUI repeat at variable intervals in an unpredictable way; intervals which constitute in their turn pieces of other pseudo-cycles. This makes the sequence, on the whole, appear as a casual sequence, while in reality it is not 'casual': it has a structure.

indefinitely, over all the pseudo-cycles in which the attractor can be divided.

The result of this behavior of a chaotic system with respect to its predictability is that, though the system is confined to the finite volume of the attractor, if at the moment t_1 , it is placed on whatever point of indefinite, quasi-periodical trajectories that compound the attractor, the next moment t_2 it could be found in another point of the attractor itself, on another pseudo-cycle, without any possibility of predicting *a priori* where and when the system will be located on a given point of the attractor. It is as if the initial uncertainty, though small, were to become amplified along the whole amplitude of the attractor, without any room for some way to predict the future. Unlike the pseudo-cycles which characterize a toroid-shaped attractor, the trajectories are here not stable at all: close points do not remain close: at every moment, they can be «shot» by the dynamic in whatever zone of the attractor, to reappear close again, who knows how and when.

Capacity of chaotic
dynamics to
generate
information



So, if one observes the temporal series that describes the evolution of the dynamic system it appears completely casual (= *chaos*). In fact, however, this casuality is constituted by a casual combination of pieces (parts of pseudo-cycles), each one determined and not at all casual, but rather with well defined properties and behavior (= *deterministic chaos*).

From an informational point of view, as it has been rightly noted,

Since in non-chaotic systems, the close points remain close during the temporal evolution, a measurement provides a certain amount of information that *remains constant in time*. This is exactly the sense in which these systems are predictable: the measurement contains certain information that can be exploited in order to predict their future course. (...) The operation of stretching and unfolding *instead systematically eliminates the initial information and replaces it with new information*: the stretching amplifies the indetermination on a small scale, the unfolding brings near trajectories very distant ones from the others, and erases the information on a large scale. Hence, the chaotic attractors behave like a sort of pump: since they bring to a macroscopical manifestation the microscopical fluctuations. (...) After a brief period of time, the indeterminacy corresponding to the initial measurement covers the whole attractor and any capacity for prediction is lost: *there is no causal link between past and future anymore* (Chrutchfield *et al.* 1987, 29).

The relevance of all this for science and modern philosophy of nature will be clear from all the rest of this work and above all from the parts

that regard, in the Second Volume, the biological and cognitive sciences. For now we limit ourselves to point to the conceptual connection that will permit these further developments.

2.7 Conclusion: end of the reductionist myth

2.7.1 Dynamics of chaotic systems

Passage to a dynamic characterization of chaotic systems

We have limited ourselves thus far to a purely *kinematic** characterization of unstable systems, and in particular of chaotic systems. The link to what we have already said about non-linear thermodynamic systems, the dissipative structure, the stable systems far from equilibrium, and thus to the study of the chemical and biological systems, will immediately become evident by extending our horizon from kinematics to the *dynamics** of chaotic systems, from a characterization of their behavior in geometrical terms, to one in terms of the forces that determine it.

Function of a single minimum potential of a stable linear system

To intuitively understand, in dynamic and not kinematical terms, the difference in behavior between a stable, linear system, and hence a system scarcely sensitive to small modifications of its initial conditions, and an unstable, non-linear system, which is sensitive to the slight modifications of the initial conditions, it is convenient to start from the graph of the *function of potential* $U(x)$ of a stable system. By means of this graph we have the integral of the generalized force of that system, «the sum» of all the forces that act upon the particles that compound the system. By representing the behavior of the system as the motion of a ball inside the function of potential, one understands that, for whatever initial condition, after a more or less long movement, the system will necessarily find itself in its more probable state, which will correspond to the minimum of its function of the potential, or *state of equilibrium* (Cf. Figure 2-12). All the linear dynamic systems are characterized by a function of potential of this kind, which will kinematically correspond to a fixed-point or limit-cycle attractor in the space of phases (Cf. Figure 2-7 e Figure 2-8).

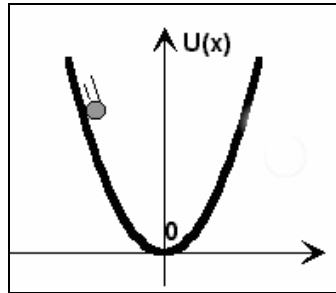


Figure 2-12. Function of the potential of a linear dynamic system characterized by equilibrium-stability. The ball, which represents the dynamic evolution of the system, will stabilize itself in a unique final position, perfectly predictable.

Function of a varied minimal potential of an unstable non-linear system

The condition of dynamic instability is instead linked to a much less regular landscape of energy than the one provided by a function of the potential of a linear system. The landscape of energy linked to a non-linear dynamic system is characterized by a varied minimal function, where small changes in the initial conditions could have a «disproportionate» influence, precisely non-linear in the long run evolution of the system. Nonetheless, the presence of many stable equilibrium-points, could make the system assume a globally predictable behavior, though it would be a more «articulated» stability than that of a linear system. For example, it could remain «entrapped», for a certain time, in one of the local minima's that characterize the landscape, as if they were an equal amount of «fixed points». Or it could assume a more articulated stable behavior by «cycling» among different these various minima's and giving rise, kinematically, to stable attractors such as, for example «limit-cycles» or «bells» (Cf. Figure 2-13). In sum, despite its non-linearity, the existence of different equilibrium points of the system would make one to think in this case about the *structural stability* (Cf. footnote 68) of the system itself.

Instead, in the case of a dissipative unstable system of a chaotic kind, the situation becomes even more complicated because the same landscape of energy could be subject to a mutation, with continuous «curlings» and «stretchings» — continuous creation and destruction of local minima's —, giving rise to the typical phenomenon of the sudden and unpredictable «jumping» behavior of the system among pseudo-cycles, or to the phenomenon, which is typical of many chaotic systems, of sudden transitions, of the «iteration» among «islands of stability» and passages to instability.

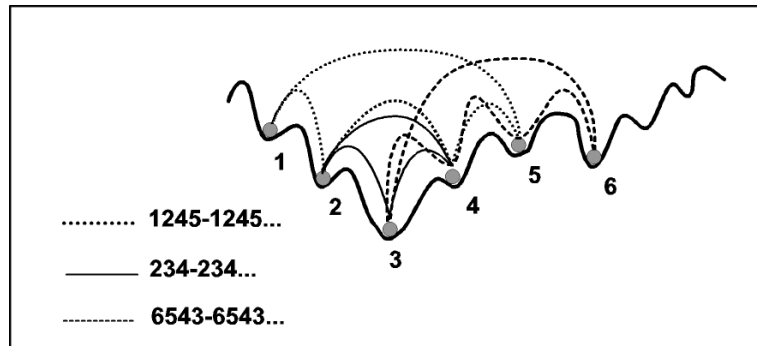


Figure 2-13. Landscape of energy of a non-linear dynamic system, which is characterized by local instability, but by a global structural stability of the system due to the presence of stable equilibrium points with respect to which the system can exhibit, for example, a complex cyclic behavior.

Basic chaotic systems for the physico-dynamic study of the dissipative structure, typical of living beings

From our more systematic point of view it is clear, however, that the non-integrable dynamic systems characterized by dynamic instability are much more realistic, much closer to the complexity of reality than those integrable and stable systems studied at the beginning of classical mechanics and modern statistics to which the physical reality was somehow expected to be reduced. In particular, the study of unstable dynamic systems and especially of the chaotic ones could lay out the appropriate route in order to comprehend the dynamic bases of stability far from equilibrium of those *dissipative structures* that, as we have already reminded in the section regarding the non-linear thermo-dynamical systems (Cf. § 2.3.4), are essential to comprehend the physical basis of self-organization and self-regulation that are typical of complex physical systems such as chemical systems and, above all, biological systems (Prigogine & Stengers 1979; Prigogine 1981; 1999; Kauffman 1992; 1995; 1999).

2.7.2 Causality, contingency and determinism

Shedding light on the relation between reversibility (determinism) and irreversibility (contingency) of chemical and biological phenomena

Due to the new discoveries concerning the dynamic systems that have followed one after the other in these last forty years, beginning from Lorentz's fundamental article, the mystery of how the *determinism* of perfectly reversible laws of classical mechanics are connectable with the *contingency* of the *irreversible* chemical processes (the wood becomes carbon, but not the other way round) and also biological ones (a cat is born, develops and dies, but never comes back to life) becomes clearer. In such a way, one gives an intelligible content to the irreducible *complexity* of chemical and organic systems: to their *qualitative* difference with respect

to the most simple dynamic systems, those characterized by equilibrium-stability.

If determinism were true, the specificity of life and the possibility of freedom would be pure illusion

Without this possibility, there would be no scientific sense in speaking both of the «miracle» of life, of its capacity to *self-organize*, to *generate information* by opposing, at least locally and temporarily, the destiny of entropic decay of all the physical systems; or of man's *freedom of choice*, if the myth of mechanistic determinism were true and there were no space for the contingency of the *reality* of the physical processes and of their laws. If all is temporally pre-determined and the indeterminacy is only the expression of the insufficiency of human representation of physical processes, the free action of men will be mere appearance, simple ignorance of the true causes that «from the beginning» determine the actions of whatever existent physical system.

Chaotic systems force a re-examination of the Humean-Kantian principle of causality

Complex systems and, in particular, the chaotic ones give an *objective, real foundation* to the contingency of physical processes, a contingency perfectly consistent with the *determinism* of the laws of classical mechanics and together with the *principle of causality*, even though it necessarily requires a different foundation than the modern Humean–Kantian one, which is too much dependent on the naive determinism of the reductionist approach of classical mechanics present at the beginning of modernity.

Steps towards the emancipation from the reductionist mythology in the theory of dynamic systems

To conclude this part and comprehend this fundamental point of the reinterpretation of the principle of causality in physics, which the theory of complexity imposes upon us, it is useful to summarize the steps of this emancipation of the theory of dynamic systems from the mythology of mechanistic reductionism, as it had been expressed by Laplace's demon-metaphor (Cf. § 1.4.2):

1. KAM theorem: integrable systems are the exception in physics

◆ KAM theorem (Kolmogorov–Arnold–Moser) has demonstrated that, among the dynamic systems, *integrable* systems, those for which it is possible to provide an *exact* analytical solution of the equation of motion, *do not exist* in nature. In fact, integrable systems are an idealized case of real dynamic systems. The phenomenon of non-integrability linked to dynamic instability, which was observed by Poincaré in the past century, is not an isolated case, nor a pathological one. The very same solar system which had raised the enthusiasm of Laplace and Kant as an instance of an integrable, deterministic, and eternally stable system, is destined to dynamic instability, even if in certainly much longer times than those at which we actually observe it (Vulpiani 1994).

2. KAM theorem: not all unstable dynamic systems satisfy the ergodic hypothesis, although such a property is unknowable *a priori*

- ◆ Once again, KAM theorem suggests that, among unstable dynamic systems, not all of them obey the *ergodic hypothesis*, unlike the ideal systems of the convex-obstacles billiard table studied by Sinai. As we know, chaotic systems are like these. KAM theorem however is not able to distinguish *a priori* between the two classes of unstable systems: thus, it is not able to predict whether an unstable system is chaotic or not. As one of the authors of KAM theorem, Valdimir I. Arnol'd (Arnol'd 1990), of the University of Moscow, has forcefully argued against the over-simplified exemplifications of the theory of catastrophes, of which he himself is one of the originators in the 1970's⁶⁷: if one were able to know *a priori* the ergodic or non-ergodic behavior of all non-linear and unstable dynamic systems, we would also possess the heart of a mathematical theory which explains deterministic chaos in the line of the theory of *structural stability*⁶⁸ in dynamic systems: a theory developed and almost completed in the 1960's and 1970's; a theory expected at that time to complete the study of stability for dynamic systems. Precisely the discovery of chaotic systems in those years revealed the existence of structures of order, even in unstable systems: the existence of ordered structures which are reproducible because they were generated by deterministic laws (differential equations) and stability (low sensitivity to the modifications of the initial conditions) yet they do not overlap in dynamic systems: the former can exist without the latter (Viana 2000).

3. Capacity of chaotic dissipative systems to generate information

- ◆ The presence of ordered structures, of pseudo-cyclic behaviors more or less complex in dissipative dynamic systems (= attractors) point to the capacity of this system to contrast the entropic decay of the system itself, by consuming energy from the outside of the system. If the dissipative system is stable, the ordered final state can be unambiguously pre-determined by even very different initial conditions. The dynamic evolution of the system has *not generated information*, on the contrary it has dissipated it, given its scarce sensitivity to

⁶⁷ Even though this theory has been developed and popularized by the french mathematician R. Thom, as if it were his own theory (Thom 1977; 1980).

⁶⁸ The stability in dynamic systems can be studied from two points of view: 1) that of the behaviour of trajectories in the space of the phases, according to which a system is stable if its trajectories for different initial conditions become closer and closer up to the point where they are attracted in a unique final state (fixed point, limit cycle, toroid), which is the fundamental way in which we ourselves have considered it; and 2) that (more sophisticated) of the *structural stability*. Such notion concerns the global behaviour of the system according to which a dynamic system is defined structurally stable if, due to a weak modification of the laws of dynamic evolution (e.g., due to a continuous change of the coordinates), the global behaviour of the same remains unmodified (Viana 2000).

the differences in the initial conditions. Vice-versa, the chaotic dissipative systems *generate information* because the structure of a chaotic attractor has dissipated and completely «forgotten», after a few steps, *the information of the initial conditions*, having amplified even small differences, by means of the mechanism of the «divergence» of trajectories. However, unlike ergodic systems, it has generated new information, through the mechanism of the «unfolding» of the divergent trajectories. In other words, points of the space of the phases of the system return to be close — never overlapping, which is why we have pseudo-cycles and not cycles — by following courses different from those, depending on the initial conditions, by means of which the same points move further apart. Through such dynamic mechanism, new structures come to existence — the space of the phases has in fact an attractor, a chaotic or «strange» attractor, — completely *unpredictable* by the initial conditions, but always exactly *reproducible*, given the deterministic character of the laws (differential equations) that govern the system. We thus are faced with neither a *stochastic system** nor a quantum system. For this reason, we are not looking at phenomena of structural stability, as Thom wrongly thought (cf. note 68), and that is why I. Prigogine has given the name of *dissipative structures* to the ordered structures that such systems can produce inside themselves. An «order», I repeat, that is not derivative from the initial conditions, but from a kind of «global» causality, independent of time, which (as we shall see in Chapter Six) from the ontological point of view, refers very closely to what Aristotle called «formal cause».

4. Necessity of a re-examination of the modern principle of causality

- ◆ In such a context, deterministic chaos poses certain new problems to the philosophy of nature and to the philosophy of science regarding the epistemology and the ontology of the *principle of causality*. Let us further examine this point of great theoretical relevance, even though we shall return to it in more detail in the next two Chapters of this work.

Beginnings with the Leibnizian reduction of the principle of causality to the principle of sufficient reason

In the modern philosophical reflection which follows the discoveries of Galilean–Newtonian science, the *logico-mathematical* notion of *law* replaces the *ontological* notion of *cause* of previous, classical philosophy of nature. The first to become aware of this was the philosopher and mathematician Gottfried Wilhelm (von) Leibniz (1646–1716), inventor with Newton of differential calculus, that explicitly referred to the reduction of the *principle of cause* of ancient philosophy of nature to the *principle of sufficient reason*, that is, to the *logical law of double implication* («B if and only if A»). His representational metaphysics — matter is a representation of the spirit, given that all of reality consisted of immaterial «atoms» or

«monads», some of them endowed with consciousness, others not — included a realistic interpretation of the logical laws: they were simply *symbolic representations of real structures*.

Meager success of the Leibnizian metaphysical interpretation of modern physics and prevalence of the Newtonian phenomenistic interpretation

But the «metaphysical novel» of Leibnizian *Monadology*, as Hegel defined it, did not have much success in modern philosophy. In the prevailing interpretation of modern scientific revolution and of the reductionist determinism which accompanied it, Newtonian *phenomenism* succeeded. Modern science is interested in measurable phenomena, explaining their variability in time in terms of functional laws (Cf. above p. 100). It is not the task for the Newtonian scientist of nature to know what are physical entities in themselves and their causal, real relations, but at most for the philosopher of nature, who generally embraced — as in the case of Newton himself when he considered himself a philosopher — an hypothesis of material, metaphysical *atomism* of a neo-Democritean sort (Cf. 5.2).

In modernity the scheme of interpretation of causality is the phenomenistic Kantian–Newtonian one

The *logico-epistemological scheme*, which prevailed in the scientific world for the interpretation of the principle of causality from modern times until now, has been the anti-metaphysical, Humean–Kantian, representational one. On the one hand, it rejected Leibniz's naive logical realism. Physical reality, which is the object of empirical observation, is distinguished from logical reality which is the object of rational reflection. On the other hand, such a scheme bore out what Kant defines as the *temporal schematism* of causality, which links empirical phenomena (the antecedent and consequent of a temporal succession) with its logical formalization under the form of a law.

Causality is linked to the mode of representation of phenomena: the logical antecedent of an implication corresponds to the temporal antecedent of an observation

If, in the naively deterministic approach of modern physics concerning 'origins', all the information that allows one to determine unambiguously the final state is in the initial state of the system, as, in a deductive system, all the information is in the axioms, the foundation of the principle of cause as an non eliminable ingredient of our experience is linked for Kant to the *mode of representation of the phenomena* and it has nothing to do with the *being* of the represented physical entities. According to such a mode of interpreting causality, the *logical antecedent* or premise of a logical implication, would correspond to the temporal antecedent of a physical process empirically observed and measured, and in concrete — in the concreteness of scientific practice — would correspond to the initial conditions of the motion of a system of particles or of «atoms of matter».

Modern causal scheme:

The logico-epistemological foundation of the truth of the claim: «the entity or event *A* causes the entity or event *B*» it is thus linked to the possibility of claiming:

1. Temporal succession among representations

- ◆ The *temporal precedence* of the representation of A , let us say A' , with respect to the representation of B , let us say B' , where A' and B' represent, in the paradigmatic case of mechanics, the initial state or the final state of motion of a dynamic system, as expressed in relative, measurable magnitudes (position and quantity of motion).

2. Necessity-
implying link as a
a priori mathematical
law, reversible

- ◆ The *necessitating link* between A' e B' in the form of an *a priori*, reversible, mathematical law of the dynamic evolution of the system (e.g., one of the laws of mechanics) that will logically assume the form of the double implication between A' and B' , $A' \equiv B'$, « B' if and only if A' », that is, to emphasize the reversibility of the relation: « A' implies B' and B' implies A' » (Cf. *infra* § 4.2.2).

The anti-metaphysical character of this foundation of the principle of causality linked to the origins of the modern scientific revolution depends on the fact that:

3. Temporal representation linked to consciousness

- ◆ The *foundation of all temporal relations* is intrinsically linked to the self-conscious human mind and to its capacity to represent phenomena *in time*, similar to the notion in Western philosophical speculation, from Plato onwards, that time does not exist without memory and consciousness.

4. Absoluteness of laws linked to consciousness

- ◆ The *foundation of the absoluteness of laws* is again intrinsically linked to self-conscious human mind, and the presumed apodicticity or non-hypotheticity of laws to their presumed *self-evidence* of a non-empirical kind⁶⁹.

⁶⁹ The «jump» between *antecedent–consequent* in a logical and temporal sense is fulfilled, in the Kantian approach, by means of the notion of *scheme* and of the temporal *schematism* of representative self-consciousness. By means of such a notion, Kant intended to solve the problem of the above-mentioned consistency between a logical and an empirical notion. More precisely, he sought to answer the question of how it would be possible to justify the assumption of *empirical images* (phenomena) of objects within the logico-formal schemes of the categories and concepts of the intellect. Very profoundly, Kant suggested that this was made possible by the fact that for each phenomenon or class of phenomena, the relative *temporal schematism* existed in the mind (Kant 1787, 155-166): i.e., a set of logico-formal rules to produce in the so called *productive imagination* a variety of images of that object. In sum, that which was it was the memory conserved of an empirical object were not the images of an object — otherwise an infinite quantity of memory would have been necessary for one single object, considering that two identical empirical images of some empirical object do not exist. The temporal rule, the sequential algorithm, was preserved to produce an infinity of these images. On this account, the Kantian theory approaches the Aristotelian–Thomistic theory of perception that dealt with the conservation of memory not of images, but of habits, that is to say, acquired formal dispositions to act, by means of which images of objects were reproduced in memory and in imagination. The difference is that while for Aristotelianism habits were inferred, for Kant such schemes were exclusively *a priori* — so condemning himself, in this way, to

From here Kant's claim that the transcendental basis of predicates is self-consciousness and not the being of the object

From here Kant's claim that the *pre-logical, ante-predicative transcendental foundation* — which gives a basis to predicates, including the numeral ones of the physico-mathematical laws of nature — of the truth of the assertions of (not only) scientific language, should not be found in the *adequacy* of these assertions to the *being of the entities*, but in the adequacy of the conscious sensations, that *hypothetically* refer to existing entities «outside» the mind, to the *way the self-conscious mind* has to represent its objects, that is, to the logical *a priori* of thought (Cf., § 5.1). In other words, there is no more *transcendental* foundation of the truth of the assertions for modern philosophy in the *being of the entities* but rather in the *self-consciousness*, the «I think» (Descartes' *cogito*) of the representational, thus self-conscious, mind. That is, if all of us converge on the truth of the statement «blood is red», it is not because the property (*accident*) that we denote by the term «red» *essentially*, necessarily, or «in itself» pertains to the physico-chemical *substance* that we denote by the term «blood»; but rather such a convenience would be based on 'common evidence', i.e., the fact that all human beings share the same way of being conscious of sensations and the same way of being self-conscious in the form of phenomena; the same way of representing sensations in an ordered form, according to certain determined logical relations, in the self-conscious mind.

Crisis of this causal scheme due to XX century scientific revolutions

The modern, Humean-Kantian, *logico-epistemological scheme* has undergone substantive mutations due to the scientific revolutions of the XX century. We have already seen in section § 1.5 how the development of logic and mathematics and the very same physico-mathematical science of the XIX century had begun to undermine the myth of the apodictic absoluteness of the laws of modern science, which are based on the *principle of evidence*, and thus on self-consciousness. We shall later see the last «episodes» of this issue from the point of view of the theory of the foundation of logic and mathematics of the XX century. From the logico-mathematical point of view, since the XIX century, one can no longer speak of the idea that the necessity of the causal relation could be *absolute*, if such a necessity has an exclusive logico-epistemological foundation.

not being able to find a solution to the problem for which he had proposed the schemes in the first place. In other words, he faced the problem of the subsumption of the phenomenal *a posteriori* into the conceptual *a priori*. If «the bridge» between them was exclusively *a priori*, how could this bridge act? If, in the Kantian theory, certain phenomena are subsumed under a certain category (e.g., that of cause) and not another (e.g., that of substance), why is the temporal scheme of *succession* and not of *permanency* in time applied to a certain group of phenomena; how could one justify such an application if the scheme is purely *a priori*? We shall return in the Fourth Part to the comparison between the Kantian and the Aristotelian-Thomistic theory of perception.

But the most significant way of giving proper importance to all this is provided by the conceptual revolutions of the new physical science of the XX century.

First of all the principle of quantum indeterminacy

The first way comes from quantum mechanics and from its *principle of indeterminacy* (Cf. § 2.4.3). In quantum mechanics, a final state is consistent not with a single initial condition but with a *cloud* of initial conditions. A variety of essays and whole books have been written on the relationship between the *principle of indeterminacy* and the *principle of causality*, to begin with the essays of W. Heisenberg, such as his famous *Atomic Physics and Principle of Causality*, of 1955 (Heisenberg 1955, 57-99), and arriving to the texts of major contemporary philosophers of science and nature. Just to quote two of them, it would be enough to mention the important book, *Causalità e indeterminismo*, by Fr. Filippo Selvaggi (Selvaggi 1964) which profoundly examines this question from the point of view of the scholastic philosophy of nature, and the much more accessible and known essay by Karl R. Popper *Of Clocks and Clouds. Essay on the Problem of Man's Rationality and Freedom* of 1972, which faces the problem against the background of the eternal dilemma between naturalistic determinism and the real foundation of human freedom (Popper 1975, 277-340).

Impossibility of an infinitely precise determination of the initial conditions

The problems, which are posed by quantum mechanics, are linked to the impossibility of an infinitely precise determination of the physical states of the system, which poses certain limits of predictability on the evolution of the system in terms of classical trajectories. In this sense, one could and should say that the final state of a quantum system is consistent with not a unique set, but with a *cloud* of initial conditions and hence — and here it is the profound meaning of the representability of a quantum system by means of a probabilistic wave function — with a possible infinity of trajectories, as taught by Feynman's formalism of *course-integrals*.

Nevertheless, in spite of the physically paradoxical character of the route-integers, the final state is perfectly predictable.

Indeed, the state of the quantum system is formally correspondent to the sum of all «histories», with respect to all of the infinite courses consistent with the *cloud* of initial conditions. It would be as if a quantum particle, to pass from state *A* to state *B*, would simultaneously pass through all the infinite possible courses («trajectories») between those two points. Feynman was the first who denounced the paradoxicalness, as well as the physical contradiction of such a logico-mathematical construction (Feynman 1989). He was the first to recognize that it would have been unacceptable to suppose that an «infinite quantity of logic» were necessary to represent in a deterministic form the change of the physical state of a microscopical particle, within an incredibly small portion of the space-time (Feynman 1984, 64). Nonetheless, it is still sur-

prising that by such a mathematical device, which gives De Broglie's wave functions such a physically paradoxical meaning, a quantum system becomes perfectly deterministic, predictable, by making the theoretical predictions converge with experimental measurements with a precision that — of course, conceived in a probabilistic sense — has not been equaled in the rest of modern physics!

Quantum systems
in the formalization
of wave functions:
stable,
deterministic,
reversible

From a dynamic point of view, all this is linked to the fact that the *stability* of quantum systems works in such a way that the system itself does not increase the indeterminacy by which its initial conditions are defined. For this, in the domain of the statistical formalism of the wave-function, quantum systems appear in the end to be *deterministic*, formally *reversible*, even if it is a different determinism from that of quantum systems studied by classical mechanics. In sum, if one uses the adequate formalism in the representation of the dynamic evolution of quantum systems — that of the wave-function and not that of the trajectories of material points —, *none* of the two fundamental ingredients of the modern principle of causality, *temporal schematism* and *determinism linked to the reversibility* of processes, is rejected.

Indeterminacy
linked to the change
of representation as
a result of an act of
measurement
(collapse of the
wave function)

In quantum systems, we will be looking at irreversible processes and indeterminacy only by making the wave-function «collapse» upon a *classical trajectory*, by means of an act of measurement that would try to precisely define the physical state of the particle. And it is at this point that the impossibility of independently determining the position and the velocity of the particle plays a role, as is taught by Heisenberg's principle of indeterminacy. The indeterminacy is thus linked to a change of representation, it is *intrinsic to the observer-observed system*, but *not intrinsic to the dynamic system as such*, which, in terms of the wave-function, let us say it again, remains perfectly deterministic. In this sense, one could say that in quantum systems, the observation enters as an intrinsic component of such systems, changing the nature (determinism/indeterminism) of the observed phenomenon (Ghirardi 1997).

Possibility of a
wholly realistic
interpretation of the
determinism/inde-
terminism passage
by means of the
phenomenon of
decoherentization

In this sense, the idealistic metaphysician could feel satisfied, although not in the subjectivist sense in which this «interference» of the representation with the represented system is popularized by genuine cultural «pushers» of «weak thought». At the same time, however, the idealistic metaphysician should not rest on his laurels, if the most recent discoveries about the phenomenon of the so called *decoherentization of the wave function* would confirm what is emerging with great clarity. Such a phenomenon appears completely equivalent to the collapse of the wave function, but, unlike this, it would result completely independent from the change of representation and/or from the interaction with the system of measurement, depending exclusively on the interaction between

the quantum phenomenon and the rest of the physical context. On the other hand, this would confirm in a different way what has already been observed with the experiment of the EPR paradox. Indeed, the quantum non-locality that expresses the ondulatory intrinsic nature of the quantum phenomenon, emerges *if and only if* the experiment occurs in a context of a practically absolute «vacuum». That is, if one eliminates every kind of interaction of the particles that make up the quantum system, the non-locality of which one is observing, with other forms of energy external to the system itself.

The fact that one of the great fathers of contemporary physics, the Nobel Laureate John Archibal Wheeler, appropriately called «Mr. Relativity», endorses this thesis, makes one hopes that this completely *realistic* interpretation of the «key-mystery» of quantum physics will soon prevail, thus definitively bringing to a close a century of theoretical uncertainties regarding this issue (Tegmark & Wheeler 2001).

The irreversibility in unstable and chaotic systems is completely independent from representation

Quite different is the case of unstable systems and, in particular, of the chaotic ones. Here we have the apparently paradoxical fact of *an irreversibility* that is accompanied, unlike the systems studied by quantum mechanics, by the validity of the *deterministic laws* of classical mechanics and by the representability of its dynamic evolution in terms of trajectories.

Connected to final determinism and reversibility of the equations of mechanics

The irreversibility of dynamic evolution is reconciled with the deterministic laws of mechanics and their reversibility through the elegant hypothesis of *Poincaré's times of recurrence*. The physical times that are required for the system, passing through all its possible states consistent with the principle of conservation of energy according to the ergodic hypothesis, in order to return to its initial state and thus to become reversible, are so huge, practically infinite, that, as a matter of fact, physically, it is as if the system would never achieve it.

By means of the theoretical hypothesis of Poincaré's infinitely long times of recurrence

All this can be clarified by means of an efficacious example by D. Ruelle (Ruelle 1984). Let us imagine that we lock a cat, with its food and what is necessary for it to survive, in a box, which is isolated from the external world. After a certain number of years the cat will die, and in the box we will have nothing more than a smelly carcass. The irreversibility of such a process is at the basis of the common belief that cats — just like any other biological system — do not come back to life. By imagining that millions of years come to pass, accepting the ergodic hypothesis, the carcass will pass down the centuries as multiple physical states consistent with the conservation of energy. But that out of it, a living cat could be recomposed, is a very unlikely hypothesis — although, always according to the ergodic hypothesis, it is not completely impossible. Neither we nor the whole of our galaxy will live so long to be able to observe

such a «miracle». Such systems are, thus, as a matter of fact, *irreversible*, albeit within a global *reversibility* of the laws that characterize their behavior.

Irreversibility in chaotic systems is not linked to the weakness of the logical principle of reversible determinism, but to the weakness of temporal schematism

The irreversibility and the aleatority of chaotic systems *qua* unstable systems would thus, in itself, be linked not to the extinction of the element of reversible determinism but to the lack of the other element of the modern scheme of the principle of causality: the *temporal schematism*. It is indeed the *instability* of the chaotic system that implies the *absolute unpredictability* of the final state of such a system and of its structure (strange attractor), a structure that, with its emergence, «breaks» the ergodicity of the system. In these systems, the temporal antecedent is *never* a *sufficient* condition to determine the temporal consequent, at least when the prediction regards a temporal interval which is *long*, but not *infinitely long*.

The antecedent, in the long (but finite) run, is not a sufficient condition for the prediction of the consequent

This notation of the always finite length of the temporal interval is particularly important. Indeed, for arbitrarily small temporal intervals, at most infinitesimal — we are here in the domain of classical mechanics and there is no principle of quantization and of indeterminacy of the physical state and hence the interval can easily move towards zero — the unambiguous dependence of the consequent on the antecedent could always be guaranteed. The dynamic instability of chaotic systems begins to play its surprising role in connection with long term predictions, which make such systems profoundly different from quantum systems.

Even in chaos the final state is consistent with a group of initial conditions as in quantum theory

In a stable, chaotic system, like in a stable, quantum system, the final state is in fact consistent with a *cloud* of initial conditions. Given the irreversibility of the process, it is not possible to reconstruct a unique trajectory that would lead to a unique set of initial conditions that can be determined with absolute precision. Nonetheless — and here lies the substantial difference with a quantum system — one can be theoretically sure that, unlike the quantum system, this *unique* trajectory does *exist*.

But in chaos one is certain that formally only one trajectory exists, even though one cannot know which one

This is the reason which explains the profound theoretical difference between the indeterminacy of a stable quantum system and the indeterminacy of an unstable chaotic system. The final state of both of them is consistent with a cloud of initial conditions. But, on the contrary, with respect to a chaotic system, one could be sure that formally one and only one trajectory exists, and that it has led the system from the initial state *A* to the final state *B*, even though it is never possible, as a matter of principle, to know which one, given that a quantum system, at least from within Feynman's formalism of quantum electrodynamics (QED), we could be sure that an infinity of possible trajectories exists, and that,

out of all of them, the system somehow chose one in order to pass from A to B .

Deep similarity
between this kind of
indeterminacy in
physics and that
one linked to
*ind decidability** in
formal systems

From here the suggestion, partly already formally proved, at least for certain chaotic systems (Agnes & Rasetti 1991; Perrone 1995; 2000), of the profound relationship that links the incompleteness of formal systems in logic and mathematics — about which we shall deal in the next chapter (Cf. § 3.1.3) — and the indeterminacy of chaotic systems in physics and dynamics. Also in formal systems, due to their essential *incompleteness*, one can be sure about the existence of a certain theorem — formally, of the existence of a logical procedure or calculus, which, from a set of premises A brings us unambiguously to a conclusion B —, although the consistency of B with A can not be demonstrated within the system. From such a strictly *finite* logico-formal point of view — thus excluding other mathematically elegant hypotheses, such as Poincaré's recurrences, but irreconcilable with the finitude of all the physical systems — what loses force in the modern causal system in the mathematical representation of the temporal evolution of the chaotic systems is the *logical necessity of the antecedent–consequent relationship*.

That is to say, once
the physical system
is brought into a
realistically finite
context

That is to say, what loses force in the chaotic systems is not only the antecedent–consequent temporal schematism, but also the other component of the modern scheme of the principle of causality, that of the *formal necessity*, based on the antecedent–consequent logico-formal relationship: a necessity that quantum indeterminacy does not undermine, even if one supposes «an infinite quantity of logic», according to Feynman's appropriate expression, to justify the determinism in the formalism of the wave-functions of quantum physics: the antecedent–consequent relationship and its reversibility-necessity⁷⁰.

But this contrast
between quantum
and chaotic
systems may come
to an end.

To conclude this section, one more consideration is necessary. In these latter paragraphs, we have dealt with the opposition between quantum and chaotic systems (particularly useful for pedagogical goals), in order to develop reflections about the actual state of the discussion in philosophy of nature and science regarding determinism and causality. This contrast could be overcome in the future, once the existence of *quantum chaos* were to be confirmed, strongly linked to the above-mentioned existence of the phenomenon of *decoherentization of the wave function*.

⁷⁰ Even from this point of view the parallelism that we have established between indeterminism in unstable systems and incompleteness in logico-formal systems would be confirmed. It is indeed well known that a way to sidestep the problems of incompleteness in formal systems, without finding a solution to them, is that of appealing to infinity-type methods of demonstration. (Cf. footnote 75).

If the existence of quantum chaos and the role of decoherentization were confirmed, quantum systems would become a subset of chaotic systems

In such a case, quantum systems would become a subset of the chaotic ones; this would come from the study of unstable systems even in the microscopic, physical systems of a quantum kind, for which the deterministic formalism of the wave function ends up in fact non-applicable. In the last ten years, different physical examples of quantum chaos have been effectively identified. Among the thinkers dealing with instability, that one who has most insisted on the change of paradigm that the study of instability implies for all fundamental physics, quantum systems included, is Ilya Prigogine (Prigogine 1981; 1999). His theoretical proposal to make the quantum systems a sub-set of the unstable ones generates, however, many perplexities from the physico-mathematical point of view. But we will not go further into this question here.

In any case, complex systems, with or without including quantum systems, require a change of paradigm concerning the Humean-Kantian principle of causality

Given our philosophical goals, we could conclude by saying that a different conception of physical time and hence of a different scheme of causal necessity, richer and more complex than that typical of the modern Humean-Kantian principle of causality, is required to interpret correctly the richness of behavior of chaotic systems and of complex systems in general. Indeed, concerning the two ingredients of the modern foundation of the principle of causality (the antecedent-consequent temporal schematism and the logic-formal foundation of the necessary relations between both of them) identified by Kant (Cf. above, p. 168), the study of complex systems put both of them in question, which was the proof that Kantian philosophy of science is only applicable to the Newtonian deterministic model of physical systems, integrable and stable, but certainly not to the actual developments of the study of complex physical systems.

Many think the Aristotelian model to be the most adequate one

Prigogine, firstly, (Prigogine & Stengers 1979; Prigogine 1981; 1999), followed by R. Thom (Thom 1990) and many others (Arecchi & Arecchi 1990; Cannata 1994; Basti 1995; Basti & Perrone 1996; Musso 1997; Oderberg 1999), have suggested that both these concepts (physical time and causal necessity) should be investigated within the philosophy of nature and science of an Aristotelian inspiration rather than of a modern one. From many points of view, the Aristotelian model of causality is suggested as the most adequate one in order to give an account of the causal relations in chaotic systems. Indeed, this model is not based on the logical relation of implication, but on the *act-potency* ontological one, that is to say, on the non-exclusive dependence of the final state on the initial cause (= efficient and material), but a dependence on global factors that control the entire dynamics and that, epistemically, can be known (partially) only after the occurrence of the final state (= formal-final cause). We shall see in the following parts of this work and in par-

ticular § 6.3.2, how much and within which limits this suggestion is reliable.



2.8 Summary of the Second Chapter

In § 2.1 we have reflected on a synthetic vision of the most revolutionary discoveries of the 20th century's «new physics». Such discoveries have reduced the ambitions of the omnicomprehensiveness and apodicticity of Newtonian physics, by limiting it to the study of *macroscopic* mechanical systems: those linked to the bodies of our ordinary experience. In the *mesoscopic* field (molecular aggregates), the principles of statistic thermodynamics, linear and non-linear, apply. In the *microscopic* field (atomic and sub-atomic level) the principles of quantum mechanics and of the theory of restricted relativity (quantum electrodynamics and cromodynamics) apply. At the *megascopic* level (phenomena on a cosmic scale) principles of general relativity theory apply. But the revolution and the strong loss of importance of a certain *reductionist philosophy*, both in a *diachronic* (reduction of the final state to the initial conditions) and in a *synchronic* sense (reduction of the complex to the simple), linked to the principles of Newtonian mechanics, has occurred in the last thirty years of the 20th century with the birth of the *physics of complexity*.

The first reconsideration of the ambition of omnicomprehensiveness of the principles of Newtonian classical mechanics occurred with the birth of *statistical thermodynamics* by L. Boltzmann (§ 2.3). The second principle of thermodynamics (§ 2.3.1), reintroduces the «time's arrow» in the statistical study of *mesoscopic* physical systems (molecular aggregates) (§ 2.3.2), by linking it, in *isolated* and *low-density* (gas) systems to the irreversible increase of *entropy*, so reducing the domain of applicability of the principles of temporal reversibility that is typical of the equations of Newtonian mechanics (§ 2.3.3). An even deeper revolution is introduced by the study of non-isolated or *dissipative* thermo-dynamical systems, characterized by a strong *non-linearity*, capable (as in those cases of certain chemical but above all biological systems) of linking the irreversibility not only to an increase of disorder (loss of «memory» of the initial conditions), but to the self-organization of ordered structures (*dissipative structures*), absolutely unpredictable given the initial conditions (= chaotic systems) (§ 2.3.4). But this is the main theme of study of the physics of complexity.

The second reconsideration concerning the domain of the equations of Newtonian mechanics occurs with the birth of *quantum mechanics* (§ 2.4) in the study of *microscopical* physical systems. In such a study, a series of

principles are introduced which are inconsistent with those of classical (Newtonian) mechanics: *the principle of quantization* (§ 2.4.1) and the consequent discrete-states model of atoms proposed by N. Bohr, by means of which a physical explanation of Mendeleejev's periodic table of the chemical elements is given (§ 2.4.2).

Later, a series of other principles have been introduced in quantum mechanics such as Heisenberg's *principle of indeterminacy* (§ 2.4.3), Pauli's *principle of exclusion* (§ 2.4.4), Bohr's *principle of complementarity*, particle-wave (§ 2.4.7), a result of the discovery of the intrinsic *ondulatory* nature of quantum phenomena, first as the product of De Broglie's work (§ 2.4.5), and then by means of Schrödinger's ondulatory equation that allows us to predict with exceptional exactitude the discrete energetic states of the hydrogen atom (§ 2.4.6). A fundamental step along this path has been Dirac's equation, which has allowed us to introduce the principle of special (or restricted) relativity — and the consequent mass-energy convertibility — in to quantum field equations, transforming quantum mechanics into quantum electrodynamics (§ 2.4.7). From here, we see the necessity of reading the particle-wave duality of Bohr's principle of complementarity as a fundamental aspect of nature at the microscopical level, with all further questions linked to the so called «reduction of the wave function» (passage from the ondulatory description to the particle one) and to its degree of dependence on the representation and/or on the act of measurement.

A further revolution concerning the conceptions of classical mechanics has occurred with the theory of relativity (Cf. § 2.5). In a Newtonian conception of space-time (§ 2.5.1), there was not, unlike Ancient, Greek and Medieval mechanics, an absolute system of reference for bodies at rest (the sky of fixed stars of Ptolemaic mechanics). Rather, on the basis of the so-called principle of Galilean relativity, there existed absolute systems of reference for inertial systems in uniform rectilinear motion with respect to each other. In this sense, absolute measurements of space and time existed.

Such an absoluteness entered a crisis when the finite speed of light (a truth which was known since Newton's times) was taken seriously into account and, above all, when physicists realized the fact that Maxwell's equations of electromagnetic systems seemed to not respect the Galilean principle of relativity: a principle, the validity of which seemed to be reduced to mechanical systems only. Instead, the young Einstein (Cf. § 2.5.2) proposed a theory that, on the one hand, extended the principle of relativity also to electrodynamics systems, and hence extended the invariance of the laws of physics; and on the other side, maintained the *absolute* character of the speed of light. It was thus necessary to replace

Galileo's transformations of inertial systems of reference with other transformations that would correct the relative measurements of space (contraction of space) and of time (dilation of time), starting with the principle of the non-surpassable limit of the finite speed of light. Such transformations were those of Lorentz, defined in order to explain the Michelson–Morley's experiment, by which the hypothesis of mechanical ether was officially rejected. Theoretically, this implied the impossibility of considering space and time as *independent* magnitudes; at least for measurements of spatial magnitudes over great distances and/or for measurements of spatial magnitudes over bodies that move at speeds that are close to that of light, given that the signals coming from the instruments of measurement cannot travel at speeds superior to that of light, and therefore it is inconceivable to consider the spatial measurements independently from temporal considerations. From here the notion of *Einsteinian chronotopos (time–space)* where time becomes the fourth, inseparable dimension of space.

But these paradoxical modifications at the level of representation of physical reality are not the most disturbing of special relativity. Even more disturbing are the last two relations — the relativistic increase of mass for accelerated bodies at velocities close to that of light and the consequent mass–energy reciprocal transformability — direct consequence of the fact that the speed of light constitutes an upper limit for all motion of physical entities. Finally, in § 2.5.3, we have shown the extension of the principles of relativity also to gravitational systems. Special relativity had made the laws of physics unchanging not only for mechanical systems — Galilean principle of relativity —, but also for electro-dynamical systems which obey Maxwell's equations. However, gravitational systems were still excluded. The special principle of relativity seemed to conflict with the evidence that the effects of gravitation are transmitted instantaneously over great distances. Einstein's ingenious solution was that of re-interpreting the role of gravity, and thus a widening of the theory of special or restricted relativity into the theory of *general* relativity. According to Einstein's re-interpretation, gravity is not a force that acts directly upon bodies, but rather modifies the structure of space-time in which the bodies move. In short, celestial bodies move along curved trajectories not because the gravitational force acts directly upon them, but because the shortest distance that connects two points on a curved surface is not a straight line, but a geodetic line. In such a way, for the first time in the history of thought, the mysterious force of gravity found its mathematical explanation: an explanation that exacted the price of replacing (in the representation of bodies subjected to gravitational force and which hence moved over *megalosopic* distances) the flat Euclidean space of mechanics and of the theory of classical gravitation,

with the curved space of Riemann's geometry. Just as in the geometry of curved space, flat geometry can be considered a zero-curvature infinitesimal of it, in the same way, *the principle of equivalence* holds in the general theory of relativity. In a small gravitational field (e.g., in an elevator which is in free fall), it is as if gravity did not have any effect, as long as we introduce a referential system which is accelerated with respect to the inertial system.

But the last and most definitive revolution in the domain of modern physics has been introduced by the study of *complex* physical systems (§ 2.6). All began at the end of 19th century with the discovery of *dynamic instability* at the very heart of the myth of «absolute sciences» of Laplacian pedigree: celestial mechanics. If one tries to calculate the results of gravitational equations at 3 and not 2 bodies, the equation ends up non-integrable and unstable. A small variation in the initial conditions is amplified by the dynamics, making the final state completely unpredictable (§ 2.6.1). The further, decisive step was made almost a century later with the discovery that certain non-linear, classical, non-quantum dynamic systems, characterized by dynamic instability violated the ergodic hypothesis: *chaotic systems*. The results, instead of uniformly distributing themselves on the space of phase (as in the classical, unstable, dynamic system of the convex obstacles on a billiard table), remain concentrated in a finite volume of the space of phase (chaotic attractor). Even though this is true, this set of solutions ends up completely unpredictable in terms of the initial conditions, since, being unstable, the system has completely lost memory of those very same initial conditions (§ 2.6.2), which is different for stable systems.

The theoretical consequences of complex systems for the modern interpretation of the principle of causality in relation to the problem of the determinism in physical systems (§ 2.7) become more evident by considering complex systems, and in particular, chaotic ones from the dynamic point of view. In such a way, the connection between chaotic systems and the emergence of *dissipative structures* in non-linear thermodynamic systems becomes very tight (§ 2.7.1). The main theoretical consequences concern the necessity to revise the Humean-Kantian model of causality, based on the possibility of identifying the temporal antecedent and consequent of a causal relation with the logical antecedent and consequent of a demonstration, so that the causal necessitating link be identified with a logical *a priori*. From many points of view, it is suggested the Aristotelian model of causality is a more adequate one to explain the causal relations in the ontology of chaotic systems, since it is based on the *potency-act* relation. In other words, this model of causality is not based on the logical relation of implication but on the potency-act onto-

logical one, namely, on the non-exclusive dependence of the final state from the initial causes (= efficient and material), given that the final state also depends on global factors that control the whole dynamic and that, epistemologically, can be (partially) known only after the occurrence of the final state (= formal-final cause) (§ 2.7.2).

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3. Research on ‘the foundations’ in the 20th century

From the failure of Hilbert’s formalist program, to the discovery of Gödel’s incompleteness theorems in the foundations of logic and mathematics, to the resulting scientific death of scientism, to the birth of the philosophy of science and the re-birth of the philosophy of nature, to finally deal critically with the epistemological and ontological problems spawned by the scientific revolutions of the 20th century.

3.1 The scientific death of scientism

3.1.1 Premise

When treating chaotic systems, we already mentioned the possible, strong relationship between the incompleteness of formal, logical-mathematical systems, and indeterminacy in chaotic systems. We now turn to the revolutionary discovery of the incompleteness of formal systems from the point of view of the theory of the foundations of logic and mathematics, thus completing the discussion at the end of Chapter One on the «crisis of the foundations», which was spawned by the discovery of *antinomies* (See 1.5.2).

The consequence of the revision of the foundations of physics: impossibility of one single theory

As mentioned, the *revision of the foundations* that took place during the 19th century did not only affect *pure mathematical sciences*, but also *applied mathematical sciences* and, first among them, the *queen* of these sciences: *mechanics*. At the beginning of the modern age it was presumed that only *one (apodictic) geometry* could exist and, at the end of the 19th century, this incorrect idea was refuted by the possibility of having an *infinite number of possible (hypothetical) geometries*. Similarly, at the beginning of the modern age, it was assumed that only *one (apodictic) mechanics* could exist (Newton’s mechanics); at the beginning of the 20th century, however, it was discovered that more than one, all hypothetical, forms of mechanics could exist: *classical, quantic, and relativistic mechanics*, each of them valid in a specific field of physical reality —respectively, the macroscopic, the microscopic and the megascopic. Above all, these were all *hypothetical* mechanics, hence replaceable with more adequate theories. Given the already mentioned *principle of equivalence* (See § 2.5.3), classical mechanics and the me-

chanics of general relativity are already unified, in the sense that classical mechanics is a subset of relativistic mechanics. One remaining problem relates to the unification of the theory of general relativity with quantic electrodynamics, that is, the construction of a substantial theory of *quantic gravity* or «theory of great unification». This theory is still far from being developed; however, even when it is formulated, it will remain a hypothetical, provisional and further perfectible theory.

Perhaps, the almost newly born *science of complexity* will bring more radical conceptual revolutions into the epistemology and ontology of modern science. Conceived within the classical mechanics studies of macroscopic systems, it appears capable of including, as its particular cases, classical (and relativistic) mechanics, as well as quantum mechanics. In this way, the science of complexity implicitly presents itself as a new, additional paradigm for a theory of «great unification». In the last subsection we referred to the most obvious among the intellectual revolutions introduced by the science of complexity: the revolution, in modern interpretation, of the *principle of causality*. This theory, however, is too recent for us to completely assess its value.

The hypothetical and incomplete character of any formal language and the meaning of the limitation theorems in logic and mathematics: «research never ends»

In any case, in the face of so many and so important conceptual earthquakes, one question comes up: is this *hypothetical*, in any case *incomplete* (we will treat below the precise meaning of this word) hence *partial* and *provisional* character of any formal theory or formalized language in science linked to a transitional moment of the history of scientific and logical thought, or is it the expression of a more radical condition? Stated otherwise, are there any precise *limitation theorems* on the power (consistency and truth) of languages and of the formal systems of science (See *infra* § 3.1.3 e § 3.1.4) that, as any other, have a *universal* and *necessary* validity, even if they are always valid only within a particular logical formalism?

Hypothetical character of a theory indicates the impossibility of absolute (as opposed to partial) truth and consistency

Clearly, the answer to this question is affirmative: there are many logical, semantic and syntactic limitation theorems of the validity of formal systems and of formalized languages that can be built in the various sciences. This does not mean that *truth* and *consistency* do not exist in the logic of scientific discourse and in rational discourse generally, particularly the metaphysical one, as the *nihilist instrumental interpretation* of these theorems —and its theological equivalent, *Fideism*—would have an inexpert audience believe. In the words of a famous expression by Popper —that he used as the title of his intellectual autobiography — science is truly an «Unended Quest», a search that never ends.

The need for «open» logical systems

Throughout this chapter (See § 3.1.5), we will see that the consequence of these limitation theorems, particularly of Gödel's, is the need to con-

sider logical systems (including scientific theories) as «open» systems, linked to the application context and able to adjust axioms, thus overcoming the static character of classical formal systems (Cellucci 1998).

This is not a new discovery, but rather the «re-discovery» of a classical theory

We should stress, therefore, that these modern reflections on the foundations of pure and applied logic and mathematics, such as the limitation theorems, do not introduce *entirely new doctrines* in the field of logical—and, more generally, metaphysical—research. To the contrary, they re-introduce, *in a rigorous and absolutely unambiguous form*, some common convictions in the highest classical logical reflection on the same issues⁷¹. In particular, they eliminate from the history of thought once and for all the excesses of *rationalism*, both in its philosophical forms (both classical and modern)—idealism, naturalism, historicism—and in its modern scientific forms—scientism, mechanicism, evolutionism.

The theological counterpart of this logical and epistemological truth

In this respect, we present a quotation from the very recent *Encyclical «Fides et Ratio»* (1998), in which Pope John Paul II, in line with the centuries old teaching of the Church, re-introduced a doctrine on the specific relationship between *philosophy* and *truth*, entirely compatible with the results of contemporary logical research on the foundations, while giving it a theological-moral as well as historical-anthropological meaning, albeit beyond the aims of a logical-epistemological reflection such as ours. As John Paul II states at # 51 of the *Encyclical*:



One should consider, in particular, that *truth* is *one*, even if its expressions bear the mark of history and, in addition, are the fruit of a human reason wounded and weakened by sin. *From this derives that no historical form of philosophy can legitimately claim to embrace the totality of truth, nor of being the full explanation of the human being, of the world and of the relationship between man and God* (emphasis added).

After this necessary premise, we now turn to illustrating the last step of this short and incomplete historical account of the reflections on the foundations, which followed the «scientific revolutions» of the 19th-20th century. This will consist in describing those *limitation theorems* on the formalized demonstration procedures of the various sciences, and also of all rational subjects, including metaphysics, as long as they try to *formalize* their statements and their demonstrations. That is, as long as they strive to «make these transparent» in relation to an honest assessment of their consistency and truth, without hiding behind words and

⁷¹ To be further convinced of this point, one should read the masterpiece of a logician such as J. BOCHENSKI, *Formal Logic*, (Bochenski 1956), which demonstrates the substantial continuity between modern formal logic (including mathematical logic) and classical (Greek and Medieval) logic—with some inclusion of oriental logic as well, particularly that from India.

ambiguities, in order not to exploit people's ignorance and credulousness.

3.1.2 The formalist approach

From antinomies to
Hilbert's formalism

Previously, we left the 19th century reflection on the foundations of logic and mathematics at the unsettling discovery of antinomies, not only in Cantor's *theory of sets* (See § 1.5.2.1), but also in Frege's *logician* approach to the *theory of classes*, as a theory of the foundations of logic and mathematics (See 1.5.2.2). The subsequent step was brought forward by one of the most important mathematicians of the 20th century, certainly the most influential of the first half of the 1900's: David Hilbert (1862-1943). We owe to Hilbert a number of contributions, not only on the foundations of mathematical logic, but also on the foundations of geometry, algebra, as well as the development of a geometric-algebraic formalism («Hilbert's spaces») that was fundamental for the mathematical formalization of quantum mechanics. This is a task which he undertook once Gödel's incompleteness theorems had demonstrated the substantial failure of his so-called *formalist* approach to the foundations of mathematical logic.

Hilbert's distinction
between languages'
logical form and
their consistency
and truth

Hilbert's «solution» to the problem of antinomies inherent in Frege's logicist approach essentially consisted in separating the issue of the formal analysis of a language from the issues of its *truth* and *consistency*. Far from being a rejection of the importance of the issues of truth or meaning for logic and mathematics, Hilbert's choice thus consisted in separating the logical-formal procedures of *demonstration* and *inference* (= logic) from the *proof of their consistency and truth* (= meta-logic).

That is, Hilbert's
distinction between
logical and meta-
logical analysis

Briefly stated, we mentioned how the fundamental result of Frege's research on the foundations was not the theory on the foundations that he proposed —logicism— but the development of a new *logical-formal method* of enquiry —*symbolic* or *mathematical logic*. Similarly, the main result of Hilbert's research on the foundations was not the formalist theory that he proposed, but the development of a *methodological distinction, fundamental* in modern logic, be it of a mathematical nature or not: the distinction between the *logical* and the *meta-logical analysis* of formal languages. This distinction brought back (in a rigorous and modern form) the classical Scholastic distinction between, respectively, *logica minor* — the logical analysis of the demonstrative procedures of the propositional forms of a given formal language (e.g., in Aristotelian logic, syllogistic analysis) — and *logica maior* — the analysis of the consistency, of the truth and meaning of the propositional forms and of the demonstrative procedures of that same language.

Simply put, the distinction between *logic* and *meta-logic* refers to a more fundamental distinction, in semantics, between *languages* and *meta-languages*.

The more general distinction between language and meta-language

In general, a «*metalanguage**» is one in which one speaks of another language, which is then defined «object-language». So, for example, if I create an English grammar in Italian (i.e., for Italian speakers), the latter is the meta-language of English, which is, in turn, its «object-language». The term «object-language» derives from the fact that, if we analyze the predicates of the meta-language, we see that they are different from those of the related «object-language». Indeed, while the predicates of the object-language have, as an argument (that is, they have as subjects, grammatically speaking) *names* that refer to extra linguistic *objects*, the predicates of the meta-language have, for argument, *names* that refer to linguistic *objects*, more precisely, names that refer to entire propositions of the object-language. Indeed, in order to correctly write meta-linguistic propositions, I will have to observe specific rules: for example, rules for a given use of the quotation marks « », or declaratory constructions, such as: «it is *true* (*consistent, meaningful, univocal, analogous...*, or any other metalinguistic predicate) that: “...”».

Let's take, for example, a statement of Euclidean geometry:

A straight angle is the sum of two right angles

Meta-theoretical character of the problem of the consistency and truth of a theory's statements

The statement of the *consistency* of this assertion with the axioms of Euclidean geometry is a *meta-theoretical syntactic* statement:

«A straight angle is the sum of two right angles» is consistent with the axioms of Euclidean geometry. [Or: It is consistent with the axioms of Euclidean geometry *that* a straight angle is the sum of two right angles]

Similarly, the statement of the *truth* of this assertion within the formal language of Euclidean geometry is a *meta-theoretical semantic* statement:

«A straight angle is the sum of two right angles» is true in Euclidean geometry. [Or: It is true, in Euclidean geometry, *that* a straight angle is the sum of two right angles]

As one can see, the statement within quotation marks works as a subject of the predicate «is consistent» or «is true». Or, in the formulation within square brackets, the proposition introduced by 'that' works as the subject of the mentioned predicates. Indeed, in the logical analysis taught in school, the above-mentioned proposition is defined as «*subjective* declaratory».

The starting point of Hilbert's reflections on the foundations was the conviction that the lack of this distinction between language and meta-language was at the roots of the failure of Frege's analysis. In particular, this failure was due to the belief, typical of the logicist approach, that, in logic, one should start from necessarily *true* (hence, reciprocally *consistent*) premises, so that the analysis of the truth and consistency (coherence) of the propositions was not a task of formal logic.

The three phases in the construction of theory according to Hilbert

Already in his pioneering 1892 work on algebraic invariants, Hilbert distinguished among three phases in the construction of any mathematical theory (see Webb 1980, 76):

- ◆ *Naïve*: corresponding to the formulation of concepts and their informal application to the solution of problems;
- ◆ *Formal*: corresponding to the discovery and development of algorithms for symbolic calculus;
- ◆ *Critical*: general existence theorems tested constructively, if possible

This distinction substantially preceded Hilbert's other, more mature and famous one, dating back to 1918 and differentiating, in the construction of any mathematical theory, between:



- ◆ An *informal* and intuitive mathematical theory;
- ◆ A *formalization* of such a theory (including a sufficient logic);
- ◆ The *meta-mathematical* study of the consistency of this formalization.

In order to understand the difference between formal *mathematics* and/or *logic* (intended as substantial sets of demonstration and calculus procedures) and *meta-mathematics** and/or *metalogsic** (intended as a proof of the logical truth and consistency of the former) —a distinction that constitutes the core of the so-called «Hilbertian program» of the foundation of mathematics and logic—, one should insert it within the developments of the mathematical-technical work carried out by Hilbert in those years.

Hilbert's formalization of Euclidean geometry

After his early study on algebraic invariants, in which he had demonstrated the potential of an algorithmic (*finite*) method of proof, one of the first and more fundamental results of Hilbert's work was the famous demonstration of the calculus of distances, which constituted the core of the author's *Foundations of Geometry* (1898-99). This work aimed at demonstrating the independence of pure geometry from numbers, by showing that all (Euclidean) plane geometry can be represented as a par-

ticular, abstract algebraic structure, called *commutative field*. Therefore, the center of Hilbert's work on the foundations of Euclidean geometry was the demonstration that *analytical geometry* does not need to presume the existence of numbers in order to justify the use of algebra within itself. This was a result that re-proposed and confirmed, by applying it to Euclidean geometry, the result already obtained by Riemann on non-Euclidean geometries (See above § 1.5.1.2).



Numbers are not so different from geometric objects, after all: as much as their epistemological origins may seem different, *the essential properties of both are expressed by the same axioms* (...). This type of result of Hilbert's geometry gave a decisive contribution to «formalism». Given that some mathematical objects that appear so different can satisfy the same laws, the natural tendency that derived from this was to *forget the objects and focus attention on the laws themselves* (Webb 1980, 78).

Absolute separation
of the logical form of
a theory from its
meaning and truth

In this sense it becomes understandable how, for Hilbert, the question of the meaning of a mathematical entity appeared irrelevant for justifying its constitution. Different *interpretations** or *models** of the latter were equally admissible, and they did not invalidate the consistency of the symbol within the formal system. To use Boole's words about his algebra:

Any system of interpretation that does not invalidate the truth of the supposed relationships is equally admissible (quoted in Webb 1980, 79).

We can then understand how, for Hilbert, what became essential for the construction of the axioms of a given formal system was not the truth of the axiom itself, taken in isolation, according to a particular interpretation or referential meaning. To the contrary, it was the *truth* and *existence* of the axiom that depended on its «compatibility» with the rest of the system. It depended on the fact that it did not contradict the rest of the system itself, or, in other words, on the fact that it was consistent in relation to the rest of the system. Hence we see Hilbert's controversy with Frege as developed in a famous exchange of letters between the two scholars.

Indeed, for Frege, the consistency of the axioms of a formal system, such as Euclidean geometry, immediately derived from its truth, from the fact that they properly referred to a set of abstract, conceptual objects (*mathematical entities*). As Frege stated in this regard:

Lack of this
distinction in
Frege's theory: truth
→ consistency

Are there ways, perhaps, to demonstrate the consistency (of a system of axioms, *editor's note*) other than to exhibit an object that possesses all the properties (defined by the axioms: again, the principle of evidence, *editor's note*)? Anyway, if one has such an object at their disposal, one

has no need to demonstrate its existence through the circular demonstration of its consistency (quoted in Webb 1980, 87).

Stated otherwise, for Frege, the consistency of axioms and of the theorems that can descend from them—that is, their consistency with the set of other objects in the domain—immediately derives from the truth of these axioms. Conversely, for Hilbert,



Hilbert's formalist approach in logic corresponds with his rationalist approach in metaphysics

This procedure of construction of an axiom by referring to its truth, and concluding from this that it is compatible with other defined concepts, is precisely the primary source of mistakes and confusion (*ibid.*)

This statement by Hilbert was proved correct by the discovery of antinomies in Frege's system. The alternative that he proposed in a famous letter to Frege is then the opposite of that of his interlocutor:

If the *arbitrarily established* axioms do not contradict one another, then they are *true*, and the objects defined through them *exist*. For me, this is the one criterion of truth and existence (*Ibid.* Emphasis added).

As one can see, to a *formalist-nominalist claim* in logic—axioms can be arbitrarily established—corresponds, from a metaphysical point of view, a *rationalist claim of a Parmenidean kind* concerning the notion of «being». Not only is 'being' reduced to the univocal character of existence and of *necessary* existence, but its foundation becomes equivalent to the demonstration of its *non-contradictory character*. At the end of this sub-section, we will come back to this metaphysical position and to its meta-logical plausibility, while, in the Second Part, we will briefly deal with the fundamental principles of Parmenidean rationalist metaphysics.



The problem of completeness of formal languages

In sum, D. Hilbert's position, in his dispute with G. Frege on the foundations of mathematics and, more generally, of formal logic, essentially consisted in arguing not only that *existence* in mathematics implies *consistency* (non-contradiction), but also that the reverse is true. Hilbert expressed all this in the form of an assumption of *completeness** of axiomatic systems.

In other words, if, through a system of axioms, we intend to define in a *univocal* and *complete* way the properties of the «primitive» objects of a given formal system (e.g., points, straight lines and planes in Euclidean geometry), it would be contradictory to add other elements to that system that cannot be derived from those axioms, and cannot be linked to the properties defined by those axioms, while, at the same time, maintaining that these do not enter in to contradiction with the axioms themselves.

For example, in plane Euclidean geometry I cannot add triangles whose sum of the internal angles does not equal 180° , without creating a contradiction. Even if I can state, in Euclidean geometry, that «a triangle whose internal angles add up to 190° » which contradicts its axioms—in particular the Fifth postulate—Euclidean geometry is complete:

- ◆ Non only if I can demonstrate that the statement «a triangle whose internal angles add up to 180° »—like all other statements that do not contradict the axioms of Euclidean geometry—is *compatible* (consistent) *with the axioms*, hence the object described in such statement is *true*, and therefore *existing* in this geometry;
- ◆ But, above all, if I can demonstrate that the statement «a triangle whose internal angles add up to 190° »—like all other statements that contradict the Euclidean geometry axioms—is *incompatible* (inconsistent) *with the axioms*, hence the object described in this statement is *false*, so that it *does not exist* in this geometry.

The consistency of Euclidean geometry demonstrated through a syntactic model in real numbers

On these bases, in the following chapter of the *Grundlagen*, Hilbert deals with the issue of the consistency of geometry that is thus axiomatically defined, and finds it by giving a geometric model or interpretation that is defined on real numbers. In this model, for example, all axioms of plane geometry become propositions relating to real numbers, that is, equations or systems of equations that can be algebraically demonstrated. In this way, what is demonstrated is not that these axioms are «absolutely» *true* but simply that they are *demonstrable in the theory of real numbers*. The truth (consistency) of these axioms, as well as of the theorems that can be derived from them, therefore, is a *relative* truth. Once the theory of real numbers is assumed as non-contradictory, all propositions that are *isomorphic* (that is, they bi-univocally correspond) to other propositions built on *n*-tuples (couples, triples, etc.) of real numbers and their relations—such as, for example, geometric axioms—are also non-contradictory⁷². The difference with Riemann should be noted: Riemann had built a *semantic model* of non-Euclidean geometry in a Euclidean geometry of curved space. Hilbert builds, in turn, a *syntactic model* (see footnote 72) of Euclidean geometry in the theory of real (algebraic) numbers, which transcends the reference to numbers as such, while simply affirming the correspondence between the *algebraic structures* defined on these numbers and the *geometric structures* defined on points.

⁷² Of the propositions defined on real numbers, it is said that they represent a «syntactic» model of the related geometric axiomatic propositions, precisely because the bi-univocal correspondence (or «isomorphism») between the two essentially relates to the structure of the relations and of the related properties, but not the *names*, hence the *meanings*, associated with the objects of those relations and endowed with those properties.

The second problem in «Hilbert's program»: a demonstration of completeness of arithmetic that is extended to real numbers

The second problem stressed in the so-called «Hilbert's program» — that is, the list of the issues that mathematics dealt with during the 20th century, most of which remained unsolved throughout the 21st century, and that Hilbert himself formulated during the Second International Mathematics Conference held in Paris, in 1900— was defined by Hilbert himself in order to overcome the problem of the relative demonstrability of the consistency of axiomatic formal systems. This issue essentially centered on the question of whether it was possible to demonstrate that arithmetic axioms —including the arithmetic of real numbers— are *compatible*.

In order to avoid infinite regression, the need for arithmetic to be able to function as the metalanguage of itself

Stated otherwise, the issue centred on whether it is possible to meta-mathematically demonstrate that, starting from the axioms of arithmetic and constructively (algorithmically) proceeding through a finite number of logical steps of demonstrations, one can ever reach contradictory results. Indeed, in order to avoid demonstrations of only relative consistency, we would need a «syntactic» model (see footnote 72) obtained within a mathematical theory whose consistency can be proved in an *absolute* manner, that is, with no reference to any other theory. In this case, a language should be able to become a *meta-language* of itself, that is, it should become *self-referential*. For example, it is possible to write an Italian grammar in Italian. Therefore, Italian can be a meta-language of itself. However, natural languages *are not* formalized languages, they are full of ambiguities and, often, of contradictions.

Therefore, the problem becomes: is it possible to build self-referential statements with formalized languages without engendering ambiguities and contradictions? And, in the case of mathematical sciences, what mathematical theory could be used as the meta-language of all mathematical theories, including mathematics itself? The stakes are indeed high: if such a research had succeeded, we could have spoken, in mathematics, of the possibility of *absolute* algorithmic demonstrations of consistency and truth, making reference only to finite (algorithmic), perfectly *certain*, formal procedures of demonstration, rather than ambiguously referring to evidence.

New form of absolute demonstration of truth and consistency, with no reference to evidence

In conformity with these criteria, the research on such an «absolute» demonstration procedure of the non-contradictory character of the different formal systems that could be built on mathematics first of all consisted in the formal illustration (using the axiomatic method and outlining the logic used) of the three most fundamental theories of modern mathematics: arithmetic, analysis and set theory. However, given the strong connection between the possibility of defining any class of numbers and the class of natural numbers, Hilbert's program, since its inception, centered on the hope that it would be possible to directly demon-

strate the non-contradictory character of arithmetic *from within* arithmetic itself.

Realization of this part of Hilbert's program in Gödel's research

This centrality of arithmetic was bolstered by Gödel's development (and, Turing's, after him) of a method for the arithmetic encoding of any formal language. Naturally, this demonstration of non-contradictory character presumed the completeness of axiomatic arithmetic, similarly to the assumption of completeness of geometry that we already know. In other words, it presumed that, given a set of axioms to formalize intuitive arithmetic, such as Peano's axioms (See footnote 20), one could derive, following a limited number of steps, in a *univocal* and *complete* way, all the properties of the «primitive» objects of arithmetic, without creating contradictory statements consistently derived from the axioms themselves.

This assumption of completeness of formal arithmetic, and the finite (algorithmic) method to be followed in order to meta-mathematically demonstrate it from within, were formulated by Hilbert already in 1900, in an essay on the concept of number (*Über den Zahlbegriff*). This essay had been the object of discussions, together with the assumption of completeness of geometry, in Hilbert's presentation to the mathematical society of Götting in 1901. Incidentally, this conference was very important for the history of contemporary philosophy, because it was attended by young Husserl who, precisely from Hilbert's presentation, drew the inspiration for his attempt to formulate a «transcendental» approach to phenomenology. This approach was free from any reference to *psychological* intentionality, and therefore able to refute the accusation of *psychologism* that Frege formulated towards Husserl's early attempts, in his *Philosophy of Arithmetic*, to create an intentional, phenomenological foundation of the concept of number and/or set.

3.1.3 Gödel's theorems

Hilbert's programme in the two characteristic points described below was realized in the brilliant work of an Austrian mathematician, Kurt Gödel (1906-1978), particularly in the first of his early studies (Gödel 1931).

Two steps:

1. The creation of a universal method of arithmetic encoding for formal systems

- ◆ First, Gödel created a *universal method* for the *arithmetic encoding* of the statements of formal languages, which was powerful enough to include the axioms of Peano's formalized arithmetic⁷³. In so doing, each statement of a given formal language was turned into a sequence of numbers; each logical operation on propositions into an operation on natural numbers; and each theorem into a theorem of the elementary theory of natural numbers.

2. Demonstration of the completeness of arithmetic from within arithmetic itself

- ◆ Second, Gödel focused on *demonstrating the completeness* of arithmetic by means of this arithmetic meta-language and finite (algorithmic) demonstration methods.

The result, which was a catastrophe for Hilbert's formalist program, is summarized in the two famous *theorems of incompleteness of arithmetic*, which we present here in a purely intuitive form:



- ◆ *First incompleteness theorem.* If consistent (see *consistency**), any formal system S , powerful enough to contain Peano's axioms of formalized arithmetic (as formulated in the *Principia*), will necessarily be *incomplete* (see *incompleteness**). Indeed, it will necessarily contain statements that are *true* but *undecidable* (see *undecidable character**) given that any attempt to demonstrate them from within the theory implies the simultaneous demonstration of their *negation*.

Failure of Hilbert's formalist program with the two theorems of incompleteness of arithmetic

In other words, if such a formal system wants to be *complete*, it will have to be *insubstantial*, because it will necessarily contain contradictory theorems. Conversely, if this system wants to be *substantial*, then it will have to be *incomplete*, because it will have to give up demonstrating all the true formulae that can be expressed in itself. That is, it will have to limit itself to containing *undecidable* statements.

- ◆ *Second incompleteness theorem.* In any formal system S , powerful enough to contain the axioms of formalized arithmetic, the meta-linguistic statement « S is substantial» is an *undecidable* statement of that system.

As one can see, if such a system S aims to be *substantial*, it will not be *self-referential**, in addition to not being complete. That is, if it aims to be substantial, this system will have to be both incomplete and *syntactically «open»*: its proof of consistency will necessarily have to be *outside* the formal language under consideration.

⁷³ In fact, Gödel arithmetically encoded Peano's formalization of axioms (Cf. footnote 20), just as they were formulated in *Principia Mathematica* by Whitehead and Russell.

Discovery of the
syntactic origin of all
logical antinomies

Gödel thus wanted to explicitly demonstrate the ultimate formal origin of all logical antinomies, both syntactic and semantic, starting from the famous *liar's paradox*. These depend on the fact that any formal system, in order to be absolutely *true and consistent*, needs to meta-logically demonstrate within itself the *falsity* of all the propositions that can be built in its logical language and that can contradict it. This is the need for any formal language to demonstrate as *false* all propositions able to *contradict* (at the level of axioms) the origin of antinomies. Now, if a formal language can never be so «powerful» (formally robust) that it can exclude *all* the propositions constructible within it that can contradict it, then all formal languages are *fragile*, they are always prone to contradictions. Their consistency will then be *limited* and always open to further proof of inconsistency.

The myth of the *absolute certainty* of deductive procedures was then destroyed. The ancient mistrust towards *inductive* methods, in favor of *deductive* ones, based on the idea that the former have limited certainty, while the latter give *absolute* certainties, thus disappeared forever, like one of the many *myths* appearing throughout the history of philosophy (Cellucci 1998).

3.1.4 Theorems of limitation

Gödel's theorems
and limitation
theorems in logic
and mathematics

It is difficult to overestimate the results obtained with Gödel's demonstrations. In particular, it is difficult in light of their extensions, which became as many *theorems of limitations* of formal languages, and whose origin is indeed in Gödel's work. These extensions are in all fields of logic and mathematics. Here, we shall deal with two of them, which had a more direct logical, hence metaphysical, value and which Gödel himself considered the most important consequences of his discoveries. The first relates to the extension of Gödel's theorems to *all formal systems* (not only to formalized arithmetic), as long as finite (algorithmic) demonstration procedures are applicable to them. The other relates to the enlargement of the *syntactic* incompleteness theorems to *semantics*, particularly to the demonstration of the *logical truth* of formal systems. If one uses the same algorithmic methods of truth testing, where truth is intended as a statement's «correspondence to objects», one arrives at the same results of undecidability. Not only must formal languages be *syntactically open* —because the test of their consistency must be made from «outside» that language — they also have to be *semantically open*. The test of «truth» or of «correspondence to facts» of that language must be made from outside them. Thus, the myth of the *absolute autonomy* of scientific languages from other types of languages, in addition to that of their *absolute validity*, was destroyed forever.

3.1.4.1 Turing's theorem

Turing's extension
of Gödel's discovery
to all formal
systems

At first, Gödel's results seemed limited to formalized arithmetic and seemed not to involve other formal systems, hence the remaining linguistic objects of the analysis of formal logic. Another 20th century mathematician, Alan Mathison Turing (1912-1954) —to whom we owe the fundamental development of *computability theory*, hence the scientific and technological development of present-day computers— extended Gödel's discovery to all formal systems and to algorithmic methods of demonstration and/or of calculation within these systems.

Church's and
Turing's theorems

This fundamental extension was made possible, first and foremost, by the work of another logician and mathematician from Princeton University, Alonso Church (1903-1995), who invented the λ -calculus, based on the hypothesis that all computable functions of logical and/or mathematical calculus can be calculated algorithmically by means of *recursive* functions. By integrating this discovery with Gödel's encoding method, A. M. Turing (Turing 1937) created an elementary diagram of an algorithmic machine or, more precisely, of *abstract calculus architecture*, the so-called *Turing Machine* (TM).

Turing Machine and
the limits of the
Universal Turing
Machine

Each TM is able to algorithmically calculate a given *recursive* function. By inserting in its «memory» the appropriate instructions (algorithm), a TM can also simulate the calculus carried out by another TM. In this way one arrives at the construction of the Universal Turing Machine (UTM), which is able to simulate the calculations of any TM. The UTM is therefore the logical diagram of a modern multi-programmable calculator. In this context Turing demonstrated that the UTM is not able to terminate its calculations. That is, it is not able to list, through a finite, halting *decision procedure*, all the consequences that can be deduced from a given finite set of axioms. This theorem of limitation is therefore also defined as the *halting theorem* of the UTM. In other words, in *any* formal system — and not only in the system of formal arithmetic to which Gödel's early discovery applies— it is not possible to determine all the valid consequences of a given set of axioms, by means of constructive or algorithmic procedures.

3.1.4.2 Tarski's theorem

Extension of
Gödel's results to
the semantic
analysis of truth in
formal systems

Alfred Tarski (1902-1983), a logical mathematician of Polish origin, extended Gödel's theorems (particularly the second) to the problem of the formal analysis of the *semantics* of any formal system. Tarski demonstrated that the «truth» of any formal system, that is, the *correspondence to objects* defined by the symbols of the formal calculus of a given object-

language, can be algorithmically decided only by a *meta-language* whose logical order is higher than that of the object-language.

Stated otherwise, not only must formal languages be «syntactically open», if they want to be formally substantial; they also have to be *semantically open* to other formal languages and, ultimately, to other forms of language—if they want to avoid the infinite regression of formal meta-languages of increasingly higher logical orders—in order to formally demonstrate their *truth*.

3.1.5 «Open» logical systems

Insufficient nature of the use of non-finitary methods to solve specific problems

In metaphysical terms, the discovery of the various theorems of limitation of formal systems and of finitary demonstration methods can be interpreted in many different ways; not all of them, however, are well founded. Many logicians and mathematicians, starting from Gödel himself, argue that the theorems of limitation simply prove the need to use *mathematical intuition*, hence *non-finitary demonstration methods* (of which Riemann gave a first example, with his Euclidean model of non-Euclidean geometries. (See above § 1.5.1.2)). In practice, therefore, all these theorems would only point to the limits of *mechanical* (algorithmic) *demonstration methods*⁷⁴. This, however, is not entirely true. On the one hand, this discovery brings back, into the heart of mathematical demonstrations, a reference to mathematical «intuition» whose consistency is difficult to define and control. On the other hand, and most importantly, it makes it impossible to guarantee that «absolute character» of demonstration procedures, when these are applied to specific mathematical problems. Indeed, infinitary methods prove generic, abstract statements, which cannot be used for the solution of specific problems (Cellucci 1998, 254). This is how Cellucci explains this point, which certainly displeases many «traditional» mathematicians:



Given $\mathfrak{S}$ as the concept of set that, according to Gödel, we perceive through intuition. Given S as a consistent system of set theory, for example, Zermelo–Fraenkel's. Intuition guarantees that the axioms of S are true for $\mathfrak{S}$. However, due to a corollary of the first of Gödel's incompleteness theorems, a concept of set $\mathfrak{S}'$ and a statement B exist, such that the axioms of S are also true relating to $\mathfrak{S}'$; however, B is true in relation to $\mathfrak{S}'$, and false in relation to $\mathfrak{S}$. (In particular, if A is the true statement, which is neither demonstrable nor refutable in S , due to Gödel's first incompleteness theorem, one can consider the statement $\sim A$ as B).

⁷⁴ Indeed, just five years after Gödel's work, G. Gentzen demonstrated the completeness of formalized arithmetic, by using non-finitary demonstration methods.

From the fact that B is true in relation to $\mathfrak{S}'$ and false in relation to $\mathfrak{S}$, it follows that the concepts of set $\mathfrak{S}$ and $\mathfrak{S}'$ are not equivalent. Now, (...) we can arrive at an intuition of $\mathfrak{S}'$. In this way, we have two different intuitions: the first tells us that the true 'set concept' is given by $\mathfrak{S}$, the second that the true 'set concept' is given by $\mathfrak{S}'$. On the basis of the first intuition, B is a true statement; on the basis of the second intuition, B is false. Which is, then, the true concept of set? *Intuition cannot give an answer* to this question (Cellucci 1998, 254).

Cellucci's example, presented above, is anything but infrequent in mathematics. In fact, it relates to a key issue in set theory. Indeed, following P.J. Cohen's demonstration (Cohen 1966) of the independence of the continuum hypothesis from the axiom of choice, hence from the concept of set of Zermelo–Fraenkel's (ZF) axiomatic theory, it is possible to construct another notion of set, that of *generic set*. Based on the latter, Cohen built his set theory, which is an alternative to ZF's. However, intuition cannot tell us which of the two is the «absolutely» true notion of set. Furthermore, Cellucci recalls, the existence of various alternative sets of axioms on the theories of large cardinals give us other examples of the inability of intuition to provide «final evidence».

Other scholars —and many philosophers among them— therefore tend to give an *irrationalist* interpretation of Gödel's results, by stating that these represented the death of «scientific reason» and of its claims to «truthfulness». However, to state that it is not possible to arrive at *absolute* truth by means of demonstrative methods does not mean that it is impossible to arrive at *any* truth, be that a *partial*, hence *perfectible* one.

Consequences of
Gödel's theorems

1. Ontological
consequences:
renewal of the
ancient
juxtaposition
between
«nominalists» and
«realists»

The conclusions are indeed different, and they necessitate a distinction between the *ontological* and the *logical* consequences of Gödel's theorems:

- ◆ *Ontologically*, it is clear that the fundamental property of self-referentiality, characterizing any logical universal as such —the capacity that any property or «quality» to which a predicate refers to has to refer only to itself; for example, the «being-red of red» or «red-ness» as the ultimate referential term, self-referring and ante-predicative, of the predicate «being-red», or formal ultimate condition of the «closing» of the class of «red objects»—, which represents the main discovery of logical thought of all times, from Plato on (See § 5.3.1) cannot be reduced to the self-referential character of a meta-logical testing procedure of an algorithmic, mechanical kind. Either logical universals do not exist —independently from the modalities of existence that the different ontologies of logic and mathematics ascribe to them— and logical and mathematical lan-

guage has no meaning in itself, viewed as a simple «syntax» of the language of science, a pure «notation without *denotation*», a simple set of «use rules» of mathematical signs; or a meta-logic of formal self-referentiality is not the means of guaranteeing meaningfulness for logical and mathematical language. If they exist, logical universals —through which one can guarantee the ultimate meaningfulness of logical and mathematical formulae— must exist «for themselves», and not because some meta-logical testing procedure justifies their existence. Gödel's theorems as such, however, do not allow us to choose between these ontological alternatives that, incidentally, bring the classical pre-modern juxtaposition between «nominalists» and «realists» of late-medieval logic back into post-modern thought. Gödel, then, opts for a *fideist* position of Platonic realism on universals.

An example of the nominalist (formalist) position



The following are quotations from two great mathematicians of the 20th century, Edward Nelson from Princeton University, and Kurt Gödel himself, which well illustrate these two ontological positions that, while opposed, can both be derived from the incompleteness theorems.

In mathematics, reality is in the symbolic expressions themselves, and not in the abstract entities that these expressions are supposed to denote. The symbol $\exists$ (the existential quantifier of symbolic logic, the «it exists» of the logic of predicates, *editor's note*) is simply a reversed E. If we conclude that a given entity exists just because we have constructed, in a given formal system, the proof of a given formula that starts with $\exists$, we do that at our risk. Syntax is the home of meaning; semantics is that of illusion.

How can I keep on being a mathematician when I have lost faith in the semantics of mathematics? Why should I continue doing mathematics, if I do not believe any longer that stochastic numbers and processes and Hilbert's spaces exist? I will answer this with another question: and why should a composer want to compose music, if this is non-representational? Mathematics is the last art that has become non-representational. Slowly, therefore, mathematics started to become non-representational. Slowly in the mathematics departments, but quickly in the computer science departments. Those that work with computers know that they invent rather than discover, and they create beautiful and profound results on the nature of possible computations. If we, who are in the traditional departments, do not want to miss the boat, we should get on a formalist horse right away. Abstract beliefs influence concrete actions. Despite the complete lack of justification, the semantic perspective on mathematics —the discovery of the properties of entities that exist in a Pythagorean world— has served mathematics well for a long period. But now the moment

has come to move forward, to discard the semantic perspective and to concentrate on the real in mathematics. And notation is what is real in mathematics, not imaginary denotation (Nelson 1997, 247).

An example of the
realist position



And here is Gödel's position on the same issue:

According to this idea (the one that reduces mathematics to pure syntax, *editor's note*), meta-mathematics is that part of mathematics endowed with meaning, through which mathematical symbols (which are devoid of meaning in themselves) acquire some surrogate of meaning, that is, the rules of use. Of course, the core of this point of view consists in rejecting any kind of abstract and infinite objects, such as, for example, the immediate meanings of mathematical symbols (universals, *editor's note*). In other words, this recognizes that only propositions relating to *concrete and finite objects*, such as combinations of symbols, have some meaning (...).

However, how could one *express* meta-mathematics *within* mathematical systems themselves, if one thinks that these consist of symbols devoid of any meaning, which acquire some surrogate of meaning only *through* meta-mathematics? (From a letter of Gödel to H. Wang, 1967, quoted in Wang 1974, 18).

We could say the same thing from another point of view, according to which, in mathematics, the most fundamental propositions are always analytical formulae of the kind $a = a$, or «red is red». Gödel starts from Carnap's definition, stating that a proposition is analytical if it is valid «exclusively by virtue of the meanings that appear in it». From this derives that such a meaning can be in turn undefinable (that is, irreducible to another, more fundamental, meaning).

So we should mention that this notion of analytical nature makes it possible again that each mathematical proposition must be reduced to a special case of $a = a$, which is admissible, as long as one knows that, if the reduction is made, *this is not by virtue of the definition of the terms* (because, then, one would introduce an infinite regression, *editor's note*), but *by virtue of their own meanings*, which can never be expressed through a set of formal rules.

2. Logical
consequence: to
forego any
ascription of
absolute truth to
deductive thought

- ◆ *Logically*, whatever the answer we give to the preceding ontological question, both scientific and philosophical thought must stop endowing deductive thought with those claims of absoluteness that, in the history of philosophy, many rationalist theories have wanted to ascribe to it. This is a claim that, more recently, has found another distinct, but equivalent, expression in the statements of modern scientism.

Logical systems as «open», rather than «closed» (formal and/or axiomatic) systems. Analogy with «open» dynamic systems, stable outside equilibrium (dissipative structures)

Inadequacy of «closed» formal systems for mathematical sciences

Relation with the consequences of incompleteness



In sum, formal languages have a relative, and never absolute consistency, that is, they have to be «syntactically open»; in addition, they also have to be «semantically open» to other languages and forms of language. Recently, C. Cellucci himself, in the conclusion of his work of synthesis on the situation of the foundations of logic at the end of the 20th century (Cellucci 1998, pp. 309ff), stressed once again that the only future for mathematics and logic is to give up the myth of «closed» formal systems, such as axiomatic ones, to start working again on *open* logical (dissipative) systems, equivalent to *open* (dissipative) physical systems: the subject matter of complexity theory (See § 2.6). Complex systems are open (dissipative) physical systems able to *generate information*, in which the latter is not entirely contained in the initial conditions; similarly, the logical systems of the future must be *open* logical systems. These are profoundly different from axiomatic logical systems in which all the information is considered to be in the unchangeable starting axioms: the assumption that all the information of the system can be determined by the initial conditions is a false one, according to Cellucci, at least concerning actual mathematical practice.

For Cellucci—as well as for us, who have argued the same for years—all this is a direct consequence of Gödel's theorems. Indeed, the discovery of the essential incompleteness of formal systems (the first of Gödel's theorems) had, as a consequence, that *formal systems are inadequate for mathematics* (and for the sciences that use them). This is the case because, quoting Isaacson,

The choice of any specific formal system must be provisional and *subject to the mathematical need* to go, at the end, beyond the choice itself (Cellucci 1998, 323).

This vision of mathematics still meets great resistance among mathematicians that, with H. B. Curry, prefer stating that Gödel's theorems only imply that mathematics cannot be expressed in one single formal system, but rather in many formal systems.

However, as Cellucci well summarizes, Curry's interpretation of the consequences of the first of Gödel's theorems is inadmissible, given that it ignores the following consequences of that theorem (Cellucci 1998, 323-6):

- ◆ *The development of mathematics cannot consist in demonstrating the theorems of a given formal system* given that, in any case, it will always contain undecidable propositions, which are true, but not demonstrable in it.
- ◆ *The development of mathematics cannot consist of the construction of a sequence of increasingly complex formal systems*, built by using, as the axiom(s) of a

given system(s), that (those) proposition(s) that had proved undecidable in the preceding system(s). Indeed, the problem of whether this sequence of systems is in turn complete implies only a partially affirmative answer for generic formulae of the kind $\forall x A(x)$, where $A(x)$ represents a decidable property, even if on the condition that this formula cannot be decided algorithmically, while the answer is absolutely negative for specific formulae—which are indispensable for the solution of problems—of the kind $\forall x \exists y A(x, y)$, where $A(x, y)$ represents true, but undecidable properties of the system.

- ◆ *The development of mathematics cannot consist of the construction of a sequence of formal systems chosen in an factual manner, that is, by means of not necessarily deductive, but also inductive inferences. It has been demonstrated, indeed, that even for such a sequence, typical of the so-called «non-monotonous» logic, one can demonstrate a theorem equivalent to the first of Gödel's theorems.*
- ◆ *The development of mathematics cannot consist of one single formal system, but must consist of more logical systems, able to interact with one another, and that can never be formal systems.* Given that each of them must obtain information «from outside» in order to overcome their own inadequacies, clearly these logical systems must not only be able to communicate with one another, but none of them, due to this fundamental character of «openness», can be a formal system. This fundamental point is made clearer by the following consequence:
- ◆ The communication among logical systems that constitute this distributed environment can be neither algorithmic, nor deterministic. They cannot be deterministic, guided by one single formal law, because otherwise everything would be reduced to one single formal system. On the other hand, the non-determinism required by Gödel's result is not the one linked to the simple non-deterministic choice of which formal rule, among the given ones, should be applied, neither to the choice of where to apply it, in relation to a set of given alternatives. Furthermore, these choices cannot even be algorithmic, because this would, once again, lead to one single formal system. In sum, —and this is the fundamental point— the non-determinism required by Gödel's result is for anyone to be able to introduce new axioms, at any stage, in a non-algorithmic way (Celucci 1998, 326).

The development of mathematics requires «open» systems, in which one can introduce, at any stage, new axioms

«Open» logical systems are not formal systems, because they require the introduction of new axioms

It is clear, then, why such logical systems cannot be formal systems: no formal or axiomatic system admits the introduction of *new axioms*. Conversely, it is precisely the introduction of «new axioms» along the way that characterizes the notion of an «open» logical system. It is also clear that the introduction of new axioms *cannot be arbitrary*; on the other hand, however, it is also clear that, contrary to what Curry believes, the formal method is not the only mathematical method. Indeed, the third consequence of Gödel's theorem excludes that the choice of new axioms can be a factual procedure that defines an ordered sequence of formal systems. On the other hand, the fifth consequence stipulates that the relations among the various systems can not be of a deterministic and/or algorithmic kind.

«Open» logical systems originate from the integration of analytical and axiomatic methods in the same system

Therefore, one only possibility remains: that the formal method is not the only mathematical (and logical) method. In this case, the introduction of new axioms will be linked to a specific *logical method*: the *analytical method*. This introduction, in other words, does not occur *arbitrarily*, as for the neo-Positivists as well as Popperians and post-Popperians: we will see this in the next chapter. In this sense, logical systems are no longer limited to the use of the pure *axiomatic method*, but will also include, as a fundamental ingredient—for the introduction of new axioms—the *analytical method* typical of classical pre-modern logic, as well as its characteristic inferential procedures (induction, abduction, abstraction, analogy) (Cellucci 1998, 348ff). In this way, the logical-mathematical inference, because it is not purely deductive, *increases* knowledge, as testified by mathematical experience. To demonstrate a new theorem, or to find a new way to demonstrate a theorem, at least psychologically, increases information, even if, from the formalistic point of view of logic—that is, in axiomatic systems—this is not absolutely true. In the case of «open» logical systems, conversely:



Inference does not only help to find logical consequences from given axioms, but also *helps to find the most adequate hypotheses to solve a given problem*. This explains why, based on this, discovering a new demonstration produces new information, i.e., discovering new hypotheses that are not already implicitly contained in the conclusion, and therefore establishing new connections between the problem and the external world (Cellucci 1998, 332; emphasis added).

Open systems characterized by being able to evolve, plasticity, modularity

Demonstrations, in such open logical (*analytical-deductive*) systems, will then be characterized by a high degree of *evolving*—one proceeds through progressive re-adjustments—, *plasticity*—one does not proceed from fixed principles, but hypotheses are found along the way— *modularity*—where each module has a particular task for the solution of a

part of the problem, even if this task can change in relation to the other parts of the logical system and to the environment in which the demonstration evolves. Anyone can observe that this notion of logical system renders the development of science better than any other, without falling in the quagmire of relativism and of «weak thought».

Conversely, it is in the name of demonstrative *rigour*, of *effectiveness* and of *productivity* of scientific thought that we are called to this radical «paradigm change», in the sense used by Thomas Kuhn (See § 4.2.2.4). Indeed, to insist on the exclusivity of formalism, on the tyranny of the axiomatic method would mean to condemn mathematics, logic and the entire *theoretical* science —*demonstrable* by its very nature— to sterility and uselessness. It would also mean to condemn the culture of the near future, as well as that of the present (that use science in an applied way), to irrationalism and «weak thought». Today's enemy, in science as well as in theology, in mathematics as in physics, is the presumed arbitrary character of the choice of axioms that makes all hypotheses equivalent.

To recover the *analytical method*, as a set of logical rules for the discovery of new axioms that are contextually *appropriate*, «adequate» to the specific problems to be solved, means to restore reason to its rightful place in the discovery and construction of a reality that is truly «worthy of Man», because it is worthy of the *being* of things. This will be the object of the next three chapters dealing with the *theoretical* issues emerging in today's philosophy of nature and of science.

Continuity of open
logical systems with
the Thomasian
paradigm of
adaequatio

To conclude, there can be no theory that is *absolutely* substantial and true, neither can there be a theory aiming to be *all-inclusive* in any field of rational knowledge, whether scientific or philosophical, worthy of the name. If we look for consistency and truth, we should endow the scientific knowledge of the future with the same *dynamism* that reality possesses. And if we want to have reason to hope that this knowledge helps us to understand *something* of reality, we must relinquish the pretense of understanding *everything*.

For this reason, the Thomasian paradigm of truth, intended as the *continuing adjustment of formal systems' axioms to reality*, appears much more «post-modern» than Descartes' paradigm of truth intended as the static *evidence* of unchangeable axioms —a paradigm that has been with us for the past four centuries. Given that only open logical systems can be *true*, all *true* theories will be *partially and not absolutely true*, because they are not reciprocally *tautological* (that is, they do not all say the same thing). That means that they will participate in the only *absolute truth* that will not identify itself with any particular theory and will somehow be *ineffable*. This is how Aquinas justifies his negation that, at least in relation to hu-

man knowledge, *one single absolute truth* may exist that is not the divine one, which certainly is not at the level of men and their capabilities.



A concrete example
of an «open» logical
system in Perrone
1996

The truth that is in the divine intellect is one only, from which multiple truths derive in the human intellect just like from one face derive many of its images in a mirror. (...) The truths that are in things, to the contrary, are many, as many as their entities. (...) From this derives that the truth that Anselm speaks of is that one only, because it is the measure of all true things (...). The truth that is in the human intellect or in things, conversely, does not relate to things as their extrinsic measure, contrary to what happens to the truth that is in the things themselves. And even these measures, *one should multiply according to the multiplicity of the measured things*, the same way that it is necessary that different bodies have different dimensional measures (two bodies having the same measures in all, would be the same body, *editor's note*) (Thomas Aquinas, *In de Ver.* I, 4c and ad 1).

Therefore, it is not a coincidence that, already two years before Cellucci's publication of his assessment of the future logic and mathematics, a proposal for an *open mathematical system* that possessed all the qualities envisaged by Cellucci had been formulated precisely in the field of the application of the principles of Thomistic logic and epistemology to the foundations of logic and mathematics (Basti & Perrone 1996) and to the study of complex systems (deterministic chaos).

Critique of
Parmenidean
rationalism in
metaphysics

This observation of the relationship between *open logical systems* and a theory of truth as *adaequatio* has a further metaphysical consequence. Any philosopher knows that the first formulation of a formalist existence principle was not made by Hilbert (see § 3.1.2), but goes as far back as Parmenides, who first assigned an ontological value to the p.n.c., by making it a necessary and sufficient criterion of *existence* of objects defined by non-contradictory statements (See § 5.1.4). Stated otherwise, for Parmenides—as for any formalist metaphysical thinker, from Hegel to Hilbert, and even including the contemporary scholar Emanuele Severino—*non-impossibility* implies *necessity*, in the ontological as much as in the logical order. Conversely, the strength of a non-Parmenidean approach to being—such as that of any theory admitting an irreducible *similarity* of the notion of «being»—is precisely the Aristotelian principle according to which any demonstration of *non-impossibility* does not imply *necessity*, whether from a logical or ontological point of view. Ontologically, the non-contradictory character of a statement simply implies the *potentiality to exist* of the entity denoted by the statement itself—something profoundly different from the simple logical possibility, even in the post-Gödelian sense of a non-deterministic choice between given alternatives.

The alternative is not the common one between Platonic realism and nihilistic nominalism

If a universality of languages grounded on the existence of one single formal language to which all others can ultimately be reduced is the scientific and *Illuministic* myth that we have to abandon, the metaphysical alternative is not the commonly stated one between realism and nominalism. In other words, it is not the alternative between a Platonic fideism *à la Gödel* on the existence of mathematical objects, on the one hand, and the equally fideistic arbitrariness of Carnap's *conventionalism* in logic and/or Nietzsche's *nihilism* on the other (See § 5.6.1).

The need to recover, in meta-logic, the core of Thomistic metaphysics: the distinction between essence and being

The real distinction and the reciprocal determination «essence»-«being» in the constitution of the entity and existence of any being—which represents the core of the Thomistic doctrine of being, both natural and logical—together with the *solely contingent* character that a non-paradoxical nature guarantees to the existence of the *referent**—that Aristotle had already argued for—allows Aquinas to point to an original way of avoiding this double fideism of realists *versus* nominalists. We will come back to this in the next three chapters, dealing with the relationship between metaphysics, meta-logic and science.

3.2 Philosophical reflections on science

The rise of the philosophy of science during our century

In this context of crisis and revision of the foundations of modern scientific thought, a new discipline of philosophical reflection, the *philosophy of science*, increasingly gained ground during the 20th century. As it has been correctly pointed out



Also for the fact of being of fairly recent origin compared to the remaining part of philosophical thought, it does not correspond to one univocal definition, given that there are different ways of understanding what it is, according to the different authors that deal with it (Strumia 1992, 9).

The same author quotes a piece from the preface of Ernest Nagel's important work, in order to historically and culturally position this discipline within the realm of knowledge, and not just in a modern sense. Hence, we present this piece here, adding a few lines to those chosen by our friend and colleague, Strumia:



[T]he record of reflection on the nature of scientific inquiry and its significance for human life goes back to the beginnings of theoretical science in Greek antiquity; and there are few notable figures in the history of Western philosophy who have not given serious thought to problems raised by the sciences of their day. In consequence, although the use of the term 'philosophy of science' as a name for a special branch of study is relatively recent, the name designates investigations continuous with those that have

been pursued for centuries under such headings for traditional divisions of philosophy as 'logic', 'theory of knowledge', 'metaphysics', and 'moral and social philosophy'. Moreover, despite the impression sometimes created by the wide currency of the term in titles given to books, courses of instruction, and learned societies that it denotes a clearly limited discipline which deals with a group of closely interrelated questions, the philosophy of science as currently cultivated is not a well-defined area of analysis. On the contrary, contributors to the area often manifest sharply contrasting aims and methods; and the discussions commonly classified as belonging to it collectively range over most of the heterogeneous set of problems that have been the traditional concern of philosophy (Nagel 1961, vii-viii).

3.2.1 The birth of the philosophy of science

Two functions of the philosophy of science in late-modernity

As one can see, the central role that the philosophy of science has acquired in contemporary philosophical reflection derives from two kinds of needs:

- ◆ The need to deepen a series of logical and epistemological problems linked to the contemporary development of mathematical and natural sciences, following the scientific revolution that took place during the second half of last century and during the present century (and which is far from over).
- ◆ The need to find some answers to a series of theoretical, but also moral, cultural and social problems that the current development of science, as an emerging cultural fact in modernity, imposes on the thought and conscience of contemporary individuals.

The misunderstanding between philosophy of science and philosophy of nature

By means of a philosophical reflection on science, therefore, we tried to find an answer to a series of theoretical problems that could give a theoretical basis to the discussion of infinite and unavoidable moral, cultural, political and religious issues linked to the question of the development of science and technology in modern society. These related to, for example, the origins of the universe; the origins and the specificity of living organisms; the origins and specificity of human life; the nature of higher human psychological functions (intelligence and freedom); the relationship between causal determinism or indeterminism in the physical realm and freedom in the ethical realm, etc.

One could think, for example, of all the moral —hence also political and religious— questions that are linked to the development of genetics and of its technological application, i.e., *genetic engineering*. Or we could

think of all the issues linked to *ecology*, hence to the regulation of technological development. Or to the problem of the relationship between *body and mind* in neuropsychological studies, and the related moral, but also industrial and social problems linked to the interaction between Man and Machine, to the development of telecommunications, etc.

3.2.2 Re-birth of the philosophy of nature

Ancillary function of the philosophy of science towards the philosophy of nature

In this sense, the philosophy of science still has, today, an *ancillary function*, relating to the lack of any metaphysical reflection on natural entities that is conceptually, hence formally and linguistically, able to compare with modern sciences—a metaphysical reflection concerning physical ‘being’ understood in its different typologies, inorganic and organic, animal and human; as well as in its relations both with the rest of the universe to which it belongs, and with men’s knowledge and actions, their well-being and development. Indeed, men are part of the physical world and yet, in some way, they transcend it, as it is shown by their moral as well as intellectual capacity—starting with the scientific one—to dominate and control the rest of nature. Briefly stated, the philosophy of science is today attributed a function that, in itself, would belong to the metaphysics of physical entities and, therefore, to the *philosophy of nature*.



Historically, this ancillary function is justified by the inevitability of some problems that science poses and the blameworthy lack of a metaphysical reflection that is adapted to the methods, formalisms and results of modern sciences. On the other hand, however, it conceals serious risks. The lack of a clear definition of the limits and methods of the philosophy of science, when this shifts from being an *epistemological* and *metalogical* discipline—whose objects are *sciences*—to being an *ontological* and *metaphysical* one—whose objects are the *entities* studied by sciences—usually creates a dangerous confusion, which becomes more serious, delicate and complex as the related issues become more profound.

The lack of a universally accepted method of *illustration* and possibly *demonstration* of the results of metaphysical and, more generally, philosophical enquiries—that Edmund Husserl already pointed to in his famous essay *Philosophy as a Rigorous Science* (Husserl 1911)—exposes contemporary culture to serious risks. These, in turn, are much more serious when they relate to the field proper of the philosophy of nature, that is, the metaphysical reflection on natural entities (both human and sub-human) as well as on the limits and the potentiality of their study and technological manipulation. Without an inquiry and demonstration method and without a universally accepted rigorous language similar to, even if distinct from, the scientific one, one always runs the risk of trad-

ing the irrefutable results that derive from scientific research with different authors' personal philosophical opinions in the interpretation of scientific data.

Without an adequate philosophy of nature, we run the risk of ideological exploitation of the results of science

In this way, first, one undermines the seriousness and rigour of scientific research, which would never feel authorized to make similar extrapolations. Second, and more importantly, such speculations become the unconscious (yet sometimes conscious) pretext for ideological operations carried out by economic and/or political groups. There is indeed an understandable (yet not justifiable) interest in falsely using the prestige of science to modify public opinion and certain philosophical, moral and religious convictions. This interest, however, has little to do with science, as well as with the intellectual ethics and rigour of scientific research.

Hence the re-emergence of the philosophy of nature during the 20th century

Therefore, it should not surprise us that, in parallel with the *philosophy of science*, i.e., the philosophical inquiry into the *methods* of natural and logical-mathematical sciences, the need for a *philosophy of nature* properly understood re-emerged in the culture of the 20th century; that is, the need for a systematic reflection on the *ontology* underlying the concepts of modern mathematical and natural sciences that is not confused with the methodological reflection typical of the philosophy of science (for a deeper treatment of these notions, see § 4.1). This need is imposed by the revolutions in the mathematical and physical sciences of the last two centuries, which we have dealt with at length in this chapter.

3.2.2.1 Pierre Duhem

The beginning of the 20th century philosophical-natural reflections in the debate between Whitehead and Duhem

This recovery of the analysis of ontological issues, linked to the scientific revolutions of the 19th and beginning of the 20th century, was initiated by one of the main protagonists of these revolutions—at least in the field of logical-mathematical sciences—Alfred North Whitehead (1861-1947), co-author, with Bertrand Russell, of the «bible» of modern mathematical logic, *Principia Mathematica* (1910-1913), that we often referred to in this work. In a famous series of lectures held in 1919 at London Trinity College, which were then collected in *The Concept of Nature* (Whitehead 1920), Whitehead argued for the recovery of an ontological approach to the reflections on the results of the «new physics», particularly in terms of the theory of relativity. This position was openly in contrast to the anti-metaphysical phenomenism of a positivist kind that was then dominant in the philosophical reflections on physical sciences, that went back to the teachings of Ernst Mach (1838-1916), later reiterated, among others, in Pierre Duhem's famous work (1861-1916): *Physical Theory: its Object and Structure* (Duhem 1914).

Duhem's
phenomenalism

In this work, the French physicist and philosopher repeated Mach's phenomenalist positions on the interpretation of the results of physical science. These positions, however, were not defended from a positivist point of view, but by that of a practising Catholic, convinced champion of the value of metaphysics. For him, however, metaphysics did not need to have a naturalist basis in physics, but rather an essentially religious foundation. From this position —a purely phenomenological physics that is the expression of the inability of this discipline to grasp reality, completely separated from metaphysics on purely ethical-religious bases— that had and still has many followers in many sectors of contemporary Catholic culture, he called for a purely *pragmatic* value of physical science. The latter, according to him, should not be judged in terms of truth/falsity, as if its theories *reflected* reality in some way, and above all, as if they *explained* its events; rather, physical science should be judged according to *utilitaristic-aesthetic* criteria of formal simplicity and economy.



A physical theory is not an explanation. It is a system of mathematical propositions, deduced from a limited number of principles whose aim is to represent in the simplest, most complete and exact way a set of experimental laws (Duhem 1978, 24; emphasis added).

3.2.2.2 Alfred N. Whitehead

Whitehead against
phenomenalism
and for the recovery
of ontology

In this cultural context of a dominant anti-metaphysical phenomenism, Whitehead's position, calling for essentiality in the philosophy of science —intended as a critical, logical-epistemological reflection on scientific knowledge as a whole— is particularly noteworthy. Whitehead, in other words, called for an adequate, modern *philosophy of nature*, that is, an adequate ontology of natural entities. In the preface to his book on *The Concept of Nature*, he firmly states that its aim:

[I]s to lay the foundations of a *philosophy of nature*: a necessary prerequisite which allows for a re-organization of theoretical physics (Whitehead 1920, ix).

Whitehead's anti-
dualist definition of
«nature»

In order to fulfil this aim, a definition of the concept of *nature* becomes necessary, one that avoids the two fundamental mistakes of the philosophy of nature in the past, both classical and modern. According to Whitehead, these mistakes can be summarized in two distinct *dualisms*:

- ◆ *The dualism between matter and spirit* that dates back to Plato (See 5.3.1) but —in the distinction between material entities and entities of thought— through Aristotle and medieval Scholastic philosophy (See 5.4.1), after Galileo's revolution in the sciences of nature,

reached modernity with the dualism between *res extensa* and *res cogitans* in Descartes' philosophy of nature.

- ◆ *The dualism between substances and accidents*, between necessary and contingent entities. This dualism—that again dates back to Plato, but was formalized by Aristotle—through medieval Scholastic philosophy and after Galileo's revolution, reached modernity with Baruch Spinoza's philosophy of nature (1632-1677).

Spinoza's metaphysics as the metaphysics of modernity

The notion of «substance» in Spinoza's philosophy of nature is profoundly different from that in Aristotle's philosophy of nature, which we will deal with in Chapter Five. Here, we should briefly mention the guiding principles of Spinoza's philosophy of nature, given that Spinozism is somehow «the» metaphysics of modernity, underpinning all great metaphysical (and ideological) systems of modernity, from Hegelianism to Marx-Engels' historical materialism.

Spinoza's philosophy of nature: from Galileo's uniqueness of physical sciences to the uniqueness of substance or «nature» in metaphysics

Spinoza's naturalist metaphysics—developed in his main work *Ethica more geometrico demonstrata*, that was published after the philosopher's death, in 1678—, contrary to Aristotle's philosophy of nature, does not speak of multiple «substances», each corresponding to a physical body, be that simple or composite, and each with its set of characteristic «accidents» or «attributes», so that the various substances or material bodies are differentiated into multiple species. Starting from Galileo's idea of the *uniqueness* of physical (mechanical) laws for all bodies, Spinoza speaks of one only *nature* or *substance*, identified with the geometric determinism of its laws, re-discovered by Galileo.

Individual entities are thus reduced to accidents of the unique substance

In relation to this, individual physical bodies are not «substances» endowed with autonomous existence—not even limited—as it was in Aristotle's system. According to Spinoza, physical bodies are *simple modalities*, or *simple manifestations* of the sole substance/nature, according to different *attributes*—of which the best known to men are *extension* and *thought*. Each of these *finite* entities-accidents has a transitional existence, *contingent*, compared to the *necessity*, hence the eternity and immutability of the only material substance, or «nature», from which any entity derives, and to which any entity goes back at the end of its existential parabola, like a wave into the sea.

Actually infinite character of Spinoza's nature/substance, containing in itself the universe of contingent physical entities (pantheist immanentism)

Therefore, we end up with a unique substance that is then *actually infinite*, like the «God» of classical theology: yet with one only, essential difference —which cost Spinoza, who was a Jew, to be banished from the Synagogue, as well as earned him the definition «the first atheist theoretician of modernity». This «god or nature» (*deus sive natura*), the foundation of all deterministic laws of physics and mathematics, is *immanent* in relation to the world, and does not *transcend* it, according to the principles of an essential theological *pantheism* (god is the universe and the universe is god), a «god» with lower-case 'g', given its immanent quality. All modern metaphysical systems during the 20th century (from Hegel's to Marx-Engels', which were dominant in the countries of «real socialism» for seventy years) are simply variations on the theme of the Spinozian approach to the reading of nature and history.

Whitehead: nature as the object independent from the subject

In order to avoid these dualisms (*matter-spirit, substance-accident and/or necessary-contingent*), Whitehead gives an initial definition of the concept of «nature» as of *object, referential end of the perceptive act*. This object, according to Whitehead —contrary to the phenomenism that was dominant in the philosophy of physics at the time— *exists independently* from the fact of being thought of or known by some subject capable of representation.



Nature is that which we observe in perception through the senses. In this sense-perception we are aware of something which is not thought and which is self-contained for thought. This property of being self-contained for thought lies at the base of natural science. It means that nature can be thought of as a closed system whose mutual relations do not require the expression of the fact that they are thought about. (Whitehead 1920, 3).

Illustration of this notion of nature in «Process and Reality»

Whatever this notion of «nature» that obeys the preceding definition and avoids the two dualisms that Whitehead wanted to elude, it cannot be understood clearly merely from the texts contained in *The Concept of Nature*. This conception clearly emerges from the endpoint of the English author's philosophical-natural reflection. This is put in a form that is not always clear, and is often convoluted, due to the novelty of the concept that was being illustrated in the work that has been correctly defined as Whitehead's «philosophical masterpiece», *Process and Reality*, first published in 1929 (Whitehead 1957).

Nature is essentially a «process», intended as ultimate, non-substantial reality

In this complex work, difficult to read, the notion of «nature» is that of a *process*, the *ultimate reality* that is non-substantial, antecedent of any duality matter/spirit or substance/accident (necessary/contingent), on which the multiple and complex articulation of bodies and physical relations, of cognitive experiences and relations depends. This articulation repre-

sents the whole of the natural and human world in its manifestation as object, referential end of men's conscious experience.

Productivity of Whitehead's teachings in highlighting the need for an ontological revision of the foundation of modern science

It is difficult to deny that this metaphysical conception of *nature as process* is much more adequate to the revolutions of post-modern science in the 19th-20th century than Spinoza's notion of *nature as static necessity* —as the immanentist surrogate of the *fatum*-divinity of pagan ancient times—that was instead well adapted to the naïve determinism and reductionism of modern science in the 17th-18th century. We should recognize that Whitehead brought modern scientists to realize that both the conceptual revolutions in the axiomatic method—with the emergence of the hypothetical-deductive method—and the conceptual revolutions in fundamental quantum-relativist physics and the physics of complexity, force us to profoundly re-think the modern concept of *nature*, which can no longer be the static one of Spinoza's vision.

The concept of nature as process is particularly useful for the theoreticians of unstable systems. Its proximity to Aristotle's «primary matter»

Today, in particular, theoreticians of unstable systems find a profound inspiration in Whitehead's work for their reflections on natural philosophy. Even if we should acknowledge with physicists such as Ilya Prigogine (Prigogine & Stengers 1979; Prigogine 1981) and mathematicians such as René Thom (Thom 1990), that precisely the notion of dynamic instability—which makes the initial conditions of a dynamic system be *in potency* to multiple *final states*—requires that the contemporary philosophy of nature engage in a serious and deeper assessment of the ancient Aristotelian physical ontology of *act* and *potency*; i.e., of matter, ontologically understood as a source of indeterminacy, *dynamis* (duhamij), standing in potency to many, unpredictable forms, intended as structured final states or *acts*, *entelechie* (enteleceiai, literally, «irreversible intrinsic ends») of the processes of an ordinary unstable material stratum, in continuous change (See § 5.1.7).

3.2.2.3 The Return of Scholastic natural philosophy

Recovery of the philosophy of nature in the context of the neo-Scholastic philosophy of the 20th century

As for this issue of the ontological implications of complex systems, Whitehead's philosophy of nature as process is not the only candidate to provide a proper theoretical background. Indeed, as stressed by many authors (Arecchi & Arecchi 1990; Basti 1991; 1995; Basti & Perrone 1996; Musso 1996), we can find that it converges with another great branch of the philosophy of nature of the 20th century, the Aristotelian-Thomistic neo-Scholastic philosophy. Therefore, it is not a coincidence that an interesting essay was recently published, underlining the common points between the philosophy of processes *à la* Whitehead and the Thomistic philosophy of 'becoming' (Felt 2001).

The branch of neo-Scholastic philosophy of nature of the 20th century includes, among its contributors, authors that have become part of the contemporary history of philosophy, such as Jacques Maritain (1885-1973) (Maritain 1959; 1977); or great scholars of ancient and modern Scholastic philosophy, but also of modern physics and sciences, such as Peter Hoenen, S.J. (Hoenen 1956), Roberto Masi (Masi 1960), Jean-Marie Aubert (Aubert 1965), Bernard Van Hagens (1983), Filippo Selvaggi s.j. (Selvaggi 1985), Mariano Artigas, José J. Sanguinetti (Artigas & Sanguinetti 1989), William A. Wallace (Wallace 1996), Leo Elders (1996), who restored prestige to the teaching of the philosophy of nature of a neo-Aristotelian and neo-Thomistic inspiration in many secular and religious universities, on both sides of the Atlantic, during the second half of the 20th century.

It is within this current of thought that we would like to see this work inserted. Briefly speaking, we can say that after a pause that lasted over 20 years (from the beginning of the 1970s to the mid-1990s), the philosophy of nature of a neo-Scholastic inspiration has come back to life in the work of many authors. This is essentially due to the input ensuing the results of complexity theory in the physical sciences, from the challenges of genetics in biological sciences, to the new perspectives opened in the cognitive sciences by the informational approach to the study of the mind.

3.3 Conclusion: after ‘scientism’

The need for a rigorous development of the two disciplines: the philosophy of nature and the philosophy of science

In this First Part of this work, the need for a rigorous development of the *philosophy of nature* and the *philosophy of science*, according to their distinct and partially overlapping fields of competence, appears clear. Between the two most traditional philosophies of nature—that is, excluding the «unique case» of Whitehead’s philosophy of nature—that developed within Western modern thought and that we briefly mentioned—the one going back to the Scholastic tradition, particularly to the Aristotelian-Thomistic one, and the other going back to the historicism of Hegel’s philosophy—we will deal in particular with the first; also because the second, due to its anti-naturalistic and anti-mathematical idealist historicism, excluded itself. This is true even if some individual doctrines of Hegel’s philosophy of nature remain highly interesting for their originality and depth.

The unique case of Ludovico Geymonat in the Italian philosophy of science and of nature

In this respect, we should mention at least one author that goes back to Hegel's metaphysics in its materialist version of a marxist inspiration, Ludovico Geymonat (1908-1991). He considers himself a philosopher of science more than a philosopher of nature; however, his passionate defence of the reasons of *scientific realism*, against the phenomenalist interpretation of Positivism and neo-Positivism and, above all, his *methodological rigour* in critically dealing with the most intricate logical and epistemological issues of contemporary science, make him a natural model also for those who, like us, draw inspiration from metaphysical perspectives very different from Geymonat's (Geymonat 1977).

The need for a formalization of the philosophy of nature, in a dialogue between the philosophy of nature, the philosophy of science and mathematical-natural sciences.

On the other hand, precisely due to the intrinsically naturalistic character of Aristotelian-Thomistic metaphysics, the philosophy of nature that took inspiration from it conflicted with the emerging Galilean-Newtonian science of early modernity, as previously mentioned. Today, however, given that the ideological reasons for the juxtaposition between these two approaches —Enlightenment 'scientism' on the one hand, and traditionalist apologetics on the other — have disappeared, only good consequences can come about for late-modernity, from the finally mature dialogue between the philosophy of nature, the philosophy of science and mathematical-natural sciences. However, the price to pay for making this dialogue possible and truly productive is the formalization of metaphysical doctrines in the philosophy of nature and of epistemological doctrines in the philosophy of science —a formalization not intended in the static axiomatic sense, naturally, but in that of the above mentioned «open», post-Gödelian logical systems (See § 3.1.5). Only in this way will these disciplines become really as useful as modern scientific doctrines, beyond every author's «membership» in different schools of thought and convictions of faith, whether secular or religious.

The limits of the modern programs of formalization of philosophy

This formalization can be reached through a twofold effort: the first essentially relating to the *logic of philosophical language*, the second relating, also and above all, to the *logic of contemporary scientific language*. That is:

- ◆ On the one hand, this means restoring demonstrative rigour to philosophical inquiry, through the renewed application of adequate logical methods to metaphysical and epistemological inquiry. This application, while keeping into account the different scientific and cultural contexts, should be akin to the degree of pre-modern formalization that metaphysical reflection had at least reached with the great medieval Scholastic tradition. This is a dream that all modern philosophical reflection pursued in vain, from Leibniz until today, and that often, due to the limits of an exclusively *extensional** logical formalization of mathematical origin, turned into anti-metaphysical reduction-

ist research programs. This is what happened, in the 18th century, with D'Alembert's *Encyclopédie*, or with the 19th century Positivist program *à la Comte*, or with the more recent attempt of the logical neo-Positivism of the Vienna circle and its contemporary followers. In this sense of a recovery of the demonstrative rigour of philosophy, the formalization of contemporary philosophical language moves towards the adequate development of a formalism—in fact, of more than one logical formalism that are adequate to the complex and multifaceted object of philosophy—that cannot only be that of the *extensional logic* of mathematics. To the contrary, it must also relate to the multiple formalisms of *intensional* logic* in its various articulations in the systems of *modal*, *deontic*, *epistemic*, etc. logic (Zalta 1988; Galvan 1990)⁷⁵.

- ◆ On the other hand, however, the very nature of metaphysical inquiry—as the research on the foundations—demands that its formalization include the development and the application of a modern *analytical* research *method* of those axiomatic propositions, within the different systems, that are adequate to («true for») a specific object of study, in a limited field of inquiry, that work as «principles» for the ensuing demonstrative procedures.

The scientific death
of scientific
reductionism



In sum, this means to also bring the notion of «open system» to bear on metaphysical reflection. At the same time, the present debate on the foundations of logic and mathematics—and also on the drive towards a more rigorous formalization coming from new fields of research such as the science of non-linear systems, chemistry, biology, cybernetics, cognitive sciences, computer sciences, economic and social sciences—demand an expansion of the formalisms of Galilean-Newtonian science.

In particular, from such a dialogue, the philosophy of nature and, more generally, metaphysics, can receive the great benefit of being drawn out of the limbo of cultural oblivion to which they are currently confined. On their part, the philosophy of science and sciences themselves, in the present era of profound revision of the foundations, can only gain from

⁷⁵Ontological reflection is one of the main fields of this philosophical inquiry that uses the tools of *analytic philosophy*. From this derived a particular discipline, *formal ontology* (that is of increasing interest for computer sciences as well), due to the numerous applications it allows of the logical method to the analysis of disciplines that, so far, only used ordinary language (one should think not only of philosophy, but also of law, ethics or medicine, etc.): a language vulnerable (with its ambiguities) to a free game of interpretations. And this is a game that, when used in disciplines such as the already mentioned ones, on which people's life, well-being, dignity, safety or even freedom (take law, for example) depend, is not sustainable within a global culture. For complete and up-to-date information on the developments of formal ontology, see www.formalontology.it.

this renewed and formalized dialogue, devoid of ideological preconceptions, with the great tradition of the philosophy of nature and of metaphysics.

3.4 Summary of Chapter Three



In these pages we have dealt with the last chapter written so far in the revolution of the foundations of logic and mathematics. This chapter was destined to radically change our way of conceptualizing science and knowledge, by declaring the «scientific death of scientism» (§ 3.1). Indeed, the profound meaning of these revolutions (See § 3.1.1) — far from denying the existence of ‘truth’ and of the possibility to (partially) attain it through human knowledge and language— highlights, with the rigour of a demonstration, the impossibility that any formal language of a demonstrative, scientific, philosophical or other kind, can claim to have exclusive right on truth, or worse, ridiculously pretend to be non-perfectible. Truly, «research never ends».

This result, linked to the demonstration of Gödel’s *incompleteness theorems* —that goes beyond the limits of arithmetic, for which it was originally designed— declared the failure of David Hilbert’s *formalist* approach to the theory of the foundations of logic and mathematics (§ 3.1.2). This approach, against Frege’s confusions, started from the important re-discovery of the distinction, already present in Scholastic logic, between *language* and *meta-language*, *theory* and *meta-theory*. This re-introduces, in modern logic, the other, classical distinction between *logica minor* and *logica maior* and, within *logica maior*, between predicates whose arguments are names taken in *suppositio formalis* (= that refer to extra-linguistic objects, for example, the arguments of «being red» are names of things such as «blood», «fire», «sun», etc.) and predicates whose arguments are names taken in *suppositio materialis* (= that refer to intra-linguistic objects, to other parts of language).

Hilbert’s formalist program, then, centred on the hope of discovering a method for absolute demonstrations of the non-contradictory character of mathematical theories. This method would not have been based on evidence, but on finite (algorithmic) methods of decision, that is, of recursive enumeration of all the theorems deducible from a finite set of axioms. In other words, Hilbert’s formalist program hoped to be able to use arithmetic as the meta-language of all formalized mathematical theories, arithmetic included, so that one would obtain the demonstration of the *completeness* of arithmetic from within formal arithmetic itself.

Gödel's two incompleteness theorems (§ 3.1.3) signalled the failure of this program. The first demonstrated that, by using finite decision methods, some true arithmetic statements, even very elementary, become undecidable in formalized arithmetic as axiomized by Peano. The second theorem demonstrated that the meta-linguistic statement affirming the consistency of arithmetic from within arithmetic itself is one of these undecidable statements. Gödel's conclusions, which demonstrated the incompleteness of formalized arithmetic, were then extended (§ 3.1.3), by Church and Turing to *all* formalized languages of science (§ 3.1.4.1), and by Tarski to the problem of a formal analysis not only of the syntax, but also of the *semantics* of formal systems, in particular of their *truth* (§ 3.1.4.2).

The *logical and epistemological value* of Gödel's, Church's, Turing's and Tarski's theorems of limitation was to force future logical and mathematical systems to be *logically open systems*, instead of becoming generic and useless due to their taking refuge—in order to maintain their «closeness», and the immutability of their axioms—in the use of infinitistic demonstration methods. In this sense, once the ancient myth of the absolute certainty of deduction *versus* the probabilistic nature of induction was abandoned, the «openness» of these systems consists in their capacity to integrate *logical invention/discovery methods (analytical methods)* for the construction of axioms adequate to their objects and always changeable, as well as *formal testing methods (axiomatic methods)* for the construction of theories that are logically substantial, even if in relative terms, thus overcoming the neo-Positivist prejudice of *arbitrariness* in the definition of axioms (§ 3.1.5).

These profound changes on the foundations of modern science brought forward, in the 20th century, a new philosophical discipline, the *philosophy of science*, as well as the re-birth of the *philosophy of nature* (§ 3.2). At the beginning, the philosophy of science often had a secondary role, related to the lack of an adequate philosophy of nature to try and answer new and urgent ontological and, above all, ethical questions, posed by the development of sciences to contemporary society and culture (§ 3.2.1). At the same time that the philosophy of science was emerging, a new form of *philosophy of nature* re-emerged, even if in a more subtle and less wide-spread way, thanks to the work of great philosophical and scientific scholars, rather than to a particular school of thought. First of all, this new philosophy of nature set to counter the exclusively phenomenalist approach to the role and cognitive function of modern science that E. Mach had ascribed to it at the end of the 19th century (§ 3.2.2).

Pierre Duhem is the first author to whom we owe the re-birth of an ontological and metaphysical reflection linked to modern science. His approach, however, still gave modern science an exclusively phenomenalist and pragmatic role, even if Duhem recognized the need for an integration between science and a metaphysics that, according to him and contrary to the Scholastics, had a religious if not expressly theological basis, instead of being grounded on natural sciences (§ 3.2.2.1).

Against Mach's and Duhem's phenomenalist approach, during the 1930s, the teaching of A. N. Whitehead took shape. Among modern thinkers, he was the first, after Galileo, to attribute an *ontological*, and not only phenomenological value to science. In particular, Whitehead recognized the need, imposed by the crisis of the foundations of mathematics and of then-emerging quantum mechanics, of a conception of nature that was no longer static, as it had been in modernity, particularly in Spinoza's metaphysics, but rather linked to the notion of *process* (§ 3.2.2.2).

Such an approach has some clear common points with the philosophy of nature of Aristotelian inspiration, which considered natural entities as entities able to *change*. In any case, the only true example of a «school of thought» in the field of the philosophy of nature in the 20th century is offered by the neo-Scholastic philosophy of nature. This has included great names of the philosophy of the 20th century—J. Maritain among them—, at least until the early 1970s. After a pause that lasted almost twenty years, starting from the mid-1980s, the philosophy of nature of neo-Scholastic inspiration is now being revived in the work of many authors on both sides of the Atlantic. This is essentially due to the drive given by the results of complexity theory in physical sciences, by the challenges of genetics in the biological sciences and by the new perspectives offered by cognitive sciences and by the computational approach in the study of the mind (§ 3.2.2.3).

To conclude this chapter (§3.3), we can say that the end of scientism requires that an adequate *philosophy of nature* be developed, in addition to a philosophy of science that is more and more adequate to its task of critical inquiry on the logical and epistemological foundations of modern science—a true *meta-science*, as some have defined it. This philosophy of nature—through an increasingly rigorous formalization of its concepts and methods, so that they integrate the *analytical methods* of invention and discovery with the *axiomatic methods* of demonstration—should be able to relate without difficulty to the formalisms of modern science. In this way, it will offer an irreplaceable contribution to present and future metaphysics, by transforming it into a system of

knowledge and language that is, in turn, really «open» to the inexhaustibility of being.

3.5 Bibliography of Chapter Three



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PART TWO

**The philosophy of nature and the
philosophy of science: a theo-
retical framework of the issue**

Capitolo Quarto

4. Philosophy of Nature and Philosophy of Science

From the distinction between philosophy of nature and philosophy of science, to the demonstrative hypothetical-deductive method of modern science. Its empiricist origins within the Neo-positivist movement, Popperian falsificationism and post-Popperian fallibilism, the ontological limits of hypothetical-deductive thought and the realist foundation of hypothesis through the integration of the analytical and hypothetical-deductive methods in the applied uses of science.

4.1 Two definitions

In the three previous chapters, we have roughly introduced the theoretical and historical framework for this work. We can now directly deal with the ‘missing link’ among all these introductory considerations, i.e., a general definition of *philosophy of nature*, *philosophy of science* and of some closely related ideas.



Definition 1

By the term ‘philosophy of nature’ we mean that part of metaphysics, which has as its object the ‘being’ (i.e., essence or nature and existence) of the entities within the physical world⁷⁷, which are the same entities that are studied - in their phenomenical aspects - by natural sciences.

⁷⁷ For the sake of information, let me recall here that, not long ago, this philosophical subject area was defined in terms of *cosmology*. Such a use of the term *cosmology* would be now highly ambiguous since, with the development of relativistic mechanics, a new branch of modern mathematical physics was born: it studies the origin and evolution of the cosmos and is, consistently, defined as cosmology.

Definitions of
'philosophy of
nature' and
'philosophy of
science'

A definition of the idea of the 'nature' of an entity will be provided later on in the work (Cf. § 6.2.1): it is indeed essential to make the definition of 'philosophy of nature' we have just suggested fully intelligible. We will deal in depth, as a matter of fact, with properly defined metaphysical issues from the next chapter onwards.



We can now proceed, instead, to suggest a temporary definition of 'philosophy of science'. Here the provisional character of such a definition depends on the fact that, as we have anticipated above, there is no agreement among philosophers and scientists about the meaning to give to such an idea, or to the subject area which is defined through this label. The definition that will be provided here draws on some aspects that can be seen as common denominators shared by several attempts of characterisation. But it is, above all, the definition that better conveys, in perspective, a certain formal rigour to this subject area; a rigour that has, instead, been seen as traditionally missing, thus contributing to back the charges of vagueness and lack of precision that have been recently addressed to the philosophy of science from different parts.



Definition 2

By 'philosophy of science' we mean the study of the epistemological and logical foundations of mathematical, natural and human modern sciences, together with their technological applications.

A definition of other related concepts, such as 'epistemology' and 'natural science', will be provided later on. However, it seems to be evident that a definition of 'philosophy of science' of this sort is not yet rigorous enough. In his handbook, A. Strumia provides the following definition of philosophy of science:

We will use the term *philosophy of science* or, more precisely, *philosophy of sciences* to point at a philosophical theory of science, i.e., a *systematic conception* of what science is, so as to point out *its foundations*, its *object*, its *methods* and *aims* (Strumia 1992, 10).

If we take into account other – relatively recent – handbooks, we will be faced with even more generic definitions, though consistent with ours. For instance, the recent book by L. Elders – which provides an introduction to the classic Thomistic interpretation of the philosophy of nature of St Thomas Aquinas (Elders 1996) – makes reference to R. Harré's classic handbook to suggest an explanation of the modern idea of philosophy of science (Harré 1972, 23ss.). Elders defines it thus:



[It is] the study of the issues raised by the new sciences of nature, i.e., a) the possibility of coming to know reality; b) the relation between the knowledge obtained through some instruments and the one provided by direct perception; c) the hypotheses that are called into question, such as the possibility of arriving to a knowledge of the universe and its order, the general application of theories, the causality principle ...; d) the meaning of the primitive concepts used in different subject areas (e.g. atom, electron, nucleus, field, flow, radiation force, gene, genetic code...); e) the scope of some significant theories, as in particular the corpuscular interpretation of reality (Elders 1996, 17).

Some examples of the ideological use made of philosophy of science

Another point made by Elders seems to be worth noticing. Immediately after having put forward this definition, he offers two examples that are meant to show the systematic misunderstanding one incurs when she comes to draw the distinction between philosophy of science and philosophy of nature, and that leads to the mistaken identification of philosophy of science with the interpretation of the outcomes of a given science, on the basis of the author's personal views. Such a misunderstanding derives from the shortfalls of philosophy of nature when compared to natural sciences, as we have already noticed earlier. The very fact that it seems to be up to some scientists – even some talented scientists – to try to suggest an answer – by means of their limited philosophical resources – to such a shortfall seems to bring to light the need of an adequate development of a *metaphysics* of natural being, which is worthy of its name. The uneasiness these scientists experience in dealing with such issues, after having devoted all their lives to science, and, at the same time, the unavoidability of those very issues are symptomatic of the uneasiness of a whole society, i.e., our society.

Denial of the principle of causality

As an example of such misunderstandings highlighted by Elders – and to which we will get back in due course – we can make reference, in the first place, to the denial of the validity, in physical nature, of the principle of causality supported by W. Heisenberg. This claim was based on a subjectivist philosophical interpretation of the mathematical relation of indetermination (uncertainty), which is inevitably encountered in the

study of microphysical (i.e., atomic and subatomic) nature⁷⁸. More precisely, such a relation can be found between the objects studied by quantum mechanics and in the operations of measurement on them. This is a relation that was made clear also by the decisive contribution of Heisenberg, who reinterpreted on its basis the fundamental concepts of quantum mechanics itself (Heisenberg 1955). We will return in due course to this idea from a more specific philosophical perspective.

The absurd character of the cosmos and its causality according to Weinberg

Another noteworthy example of confusion between the scientists' personal beliefs in the philosophical interpretation of scientific theories on the one hand, and philosophy of science, as the study of the foundations of the scientific notions and subject areas on the other, can be found in another fundamental text. This is a book written by the Nobel Prize winner for Physics (1979) S. Weinberg, and it is considered to be, from a strictly scientific point of view, one of the most valuable contributions to the divulgative literature on cosmology (Weinberg 1977). Despite the extreme complexity of his writing, when, in the conclusion to the book, S. Weinberg offers a few closing considerations on the philosophical aspects of his investigation, he maintains that the only reliable conclusion that can be derived from the study of cosmology is the absurdity and the absence of meaning for human beings and for their existence in the immensity of the cosmos. In particular, what appears to be meaningless for the existence of humans is the 'exceeding' number of celestial bodies in the universe. Or, more significantly, the number of celestial bodies in the different universes that are thought to exist simultaneously, according to a model that represents an alternative to the one characterised by the 'entropic principle', which we are about to address within the boundaries of the progressive construction of the cosmological quantum-relativist theory.

In Weinberg's words:

Despite the way we may decide to solve all these problems, and despite which cosmological model may be the correct one, the solution we may reach will not be able to give us any consolation. Human beings have an almost uncontainable need to believe that we are in a somewhat special relation with the universe, that human life is not the mere – more or less peculiar – result of a chain

⁷⁸ As Heisenberg wrote at the end of his seminal article in 1931, 'Since all the experiments are subjected to the laws of quantum mechanics and, consequently, to relations of indetermination, the invalidation of the causal law is conclusively confirmed by the quantum mechanics itself' (Heisenberg 1931, 182). As we will make clear later on in this work, the discovery of the relation of indetermination may cause rather serious problems to the Kantian foundation of the principle of causality. It would, instead, be different with another kind of foundation, e.g. a non-representational one.

of accidental events which goes back to the previous three minutes, i.e., that our existence was already somehow predefined since the beginning. While I am writing these words, I am on a plane flying at 9,000 meters from the ground in the Wyoming sky, going from San Francisco to Boston. Below me the earth seems to be pleasant and comfortable: I can see, here and there, fluffy clouds, that the descending sun colours in pink; the countryside is crossed by straight paths that link one town to the other one. It is very hard to realise that all this is only a tiny little part of an extremely hostile universe. It is even harder to realise that the current universe developed from unbelievably diverse conditions and that it is doomed to an extinction characterised by an inestimable freezing, or by an unbearable heat. The more the universe seems to be comprehensible to us, the more it appears to be aimless (Weinberg 1977, 170).

Frankly, the logic of such reasoning is rather hard to grasp. The fact that men have 'an almost uncontainable need' to believe they are in 'a somewhat special relation with the universe' is fully understandable if we think that men are part of the same universe and – at least, up to now – they are the only part of it that is capable of giving the universe a certain consciousness of its self and its origins, to use a neo-Hegelian metaphor. To look more carefully, it is Weinberg himself who recognizes this in what follows:

But if there is no consolation in the results of our research, there is at least some consolation in the research itself. Men and women are not satisfied with myths of gods and giants, nor are they content with collapsing their thoughts on everyday life's affairs. They even build telescopes, satellites, accelerators; they sit at their desks for unending hours in the attempt to understand the data they have gathered. The effort to understand the universe is one of the very few things that lifts up the human life over the level of a farce, conferring it some of the dignity of a tragedy (Weinberg 1977, 170).

Barrow and Tipler's
entropic principle
and finalism in
cosmology

As to the claim that our way of being, our human life would only be the more or less peculiar 'result of a chain of accidental events', it seems to be absolutely arguable from different perspectives. There is, for instance, another utterly different stream of interpretation of the same cosmological data considered by Weinberg, which arrives at a conclusion that is just like the opposite of the one he suggests. In cosmology, the supporters of the so-called *entropic principle* argue, as a matter of fact, that without that chain of accidental, highly unlikely, events that are supposed to have occurred in the very first moments of life of the universe, the evolution of matter would not have led us to the appearance of life in general, and human life in particular; the history of the

universe would have, instead, followed a completely different course (Barrow & Tipler 1968).

In other words, it is like if, at any crossing point of its history, the developing universe decided to always and exclusively pick, among all the possible alternative ways to follow (every unpredictability logically implies indeed a multiplicity of alternatives), that route which alone could have taken it towards human beings. Human existence is hence what *a parte post* (i.e., from the end of the process) can cast intelligibility on the evolution of the cosmos. This is, in a nutshell, the core of the 'entropic principle'.

The cosmological theories that draw on the 'entropic principle' are numerous and some of them are not very convincing from a logical viewpoint. As a matter of fact, if we want to take the principle as an explanation of cosmic evolution (i.e., if we are to take the entropic principle in its stronger sense), it risks exposing the theories that endorse it to logical inconsistencies, which can be seen as equivalent to those deriving from the Neo-platonic model of finalism, as opposed to the Aristotelian and/or Thomistic one. (See *infra* § 6.3.2.3). On the contrary, if the 'entropic principle' is not considered as an explanatory theory, but rather as evidence to be explained, then it no longer poses any problems in principle, as any physicist could certainly accept (Hawking 1988, 146)⁷⁹.

The deterministic character of the cosmos according to Hawking

But this is not all. There is also a third philosophical interpretation of the same cosmological data, which is opposed to these two interpretations, even more than they are opposed to one another. This interpretation, supported by the renowned physicist and mathematician S. W. Hawking, denies the common premise from which both of the two previous theories instead move: the idea of the pure accidentality, of the pure casuality of the origin and development of the universe (or, of the different universes), from the 'singularity' of the *big – bang(s)* – from which it/they would originate – up to the 'gravitational collapse' to which it/they seem to be doomed.

Hawking rightly points out that, since none of the available cosmological models offers a satisfactory justification of the reason why the initial configuration of the universe did *not* produce anything different from what we currently have in the same universe,

⁷⁹ J.J. Sanguineti (Catsagnino & Sanguineti 2000, 158) arrives at the very same conclusion. In the scientific part of the book edited by M. Castagnino, one can also find a valuable presentation of the fundamental scientific notions that lie beneath these different interpretations of time and its evolution, within the domain of the theories of quantum gravitation (*Ibid.*, pp. 361ff).



Are we to look for an explanation in the entropic principle? Everything was then produced by a fortunate accident? This seems to be a highly unsatisfactory conclusion, a denial of all our hopes to understand the universe's underlying order (Hawking 1988, 157).

This seems also to be a denial of the weak hopes of a powerful supporter of the theory of causality as Weinberg is, i.e., the hopes of making some sense, thanks to physical research, of what he has defined as the 'farce' of human life on Earth. At the basis of the unpredictability of the single events, there would be, according to Hawking, the *rigorous determinism* of the geometric structure, reproduced on the space-time larger scale. This would be consistent with the theory of relativity and with the following hypothesis on the 'complex'⁸⁰ nature of time, when we deal with the integration of quantum phenomena within the framework of the relativistic theory of gravitation (Hawking 1988, 158ff. See *infra* § 6.3.3.1).

'God does not play dice'

All this is, none the less, consistent with the strictest possible interpretation of the Einsteinian deterministic motto: 'God does not play dice' (Hawking 1993). If compared to both of the two previous opinions, Hawking's hypothesis has, accordingly, the merit of being constructive,

⁸⁰ The idea of the 'complex' nature of time suggests that it cannot be defined only by means of real numbers, as physicists are used to do. If we take, instead, the quantum theory in the interpretation given by Feynman, we will notice its 'strange' feature, that we will address later on, according to which time needs to be defined on the basis of imaginary numbers. Imaginary numbers are those numbers that if multiplied by themselves always give a negative result, which is different from what happens with ordinary numbers, or 'real' numbers, that despite their being positive or negative, when multiplied by themselves always give a positive result. Thus, on the real axis, $(-2) * (-2) = 4$. Conversely, on the imaginary axis, the product of two negative imaginary numbers will always be negative. The imaginary unit i when multiplied by itself gives -1 , hence $2i * 2i = -4$. In this way, if imaginary numbers are used to define time, the *inversion of the temporal arrow* on a cosmic scale becomes possible (= temporal reversibility). This idea can be extended to the 'curved' space-time of the theory of general relativity to describe, within a formalism *à la* Feynman, the 'path integrals' of infinite possible universes – whilst *within* each universe time continues to be defined on the basis of real numbers, as always rising, consistently with the principles of thermodynamics. Along these lines, this kind of cosmological meta-universal time acquires a reversible structure, just like space, thus removing one of the main theoretical impediments to the unification of quantum and relativist mechanics. From a metaphysical perspective, this would mean to propose the possibility — we are indeed only dealing with a mere physical-mathematical hypothesis — of a view on the cosmos as eternal (co-eternal with God, in natural theology), thus providing a modern example of that principle of Thomistic metaphysics, according to which the issue of the eternity of the world is not something that can be settled in metaphysical terms, even if there is always the need of an external *foundation* of its existence — which would be, in such a hypothesis, co-eternal with God. We will get back in Chapter 6 to the tricky question about the relations between physics, metaphysics and theology. See also (Basti 2000).

even if it derives from the ideological conviction of a geometric determinism of nature that we are tempted to define as old-style-Newtonian⁸¹. Or better, Hawking's hypothesis has the merit of being trying to suggest, from an internal scientific perspective, a contribution to the development of the theory of the so-called 'quantum gravitation'. This is indeed the common goal to which different efforts of this kind are striving, so as to construct a 'theory of big unification' between relativistic and quantum mechanics. We will get back to this later.

Weinberg's *casuality*, Barrow's *finalism* or Hawking's *determinism*? We have suggested here only one example of this limited use of philosophy of science, when it tries to take the place, without an appropriate critical work of foundation, of the discipline that is really missing in our time, i.e., philosophy of nature. Other, even more worrying examples could be suggested from different fields of modern sciences as, for instance, from biology, or also neurosciences, to begin with. We will hint at them in the course of this work. Is philosophy of science reduced, then, to the discipline of the free 'game of interpretations' of data and of, more or less founded, scientific hypothesis, according to an expression borrowed from P. Ricoeur's hermeneutics? This would be a very small and miserable condition, despite the fact that all philosophical opinions are worthy of respect, especially those of well-established scientists. But these opinions demand our respect as long as they stay what they are, i.e., *opinions*. In other words, they are respectable only as long as they do not claim the status of a 'scientific doctrine', maybe by virtue of an undue, and not always unconscious, extrapolation of the prestige acquired by these big names in science – thanks to the demonstrative rigour they have shown as mastering in their disciples – and of its extension to philosophical, or even metaphysical domains.

Necessary
distinction between
philosophy of
science and
philosophy of nature

Going back to our point of departure, only if we accept the definition of 'philosophy of science' provided above (See Definition 2) as a theory on the foundations of natural sciences, and not of the being of the entities studied by it – in other words, only if we accept the distinction between philosophy of science and philosophy of nature (See Definition 1) — can we prevent philosophy of science from turning into a mere gallery for the exhibition of philosophical and/or personal and ideological (respectable) opinions. At the same time, it will become much easier to emphasize the relation between philosophy of nature and philosophy of science, which is the very original goal of this work. This will become

⁸¹ It is not a mere coincidence, then, if Hawking has now the same prestigious academic position that Newton used to hold at the University of Cambridge.

clear as soon as we will have also suggested a definition of ‘metaphysics’, since philosophy of nature is a part of it.

4.2 The scientific method

4.2.1 Modern science and demonstrative method

The language problem in the understanding of classical thought

Metaphysics is defined by Aristotelian-Thomistic philosophy as ‘a science of being *qua* being’, or as ‘first philosophy’, so as to underline its character of a ‘science of the first causes’. All these expressions do not have much to say to the modern theorist, but they can become relevant to him/her once all the terms involved in them are defined. Once we will have done that, we will certainly be free of either accepting or changing such a terminology to make it more understandable and, above all, not ambiguous for the modern theorist.

The first step to take to clear up the high number of ambiguities that have recently grown around these terms is to try to understand what the term ‘science’ means, according to an Aristotelian-Thomistic understanding of it. We will be then able to see that this meaning is a very extensive one, and that it also includes the modern understanding of such a term, which, in turn, identifies what science is only within an (either natural or human) *empirical* and/or *logical-mathematical science*. Moreover, through the definition of the strictly related notion of *epistemology*, this part of the work will provide the basis to understand in what sense and why epistemology has tried, in the modern era, to stand in for metaphysics and meta-logics in the foundation of sciences. This will, subsequently, help us to understand why philosophy of science has tried to stand in for philosophy of nature. But let me start from the beginning, i.e., from the notion of *epistemology*.



Definition 3

By epistemology we mean that logical and philosophical subject area that studies the methods of ‘scientific knowledge’ (from the Greek *epistēmē*, *epistēme*) and in particular the basis of its being *scientific* and *certain*. Epistemology is not the same as *gnoseology*, or ‘philosophy of knowledge’, which, instead, studies knowledge in general

terms, in its variations and in its psychological bases within the human mind. It is also different from *meta-logics* and *meta-mathematics* that study the foundations of logic and mathematics, i.e., of such semantic notions as the *consistency* or *truth* of formal languages.

We will get back to the issue of the relation between epistemology and the modern understanding of science after having defined the notion of *science* in a broader sense, which is capable, that is to say, also to include a reflection on the foundations, i.e., a metaphysical and meta-logical reflection.

The idea of
'science' according
to classical thought

To get closer to such a broader definition of 'science', it seems interesting to take Strumia's suggestion (Strumia 1992, 16) and begin by quoting from J. Maritain's famous work, *Distinguish to Unify: The Degrees of Knowledge*. Among the Neo-Scholastic thinkers of this century, it was this talented philosopher who tried to make the Moderns understand the classical characterisation of the different forms of knowledge from a non-religious perspective. At the same time, this quote can also show us the limit of these eager Neo-Scholastic interpretations of the relation between metaphysics and science. In Maritain's words :



What kind of idea can we have of science in general terms, according to the form-limit that the spirit is aiming at, when it is conscious of its efforts towards what men qualify as knowledge? The idea that Aristotle and the Ancients had was very different from the one the Moderns endorse, since for these latter it is the eminent dignity of experimental sciences (...) that includes the notion of science. According to the Ancients, instead, it was the eminent dignity of metaphysics that used to characterise such a notion. It is necessary, then, to be careful in avoiding to apply, without precautions, the Aristotelian-Thomistic notion of science to all that huge noetical material that our contemporaries are used to call by the name of science; this would indeed cause very serious mistakes. None the less, both for the Ancients and the Moderns, the most clear, polished kind of science, which is also perfectly at our reach, is represented by mathematics. And we can also think that *in its being satisfactorily explored in depth and purged, let alone in its being correct and adapted, the intellectualistic critical, or realistic critical theory of science – whose principles were established by ancient and medieval metaphysics – is the only theory that can suggest us a way to understand epistemological problems, that have become now very confused and confusing*. How can we then define science in general terms and in accordance with its ideal type? We would say that science repre-

sents the knowledge of a perfect world or, more precisely, a knowledge in which – under the constraints of *evidence* – the spirit assigns the reason of being to the being of things, since the spirit cannot be satisfied until when it touches not merely any thing or fact, but also what grounds that datum, both in its being and its intelligibility. The ancients used to say: *Cognitio certa per causas*, i.e., knowledge *by demonstration* (or, in other words, *on average evident* knowledge) and *explanatory* knowledge (Maritain 1959, 44 ff., emphasis ours).

The classical notion of 'science' and its character that includes the modern notion of it

As we mentioned above, this quote is rather significant, not only because it represents a fundamental stage in the recent history of the relation between philosophy of nature and philosophy of science, but, above all, because it unconsciously underlines some of the limits of a certain Neo-Scholastic philosophy of this last century. We will analytically address this essential point later, in order to understand in what sense we can correctly set, in our time, the relation between metaphysics and meta-logics, on the one hand, and science and philosophy of science, on the other. But before doing so, let me focus on the definition of *science* that Maritain suggests here; a definition which is taken from scholastic philosophy, but that can also be accepted by a modern, since it includes both the notion of 'natural science' and of 'mathematical science', and also that of 'physical-mathematical' science⁸². There are, indeed, two main characters of scientific knowledge and language that emerge from such a definition, in comparison with other forms of knowledge, and these are the *demonstrative* character and the *explanatory* character.



Definition 4

By 'scientific theory' we mean a doctrine, or a system of definitions, demonstrations and methods of investigation, which does not limit itself to the description of its objects of study, but that tries to explain them, according to a *demonstrative procedure*, which is typical of its method of analysis,

⁸² As it is known, according to Aquinas, who was educated at Albert Magnus' school in Köln, among the Dominicans (like Roberto Grossatesta and Roger Bacon), Franciscans and their contemporaries at Oxford, there were also those involved in the physical-mathematical sciences. These were defined by Aquinas as intermediate sciences (See Thomas Aquinas, *In Post. An.*, I, xli, 358; *In de Trin.*).

so as to give to its explanations a *universal* and *necessary* value.

Here ‘universality and necessity’ suggest that anyone who will study these objects, following the same method of investigation and moving from the same *postulates* or *axioms*, will always and everywhere arrive at the same conclusions. The universality and necessity of the explanations given by a scientific discipline and its demonstrative procedure is a completely different question from the one on the hypothetical nature of its axioms and of its rules of inference and, consequently, from the foundation of the *truth* and *consistency* of the scientific theory that, as we know, can never be expected to be *absolute*, yet always *valid within well defined boundaries*.

To sum up, we can say that a *discipline* (= set of doctrines with a well defined method) can be classified as ‘scientific’:

1. Demonstrative character of every scientific subject

- ◆ As long as it does not merely *describe* its objects of investigation (entities or events), but tries to *explain* them. Namely, it looks for:
 - a.** in the case of *natural sciences*, the *causes* and/or the physical laws, as far as *natural* objects (i.e., physical entities/events) are concerned, or
 - b.** in the case of *logical-mathematical sciences*, the *laws* and *logical reasons* (= *conditions of think-ability/ realizability*), as far as *logical* and/or *mathematical* (languages, theories and so on) and/or *artificial* (i.e., machines, artefacts and so on) objects are concerned, or
 - c.** in the case of *metaphysical* and *meta-logical* sciences, the ultimate *metaphysical* and *meta-logical* foundations shared by all the previous cases, or, in other words, both by the natural entities and by the logical and artificial ones.

2. Explanatory character of every scientific discipline

- ◆ As long as it reaches and defines its explanations according to a rigorous *demonstrative* procedure. In this way, scientific explanations always have a *universal* and *necessary* validity – even if it *never* is an *absolute* one – otherwise we would fall into the mythology of rationalism in philosophy and of scientism in natural sciences. Even better, the *perfectibility* of a specific knowledge is an indicator of rationality and scientificity. The *full* and *consistent* scientific knowledge of the world is the anthropocentric myth of the Enlightenment, which is at the basis of all theoretical errors of modernity (see *incompleteness**).

Pre-scientific character of every descriptive and/or phenomenological discipline

Those disciplines, instead, that limit themselves to describe their objects of investigation without explaining them are referred to as *descriptive* and/or *phenomenological* disciplines. Usually, the careful description of phenomena that are to be explained is the first essential step towards their actual explanation; in other words, it is essential to shift from a descriptive subject area (e.g., geography) to a scientific one (e.g., geology).

Universality, necessity and incompleteness of the scientific language

By all this we mean to suggest that any statement that is demonstrated in a consistent (= non-contradictory) and unmistakable (= without ambiguities in the definitions and procedures) way gains an *eternal* (universal-necessary) validity, even if we will never be able to say that it is a *conclusive, comprehensive*, non-perfectible and, consequently, 'absolute' assertion of that field of investigation. For instance, the Pythagorean theorem is universal and necessary since it can strictly be demonstrated within the postulates of Euclidean geometry. Those who accept those postulates, that is to say, regardless where they are, cannot but recognize the formal validity of that theorem. And this is true also for any other theorem that can be demonstrated within Euclidean geometry. At the same time, despite what one could have imagined before the creation (in the 1800's) of non-Euclidean geometry, Euclid's theorems of geometry do not represent geometric science as a whole. The incompleteness of scientific knowledge is due to a twofold reason:

Necessity of a foundation for the axioms

- ◆ The fact that the *postulates* or *axioms* (= the initial propositions and definitions of a demonstrative procedure), on which the consistency of the demonstrative procedure depends, have to be in turn *founded*, since we cannot rely anymore on the myth of *self-evidence* (See § 1.5).

Incompleteness of formal languages

- ◆ The fact of the logical *incompleteness** of every formal language. According to this, every formal language *necessarily* contains propositions that are compatible with its initial axioms, yet *undecidable* within the system (§ 3.1.3)

We will get back shortly to the problem of the *foundation of the axioms*. At the moment, it will be enough to anticipate that:

The problem of the foundation of the axioms

- ◆ The different axioms at the basis of the theories belonging to the various *natural and human sciences* (= physical, biological, psychological, sociological... laws) needs to be founded:
 - *By mathematical sciences*, as long as natural and human sciences make use, as in the case of modern sciences, of a formal language of a mathematical kind to construct their demonstration. Among the axioms in the natural and human sciences there will

thus be a series of mathematical propositions, whose validity is assumed, having been demonstrated elsewhere.



- *By the reference to the empirical object of investigation* of a given science. Through such empirical axioms, the ‘generic’ mathematical *formal system** (e.g., a certain class of functions or equations), which is used in a given demonstrative procedure of a natural science, is ‘interpreted’ as a *model** that is applicable to that specific research. Thus, for instance, the Maxwellian theory of electromagnetism is nothing else but a particular model of a certain class of equations that is applied to the representation of the movement of electric charges in a field of forces. The same kind of equations can also be applied to the representation of the movement of the particles in a fluid, in accordance with another model, or ‘interpretation’ of the same formal system. In fact, from a historical point of view, Maxwell actually borrowed his electro-magnetic theory (model) from such a hydrodynamic model and, only at a second stage, the formal system from which the both of them derive was defined.
- ◆ The axioms of *mathematical sciences* need to be in turn founded within a foundational reflection of a *meta-mathematical* kind. In other words, they need to be founded on theories that belong to the so-called *meta-mathematical* and *meta-logical* subject areas and that properly mirror a post-modern understanding of metaphysical and ontological sciences.
- ◆ The axioms of *metaphysical and ontological sciences* also need to be founded. These are, first of all, the *first principles* of every demonstrative procedure, i.e., the meta-logical principles and their metaphysical *interpretations**, e.g. principle of non-contradiction (p. o. n. c.), identity principle (i. p.), principle of excluded third (p. o. e. t.), causality principle (c. p.), etc. in their application to that specific object of investigation that is *being*, in all its different meaning. These axioms need to be founded by their reference to the *being* of the entities that are being studied and some of them like the p.o.n.c., can be demonstrated only *ab absurdo*. For instance, anyone wishing to deny the p.o.n.c., simultaneously, from a meta-linguistic perspective, proves it whilst s/he is trying to refute it, since its denial implies its affirmation and thus, in order to deny its validity, one has to accept it.

From the apodictic method to the hypothetical-deductive one in modern science

From this perspective, a significant difference can be noticed between the method followed by *modern natural sciences*, and the *analytical-synthetic* method, which is typically used by Thomistic-Aristotelian epistemology. While the former, which was originally – within the Newtonian physics – *axiomatic*, but *apodictic-deductive*, at the beginning of the last century became hypothetical-deductive, though preserving its *axiomatic* nature, the latter, which is, instead, based on syllogism, appears to be apodictic – or ‘categorical’ – in metaphysics, and hypothetical in physics. (See. § 6.3.2.2). With Galileo and, especially, with Newton – and his invention of calculus – natural sciences, and above all physics, stopped being interested in the study of the different ‘natures’ of physical entities (both bodies and events) and in the ‘causes’ of their *existing* and their *becoming* (= local movement and intensive and extensive modifications of certain distinctive features, e.g., temperature, size, position, ...) to take the shape of sciences that were uniquely concerned with the *phenomenical representation* of these entities – through rigorous measurements of their particular features – and with the predictability – in terms of analytical calculation – of the variations of such features.

Empirical models and ontological models in modern science

This certainly does not exclude the possibility of advancing *ontological models* of purely representational scientific theories. In other words, we cannot exclude the possibility of passing from a purely phenomenical interpretation of scientific theories – according to the principles of the logic of propositions and the hypothetical-deductive method – to an ontological interpretation, in terms of models that describe certain *essential properties* (= the *nature*) of physical objects (both entities and events) and of the causal relations that determine their existence, according to the principles of the *logic of predicates** in its intensional interpretation and the analytical-synthetic method, which is typical of Aristotelian syllogistic, and that is being currently reinterpreted by means of the modern tools of a symbolic non-intensional logic (See Wallace 1996; Basti & Perrone 1996; Cellucci 1998)⁸³. To look carefully, from an epistemological point of view, every experimental scientist operates in light of the well-grounded belief that his/her studies also involve entities and causal

⁸³ The main difference between *intensional* and *non-intensional* logic of predicates is that the former interpretes the predicates in terms of the *properties* that are inherent in certain objects, characterized by the subjects of the related propositions. The latter, instead, interpretes the predicates supposing the existence of the classes of elements, which constitute the domain, and hence the extension, of the related predicate. The interpretation of predicates in intensional terms is the basis of both descriptive and formal ontology, and therefore constitutes the *organon*, i.e., the appropriate logical tool, for any metaphysical theory (See: Bochenski, 1995, 116). The distinction between these two ways of dealing with predicates is at the basis of the distinction between *intensional* (or *contentutistic*) and *extensional* (or *formal*) *logic*.

relations, and not only logical-formal relations among his/her mental and/or linguistic representations of hypothetical objects. It is only at the level of the logical formalization of the theory that there exists – and must exist, to avoid some mistakes and confusions that took place in the past – the aforementioned distinction between hypothetical-deductive *phenomenical models* and analytical-deductive *ontological models* of the matter of study of modern science. Following this line of reasoning, every authentic scientist is also a philosopher of nature: what really matters is that these two roles are kept well separated. This is the only way in which they can be fruitfully combined, preventing them from suffering from the ideological exploitations of modern science.

In such a light, we can anticipate that it was only a certain dogmatism deriving from the cultural domination of a purely *empiricistic* epistemology and of a purely *representational* ontology of science that drove many philosophers of science in the 20th century to exclusively affirm the *hypothetical-deductive method* as the only demonstrative method for modern sciences. But such a shortfall — which especially characterised the last ten years — seems to have been now overcome, both for cultural reasons — it has indeed created a gap, that sometimes is very hard to bridge, between the experimental scientist and the philosopher of science —, and for theoretical ones — i.e., the end of scientism.

The explanatory character of modern physics within the hypothetical-deductive method and the concept of 'law'

However, we have decided to limit our analysis to those physical sciences that constitute the paradigm of modern science, and to stick to the use of the hypothetical-deductive method, which none the less represents the method by means of which most of contemporary physical-mathematical sciences were formalized. From this perspective, the scientific 'explanation' acquires a particular connotation, when compared to the pre-modern view of science that was of an exclusively ontological kind. In other words, explanation, in modern science, is not identified with the definition of the cause of a certain event, but it is rather identified with the definition of the *geometric law* (= function) and of the related *algebraic equation* (= polynomial) that govern the variation of a certain quantity (= dependent variable) in relation to the variation of another quantity (= independent variable). Accordingly, 'to explain' means, for modern physics, 'to refer to a universal law' a given physical event/process as characterized by certain *measurable quantities*.

We can define as follows the concept of 'law' as a substitute for the concept of 'cause' in the explanations given by modern natural sciences, in accordance with the hypothetical-deductive method:



Definition 5

By ‘scientific law’, physical hypothetical-deductive sciences mean a general assertion, which is normally expressed in a mathematical form (equation, function), by which one can define, explain, and, above all, predict the behaviour of a physical system, in accordance with experimental measurements, which make reference to situations of the same kind.

The use of such geometrical-algebraic formalism for the representation-determination of local motions, and/or of modifications of bodies, suggested to the modern scientist, since the origins, the possibility to look for the ‘ultimate’ explanation, not at the level of *universal* physical causes of all physical events, but rather at the level of the *universal* laws and postulates of motion, from which all particular laws and relations were to be derived as theorems, just like, in the geometry of flat space, all the theorems and equations, that are related to the different geometric figures and to their relations, can be univocally deduced from Euclid’s postulates.

The three laws of Newton’s dynamics and their paradigmatic character

As we have anticipated in Chapter 1 (See: *supra*, § 2.2), we owe to Newton the definition of the *three fundamental laws of dynamics* (the principle of inertia, the principle of proportionality between force and mass multiplied by acceleration, the principle of action and reaction) that are universally valid *a priori* for all the motions that are studied by mechanics (= the science of the motions of physical entities and of their laws), as logical-formal conditions of applicability of the infinitesimal/differential calculation to the description-prediction of all the motions of the physical order, according to a strictly both *phenomenical* and *apodictic* method that is founded on the presumed self-evidence of the laws of motion.

Centrality of the evidence principle and the epistemological – not ontological – foundation of the modern science of origins

The extraordinary fortune that the ‘new’ Newtonian method had at the beginning forced *modern realistic* epistemology and metaphysics to become *empiristic and/or rationalistic*. This means that this forced them not to look for the foundation of ‘explanations’ and of ‘postulates’, for the demonstrative procedures of each single science, in the *being* and in the different *natures* of the different entities, but rather to do so in the contingent *empirical evidence* of measurements, and in the *apodictic, absolute* evidence of the postulates that are placed at the basis of the various scientific procedures of demonstration: i.e., Newton’s three laws of dynamics. Now, since the evidence is a property of thought and, more precisely, it

is a property of *conscious* thought (in view of this, to demonstrate a theorem would mean to make something logically evident, i.e., to make clear to a conscience the consequences that are implicit in the premises), it is obvious that the ultimate evidence of postulates and of first logical principles – if not grounded on the natural being of different entities – will have to be based on a presumed *immediate evidence*, or *self-evidence* to the conscience and, consequently, on the ability of the conscience to become aware of such an evidence. In a word, it will have to be based on self-conscience.

The foundational issue: the transcendental of self-conscience vs. the transcendental of being

Hence we have the main difference between modern and classic metaphysics, in the search of the ultimate foundation for the universal concepts of reason and of the ‘first’ propositions or, better, of the *principles* of every demonstrative procedure, not in the *objective transcendental* of the being of an entity, of its *properties* and of its *real relations* (= causes), but rather in the subjective transcendental of the ‘*I think*’, of the Cartesian and Kantian *cogito*, i.e., in the *self-aware* thought *qua* foundation of the *logical relations*, through the principle of evidence. According to the Kantian scheme, these self-evidences (that were conceived of as the *pure a priori forms*, without any content, of the self-aware thought) are what *gives an order*, in a *universal* and *necessary* way, to the contents of the *a posteriori empirical evidences* that are contingent to the observation. In doing so, they give to the statements of modern physical science that both empirical and logical touch — that Kant used to define in terms of *synthetical-a priori* element — that grounds the universality and necessity of its purely phenomenal explanations (See: § 5.6.1). We will deal with such metaphysical and epistemological issues in the next chapter, since they will be extremely useful in order to define a modern notion of philosophy of nature, which is adequate to the results of contemporary studies. None the less, before directly tackling such issues, let me cast a brighter light on the features of the *hypothetical-deductive* method, which is typical of the logic of modern, post-Kantian science.

4.2.2 The hypothetical-deductive method

4.2.2.1 Ludwig Wittgenstein

From epistemology to philosophy of language

Philosophy of science, conceived of as the methodological reflection on science, dates back to the origins of Western thought (Oldroyd 1998). The development of philosophy of science as a freestanding discipline, at the beginning of 20th century, corresponds to the growth of a necessity to reconsider the foundations of modern science. A shift was needed from the reference to the analytical philosophy of the cognitive

act and of the modern principle of evidence, as an expression of the thought of a *knowing subject*, to the objectivity of the logical analysis of *scientific language*.

Ludwig Wittgenstein
and the *Tractatus*

The beginning of the modern philosophical-scientific study coincides with the attempt carried out by Ludwig Wittgenstein (1889-1951) to extend to epistemological and linguistic analysis the results obtained by the logical axiomatization of mathematics that were achieved through *Principia Mathematica* (1900-1913) by Whitehead and Russell (whom he met at Cambridge in 1911-1913). The *manifesto* of such a revolution gave life to the main work by Wittgenstein, i.e., the *Tractatus Logico-Philosophicus* (1922), which was published with the support of Russell and that immediately attracted an international interest.

Philosophy of
language as
linguistic therapy
and the birth of
analytical
philosophy

The core of the *Tractatus* is not actually devoted to philosophy of science, but rather to philosophy of language, which is presented as a 'drug' to heal ordinary language, and above all philosophical language, from its 'illness'. The *Tractatus* thus represents the inaugural *manifesto* of that school of philosophical thought that falls under the name of *analytical philosophy*. This latter is characterized by the attempt to systematically apply symbolic logic to philosophical analysis, so as to make its presuppositions and methods clear, against all ambiguities and inconsistencies⁸⁴.

Reductionist spirit of
the *Tractatus* and
the anti-
metaphysics
dispute

However, the *Tractatus* endorses a sort of reductionist outlook because of the underlying prejudice that used to identify symbolic logic with mathematical logic, and the analysis of reference with the empirically based analysis suggested by sciences. Accordingly, the illness of language is, according to Wittgenstein, related to the non-critical use of propositions *with no meaning*, the meaning of such propositions being the *method of their empirical verification*. It is rather clear, then, that most of metaphysical propositions, exactly because they cannot be empirically verified, come out to be meaningless; just so, the problems they make reference to seem to be only *false problems* (i.e., non-ontological problems, but rather grammatical problems that are related to a false use of language). The kind of language that is really meaningful to Wittgenstein essentially is the language of natural and mathematical sciences.

⁸⁴ A classic introduction to themes, problems, and methods of analytical philosophy, in their relation to traditional philosophical and metaphysical issues, can be found in (Strawson 1992). Another collection of bibliographical references, dealing with the connections existing between analytical philosophy and contemporary formal ontology, can be found on-line at the following URL: www.formalontology.it.

Program of
Wittgenstein's
logical atomism

Taking into consideration the tautologicity of mathematical and logical propositions and, consistently, their logical-formal truth, the program of linguistic therapy that Wittgenstein proposed is also defined in terms of *logical atomism*. A truly meaningful language is structured by following the rules of definition and of inference suggested by mathematics and logic, through which it is possible to create meaningful propositions and sets of meaningful propositions (discourses, demonstrations). These constructions use as 'bricks' the *primitives* of different *formal systems**. The 'primitives' correspond, according to Wittgenstein, to *simple* or *categorical propositions* (subject–predicate) that mirror *basic facts* of the physical world. They are, in other words, proper *linguistic atoms* of a sensate kind. In this way, the meaningful language becomes a proper 'symbolic mirror' of reality and of its structures⁸⁵.

Reductionist spirit of
Wittgenstein's
program and its
therapeutic function

This does not want to suggest that Wittgenstein was not aware of the reductionist character of this program, i.e., that he was not aware that, in this way, the use of language was kept apart from a number of problems and objects that constitute most of current linguistic uses. One can think here not only at metaphysical language, but also at religious, artistic, political, or literary languages. To all these uses of language, Wittgenstein applied that last famous maxim, which is placed at the end of the *Tractatus*: "what we cannot talk about we must pass over in silence". Despite the reductionist use that is attached to this maxim here, it is none the less a precious indication, which should really be applied to several contexts of today's communication! Language would indeed need a strong therapy in these days, as the one proposed by Wittgenstein, even though this would also probably need different and more inclusive 'drugs' than those suggested by him.

⁸⁵ Incidentally, a sort of bi-univocal correspondence can be found between the scheme of logical atomism and that of Kantian transcendental phenomenism. According to Kant, phenomena are the result of the composition between the *a priori*, self-conscious forms of sensation (space and time), which give an order to single "atoms" of sensation, and the conscient "impressions" – that remind us of Hume – received by the sense organs. Logical atomism brings the same scheme into the linguistic domain: the linguistic atoms of simple propositions correspond to Kantian "sensations" of the sensible conscience. Vice-versa, the tautological nature – as an expression of formal relations of self-identity – of logical-mathematical axioms, and their purely formal truth (from which they derive their role in the logical organization of simple propositions – i.e., atoms – in view of the creation of complex propositions – i.e., molecules – that are significant and consistent), correspond to the foundational function of Kantian self-conscience. This latter, with its *a priori* forms and its categories, organizes sensations into phenomena, and phenomena into judgements. On these relations between logical atomism, on the one hand, and Neo-positivism and Kantian epistemology, on the other, see (Barone 1977) and (Pera 1981).

The 'second' Wittgenstein and his teaching at Cambridge

To look carefully, it was Wittgenstein himself who began first to go in this direction. After having written the *Tractatus*, even before its publication, after the first World War (in which he took part and by which he was profoundly affected), Wittgenstein spent a long period of time (1920–1926) on his own in Carintia. Since his family was very rich and he did not consequently have any economical problems, he began to teach at a primary school there, and he also thought for a while about becoming a monk. It was Russell who persuaded him to get out of this sort of voluntary exile. In 1926, Wittgenstein got his degree in philosophy (*sic!*), when he was already considered one of the most influential contemporary philosophers, and in 1929 he took what used to be Moore's job at Cambridge. To witness this phase of his life, we only have a collection of essays that was published after his death with the title of *Philosophical Investigations* (1953).

The concepts of 'linguistic game' and 'universal grammar'

In these writings, though remaining faithful to his therapeutic program for the philosophy of language, his criticism was focused on the improper uses that were made of different kinds of languages, having already abandoned the rather naïve idea, according to which only one kind of meaningful language was available. The meaning of a language is indeed connected to the different uses of such language and to its rules. Accordingly, when an improper use of a language is made, this is due to the application of linguistic constructs that lie outside the *linguistic game*, i.e., outside the set of rules, in accordance to which they were originally conceived (as, for instance, when some constructs that belong to religious language are used in a scientific context, or *vice-versa*). The presumed 'universal grammar' through which it should be possible to compare the different uses and linguistic games was considered by him

- ◆ on the one hand, as a *reality* — otherwise we would not be able to grasp the capacity human beings have to understand each other and to switch from a linguistic game to another one;
- ◆ on the other hand, as something totally *implicit and non-thematizable*, thus excluding the possibility, in light of the theorems of limitations that he perfectly was aware of (See: § 3.1.4), both of creating an explicit *universal meta-language*, and of *comparing and translating* the linguistic constructs, belonging to different linguistic games, in a scientifically, logically and linguistically satisfactory way.

4.2.2.2 The Neo-positivist movement

The 'Vienna Circle' and the birth of the neo-positivist movement

The *Tractatus* exercised a fundamental influence on the philosophical and scientific thought of the time, and represented the starting point for the development of the first group of philosophers of science of the 20th century, which is known as the 'Vienna Circle'. Members of such a Circle were such young philosophers, logicians, mathematicians and physicists like Kurt Gödel, Otto Neurath, Herbert Feigl, Rudolph Carnap, Philip Franck, Karl Popper — and less directly, Hans Reichenbach and Carl Hempel, who founded a similar movement in Berlin. They started meeting in the 1930's, thanks to the seminars of 'inductive sciences' given by Moritz Schlick (1882-1936), who was holding at that time the position at the University of Vienna that used to be held by Ernst Mach. Schlick was murdered by a Nazi student because of his fierce opposition to the *Anschluss* of Austria to Nazi Germany.

The application of the principles of logical atomism to philosophy of science by M. Schlick

Despite the fact that Wittgenstein never took part in those seminars, Schlick contributed heavily to the diffusion of his ideas, hence providing the basis of the so-called *Neo-positivist movement* — also known as logical *Neo-positivism*, or *logical empiricism*, according to the different conceptions and further changes in the original program that however cannot be addressed here⁸⁶. Consistently with Wittgenstein's logical atomism, Schlick proposed a theory of scientific language based on the rules of construction and inference of mathematical logic and on the existence of the so-called *protocol sentences*, i.e., observation statements that express in a rigorous form — which can thus also be interpersonally reproduced and controlled — the results of single *operations of measurement*.

Logical-linguistic analytical function of scientific propositions operated by the philosopher of science.

Such statements can represent both the *primitives* that are needed for the construction of axiomatic theories in physical sciences — in particular, in terms of *ostensive definitions* of their empirical objects of study — and the statements needed for the *empirical verification* of an inductive kind (enumerative induction) of propositions that would have to be deduced otherwise from the hypothetical axioms of the theory. While natural sciences are thus asked to empirically test the factuality of hypotheses to ground the scientific theories on strong hypothetical bases, the philosopher of science is expected to investigate the meaning of the constructions of scientific languages, by means of a logical-linguistic analysis

⁸⁶ Let me make reference, in this respect, to (Barone 1977), which I consider to be the most complete work of reconstruction — not only within an Italian context, but also internationally — of the Neo-positivist movement of the 20th century, in its different aspects, from the origins down to its "death" by hand of K.R. Popper's philosophy of science.

which involves both such constructions and the consistency of the procedures of inductive verification created by the scientist.

4.2.2.3 Karl R. Popper: falsificationism

Falsificatory use of the hypothetical-deductive method and its consequences.

Popper's falsificationism and the critique of the 'verification' use of observation propositions.

It is exactly on the logical-formal inconsistency of the use made of observation statements for the *inductive verification* of propositions within essentially hypothetical theories, that the Popperian critique of logical Neo-positivism and its inductionism is based. In such light, especially if we consider observation statements in the sense of Schlick's protocol sentences as representing the result of an operation of measurement, it is clear that such statements need to be considered as a consequence of the axioms of the theory itself. This latter, like any other model, supposes indeed the definition of certain *metric axioms*, i.e., of axioms that define the metrics that will be used within the same theory to make the formal system capable of including within it empirical statements, related to experimental measurements. In rough intuitive terms, if I build up a hypothetical-deductive theory of atomic physics, I have to set a metric axiom that defines that the quantities I am going to measure correspond to 10^{-8} cm. I will also have to define a series of procedures for the definition of tools that are adequate to carry out that kind of measurement.

Some valid and non-valid modes in hypothetical arguments

This means, in terms of propositional elementary logic, that an observation statement is always a certain consequent q , materially implied from a certain premise p , according to a relation $p \supset q$. Using the truth of q to verify p is consequently a logical mistake, a mistake which is known as the 'fallacy of the consequent':

$$((p \supset q) \cdot q) \supset p \quad (1)$$

Vice-versa, the argumentation of the *modus tollendo tollens* of the hypothetical syllogism is correct :

$$((p \supset q) \cdot \sim q) \supset \sim p \quad (2)$$

This leads to the claim that the empirical test conducted through observation statements, deduced from the theory, can be exclusively considered as a *method of falsification* of the theory itself, and not as a strategy for its verification. The logical-formal basis of such a claim is well known. Formally, q is the *necessary but not sufficient* condition for the truth of p . Correspondingly, p is the *sufficient but not necessary* condition for the truth

of q . This means that not only (1) is false, but that also the correlative ‘fallacy of the negation of the antecedent’ is false:

$$((p \supset q) \cdot \sim p) \supset \sim q \quad (3)$$

Vice-versa, the argumentation of the *modus tollendo ponens* of the logical sum is formally correct:

$$((p \vee w) \cdot \sim p) \supset w \quad (4)$$

In other words, if, say, on the basis of the hypothesis p taken from the Newtonian theory of gravitation, I affirm q , i.e., that a certain planet at the time t has the position z ; if when pointing my telescope at the time t towards that position z I falsify q ($\sim q$), i.e., I do not see the planet, on the basis of (2), the theory p would come out formally falsified. *Vice-versa*, the fact of actually finding the planet in z , i.e., the verification of q , in light of the fallacy of (1), does not verify p . Another theory different from p (e.g., the theory of general relativity) could indeed equally imply q ⁸⁷.

The demise of
logical Neo-
positivism by
means of
Popperian
falsificationism

This is the core of the falsificationist theory proposed by Karl R. Popper (1902-1994), which represented the logical-formal ‘weapon’ by means of which this member of the Vienna Circle carried out his ‘killing of logical Neo-positivism’, as Popper himself confesses in his *Unended Quest: An Intellectual Autobiography* (Popper 1978), on the basis of the work which made him famous, i.e., *The Logic of Scientific Discovery* (Popper 1934). Through the appropriate application of rules of empirical falsification of theories, sciences are expected to develop as self-correcting procedures of approximation to truth. Hence, we see his conception of the history of science, which can be found in his *Conjectures and Refutations* (Popper 1963), as a sequence of falsifiable conjectures, followed by their confutation, in order to subsequently propose new and more powerful falsifiable conjectures, and so on and so forth.

Falsifiability as a
criterion of
distinction between
science and non-
science

According to Popper, what characterizes scientific theories, and distinguishes them from what does not belong to the realm of science (especially from those ‘metaphysical theories’ that are conceived by Neo-positivists as the negation of science) is not the fact that scientific statements come always together with the definition of a method for their

⁸⁷ The example is not totally convincing since, as it is well-known, thanks to the “principle of equivalence” (See above, § 2.5.3), the gravitational theory of relativity implies the Newtonian theory as one of its particular cases. Therefore, strictly speaking, it is not the denial of p in our example, but rather an additional r , which implies p . In the development of science, new conjectures always have to include the previous ones: only in this way there is actual scientific progress.

Ontological and
logical limits to
falsificationism

experimental verification, but rather the fact that they are paired with a method for their experimental ‘falsification’. What we define as ‘science’, within different historical periods, is therefore only a set of falsifiable hypothetical argumentations that are not (yet) falsified.

Despite the appeal of Popperian theories to the common attitude of people of our time towards science (an appeal that is also related to the consequences in political philosophy that such theories have justified⁸⁸), two consequences of his falsificationist theory need to be underscored:

- ◆ *Ontological limit*, i.e., the impossibility of science to positively deal with the real. According to the falsificationist theory, in Popper’s view, our scientific theories can be sure of having ‘touched the real’ only when the empirical control falsifies the theory, and not when it confirms it. ‘Our falsifications show the points in which we have, so to speak, touched the reality’ (Popper 1969, 42. See also Popper 1963, 201). Thus, paradoxically, according to falsificationism, science is supposed to express in a formally correct way only what reality is not, never what it truly is.
- ◆ *Logical limit*, i.e., the impossibility of devising consistent logical procedures to define new falsifiable hypotheses following the falsifications of the previous ones. The definition of new hypotheses is paradoxically abandoned by falsificationism to irrational procedures that are never logically formalizable. On this, Popper maintains:



Theories are free creations of the mind, they are the result of an almost poetic intuition, of an attempt to intuitively comprehend the laws of nature (Popper 1963, 330).

As far as I can see — for what it is worth — there is no logical method to come up with new ideas and no logical reconstruction of this process is available. (...) If, in accordance with Reichenbach, we distinguish between a ‘finding procedure’ and a ‘justifying procedure’ for a hypothesis, we have to admit that the former

⁸⁸ A part from their cultural merits, the popularity of Popperian theories is certainly bound to the fact that he made use of his anti-metaphysical falsificationism as a justification of a *free-trade* conception of society, that allowed him to become lecturer at the prestigious *London School of Economics*. This made it possible for his theories to be advertised for his purposes of economical *deregulation* from ethical and metaphysical constraints, i.e., from the so-called ‘strong powers’ of the free-trade and hyper-free-trade economy of these last twenty years. In his famous book, *The Open Society and Its Enemies* (Popper 1966) he maintains that from Plato to Marx, through Christianity, all those who opposed the ideals of the modern liberal society have always used an ‘essentialistic’ conception of truth, i.e., the presumption to be able to reach — through their non-falsifiable ‘metaphysics’ — a truth that is, somehow, ‘eternal’ and unchangeable, by means of which the progress of modern science, politics and economics can be halted.

procedure, i.e., *the finding procedure of a hypothesis H, cannot be rationally reconstructed* (Popper 1959, 11; 349).

Once we are faced with a problem, we can proceed with two kinds of attempts: *we try to guess, or to make conjectures* on a solution for our problem, and we can try to disprove our solutions that are normally quite weak (...). The best method, if not the only one, to learn something about a problem is to make an attempt to *solve it, in the first place, trying to guess*, and thus to single out the mistakes we have made. (Popper 1963, 139-40).

Problematic applicability of the falsification principle to scientific theories

We will soon examine the theoretical reasons of Hans Reichenbach's failure in the attempt to ground a 'logic of discovery' and not only 'of the proof' (or 'of the justification') from within the neo-positivist movement. We can, instead, now suggest a clarification – that was recently proposed, among others, by W. Wallace in his book *The Modelling of Nature* (Wallace 1996, 249) – on the capacity of the falsificationist interpretation of the hypothetical-deductive method to provide a procedure to 'single mistakes out'. The so-called 'Duhem–Quine thesis' excludes that falsificationism has such a capacity. The complexity of scientific theories excludes, indeed, that the falsificationist method can be reduced to the over-simplified scheme of the *modus tollens* in (2). Because of the multiplicity of the existing premises for a hypothetical-deductive procedure, the conditional argument assumes the following form:

$$(((p_1 \cdot p_2 \cdot \dots \cdot p_n) \supset q) \cdot \sim q) \supset \sim p_i \quad (5)$$

Which means that failing the verification of q , we are left in principle with no useful information to identify which and how many of the different p_i are falsified.

Logical condition on the use of empirical propositions as a criterion of verification and not only of falsification

It is the same Wallace who correctly suggests that, if one wants to use the empirical reference to either *verify* or *falsify* some hypothesis, i.e., if one wants to validate not only the conditional arguments (2) and (4), but also the conditional arguments (1) and (3), it is necessary that *in all* of them, the premise of the conditional argument is rewritten in the *biconditional* form, $p \equiv q$, thus affirming the equivalence of p and q .

$$((p \equiv q) \cdot q) \supset p \quad (6)$$

$$((p \equiv q) \cdot \sim p) \supset \sim q \quad (7)$$

However, in this case, as Wallace correctly notices, the empirical statement is no longer a statement that works as only the necessary

condition for p , but it also represents the sufficient condition, as it seems to be evident when we put together (6) with the rewritten version of (2), and use as a premise the bi-conditional form used in (6):

$$((p \equiv q) \cdot \sim q) \supset \sim p \quad (8)$$

Necessary move implied by the verificative use of empirical propositions; from the logic of proof to the logic of discovery

Here the empirical statement q , characterised through such a construction as the *basis of the truth* of the hypothetical proposition p , formally becomes both the necessary (Cf. (8)) and sufficient (Cf. (6)) condition of the truth of p . This means, however, that we are no longer moving within the context of the 'logic of proof' of the hypothesis, but rather within the context of the 'logic of the constitution' of the hypothesis. The empirical statement is no longer used for a control that comes after the constitution of the hypothesis p . It becomes, instead, an indirect expression, in the inadequate terms of propositional logic, of a procedure of *hypothesis constitution*, i.e., that which Popper, rather superficially, claimed to be impossible to logically define in any way whatsoever.

Necessity to shift from the logic of propositions to the logic of predicates

The reason of the inadequacy of propositional logic in dealing with problems of hypothesis constitution, is that we are working here on the internal constitution of the propositions. Accordingly, it is necessary to shift from the formalisation of scientific languages in terms of the propositional logic alone, to their formalisation within the logic of predicates. More precisely, we need to step into the realm of the syllogistic procedures of the *veritative* foundation of the premises of the hypothetical arguments for specific contexts.

Constitutive induction of hypothesis vs. enumerative induction of already constructed hypothesis

In such a context, as Aristotelian logic had already understood – even though in a deficient and incomplete way – it becomes possible to logically define valid procedures for the *constitutive induction* of true empirical statements, that may work as premises for subsequent hypothetical arguments — and, in specific cases, also for apodictic arguments, that are no longer based on the shaky basis of *evidence*. As we shall see, this does not represent the logical equivalent – in accordance with a *material, content-oriented* logic, and not only with a *formal* one — of what is defined, by epistemology and psychology, as the *abstractive procedures of logical universals* for the intentional mind.

The limits of enumerative induction and Carnap's and Reichenbach's failures

If Hans Reichenbach failed in his attempt to suggest a logical justification for the 'procedure of discovery and/or of invention' and for inductive methods, this was because, in view of his empiricistic epistemology, he tried to ground his discovery procedures on *enumerative induction*. The limits of his attempt are thus the same as those that Rudolph Carnap encountered in his, equally failed, attempt to use induction, in the domain of the Neo-positivistic 'logic of justification', as a 'strengthening

method', or as a 'probabilistic verification' of the hypothesis, in relation to the hypothetical-deductive method of modern science.

Strengthening use of the hypothetical-deductive method and its limits.

Extension of the 'weak' hypothetical-deductive method to a method of verification

Despite all its failures, the Neo-positivistic movement managed to impose, in the philosophical attitude of the 20th century, the hypothetical-deductive method as *the* method of contemporary modern science, both in natural and in human sciences. This is due to the fact that the scientist, in the concrete use of formal languages that the mathematical logicians of the 20th century managed to make available to him/her, does not make use of the empirical proof exclusively as a procedure of falsification of the hypothesis, as would be formally correct, but also as a procedure for the (partial) *verification* of the hypothesis, despite the fact that such a procedure may sound partial, insufficient and formally not grounded from the perspective of propositional logic. The scheme of such a verificative use of the hypothesis, within the hypothetical-deductive method, can be synthesized as follows (Wallace 1996, 248):

$$\Diamond (((p \supset (q_1 \cdot q_2 \cdot \dots \cdot q_n)) \cdot (q_1 \cdot q_2 \cdot \dots \cdot q_n)) \supset p) \quad (9)$$

The extension is made possible through the use of modal operators

Where ' $\Diamond$ ' is a modal operator which means 'possibly' — as opposed to 'necessarily' ' $\Box$ ' — so that the schematic formula $\Diamond p \supset q$ means that '*possibly* if p then q '.⁸⁹ As is obvious, within the hypothetical-deductive method, the bigger the number of the empirical consequences that are verified, the bigger the possibility that p is true. In these days, scientists actually operate according to this scheme: the more the number of the experimental verifications of a given theory increases, the more confident scientists are in the truth of the hypothetical axioms of the theory itself.

Verificative use of the hypothetical-deductive method and the calculation of probabilities

Rudolph Carnap (1891-1970) is the representative of logical Neo-positivism who most significantly strove to give scientific dignity to this widely spread verificative use of the hypothetical deductive method, trying to apply to it the principles of the mathematical theory of probability. In the elementary calculation of the probability, we try to define the probability P that a certain event u happens (e.g., the fact that a certain number, say number 3, comes out) given another event v (e.g., some dice are rolled). This is usually expressed by defining the function of probability $P(u, v)$. It is evident that, in our example of the rolling of dice, the *a priori probability* that our event takes place is equal to $1/6$, given that each of the dice has only six sides.

⁸⁹ In ordinary propositional logic there is no need to make these operators explicit, since the only mode that is allowed is that of necessity: '*necessarily* if p , then q '.

For all this to be true, the following conditions are needed: (i) that all the possible outcomes can be counted in a *finite* number (the six sides of the dice), and (ii) that the events u are independent of each other (the dice are not loaded). One of the greatest laws for the calculation of probabilities is the so-called 'law of large numbers'. If I were to roll the dice a limited number of times, it is obvious that the *occurrence* F , or a *posteriori probability*, with which number 3 actually comes out could significantly vary from that calculated *a priori*. For instance, out of twenty times I roll the dice, number 3 may actually never come out ($F = 0$), or come out only once ($F = 1/20$). It is equally evident that if I increase the number of the times I roll the dice, the *a posteriori* probability would get closer to the *a priori* one. The 'law of large numbers', which was discovered by Gauss and that lies at the basis of the entire modern theory of probability, guarantees indeed that given a very large number, even an infinite number of events, i.e., when $v \rightarrow \infty$, $F = P$.

From the calculation
of probabilities to
logic

Now, if we make reference to propositions rather than to events, the 'possibly' of (9) could significantly be replaced by the 'probably' of the calculation of probabilities. In this way, we could calculate, in principle, the function of probability of the truth of the proposition p in relation to the number of experimental evidences q that are referred to the theory. In other words, we could calculate the probability of the truth of p given the truth of q , $P(p, q)$. This becomes symbolically clearer if we replace p with h for 'hypothesis' and q with e for '(experimental) evidence', $P(h, e)$. But, we can also replace P with C which, in accordance with Carnap, means 'confirmation' or, better 'corroboration'. We will have then $C(h, e)$, which is a function that expresses the degree of corroboration (or confirmation of truth) of h , given e .

Application of
Bayes' laws to the
procedures of
verification and its
limits

At this point, to fully express (9) in this form, we need a law for the calculation of probabilities that expresses the function of probability P of an event u , as the effect of the connected events $v_1, v_2, \dots$ that are, that is to say, given together. This law is the so-called 'Bayes law', named after its discoverer Thomas Bayes in 1763. Considering an oversimplified case, where there are only two of these events, such a law of the connected probabilities will be written in this way $P(u, v_1 \cdot v_2)$. Along these lines, the isomorphism between (9) and $C(h, e_1 \cdot e_2 \cdot \dots \cdot e_n)$ would be completed. We would thus have a function that calculates the variation of the degree of the probability of the truth — i.e., the degree of confirmation, or *corroboration* — of an hypothesis h in relation to the full set of experimental evidence that is gathered time after time.

However, this is not true. The assumptions that validate Bayes laws can hardly be satisfied in the case represented by Carnap's corroboration function (Wallace 1996, 262).

- ◆ First of all, it is very hard to understand how propositions, and propositions that make reference to theoretical concepts like *h* does, can be formalized as *simple events*.
- ◆ Secondly, it appears to be hard to satisfy the condition that such propositions stay *within a finite number*.
- ◆ Finally, it is very hard to suppose that they satisfy the *equiprobability condition*.

Wallace is right to show his surprise about the wide application the Bayesian method has, despite these limitations, in every field of both natural and human sciences. But this is the case: the need to verify hypotheses is uniformly spread among all the fields of science, and the hypothetical-deductive method cannot offer anything that is significantly better than this.

The conventionalist background of empiricist ontology

The alternative to this would be to endorse a *radical conventionalism*, which may satisfy the philosopher of science, but certainly not the philosopher of nature. This latter always engages with the real, but unfortunately, in light of the hypothetical-deductive method, he has no valid argument to oppose to the conventionalist to justify his conviction of realism. We can then understand the declaration of radical conventionalism in the choice of the hypothesis that Rudolph Carnap presents in his *Logische Syntax der Sprache* as *Toleranzprinzip der Syntax* ('principle of toleration of the syntax') or 'principle of conventionality'.



We do not want to forbid anything, but to establish standards (...). There is no morality in logic, everyone can define as s/he wishes her/his logic, i.e., his/her form of language. But if s/he wants to discuss it with us, s/he needs to say how s/he wants to do that. In other words s/he needs to establish syntactic determinations, instead of philosophical discussions (Carnap 1934, 44s.).

Similarly, some years earlier, in the 'Scientific Manifesto' of the Vienna Circle, that was published by Carnap together with Otto Neurath in 1929, it was explicitly stated that:



Everything is open to human beings, and they are the measure of all things. In this we can see a similarity with the Sophists, and not with the Platonists, with the Epicureans, and not with the Pythagoreans, with all the supporters of the mundane and of the earthly.

4.2.2.4 Popper's followers

In line with the same kind of criticism of Neo-positivistic induction, and in accordance with an essentially conventionalist ontology in relation to science, the three main followers of Popperian falsificationism adopted an *historicist* approach to philosophy of science: these are T. S. Kuhn, I. Lakatos e P. K. Feyerabend.

T. S. Kuhn's Structure of Scientific Revolutions

The leading historical and cultural factors in the history of science within Kuhn's theory

Thomas Samuel Kuhn (1922-1996) was one of the main historians and philosophers of science of the late 20th century. He taught at the most prestigious American Universities (such as Harvard, Berkeley, Princeton and MIT in Boston), and he contributed to give to the philosophy of science of Popperian falsificationism an essentially historicist character. He got his education in physics and, in his reconstruction of the history of science, he underlined the link between this latter and the history of culture, focusing on the psychological and social factors that determine the development of the scientific enterprise, much more than the contact with reality can achieve.

Scientific paradigms and the change of paradigms

In his most important methodological work, i.e., *Structure of Scientific Revolutions* (1962), Kuhn presented the history of the scientific knowledge as a series of periods of research (defined as periods of *normal science*), based on some acquisition derived from the past (paradigms). The community of scientists recognize these latter as the foundations of its further activity of research. Such periods of *normal science* are interrupted by some other periods of crisis (defined as periods of *extraordinary science*), during which anomalies, that cannot be solved by the existing paradigm, lead to its replacement with a new paradigm (*scientific revolutions*. See (Kuhn 1962)).

Incommensurability of scientific theories and decisive role of psycho-sociological factors in the changes of paradigms

Kuhn argued for the *incommensurability of scientific theories* in light of the impossibility for hypothetical-deductive science to make reference to the real, so as to use it as a criterion to judge the validity of opposed theories. The thought behind this claim is that the hypothetical axioms of opposed theories are not derived from the consideration of reality. Moreover, the control of hypotheses through empirical measurements is 'theory-laden', since it fundamentally depends on the axioms of measurements that characterize each theory, and in view of which the instruments of measurement were defined and determined. Given all this, Kuhn claimed that the choice of theories, during conceptual changes, is fundamentally guided by social and psychological factors.

I. Lakatos's Scientific Research Programs

Decisive logical and rational factors for the development of science

The Hungarian historian and philosopher of science Imre Lakatos (1922-1974) used to be a student of Popper's, and he applied the principles of Popperian falsificationism to the history of mathematics. Despite his being situated in an essentially historicist epistemological context, he underlined, unlike Kuhn, the *logical-rational* aspects of scientific research in the role they have in its development, instead of focusing on psychological and social factors.

The theories of the 'research programs' and the critique of the dogmatic use of falsificationism

To this purpose, in the '70s, he elaborated an original theory of the development of science, which is based on the notion of *research programs* (Lakatos 1974; 1998). According to this theory, science does not develop following a chain of single refutable theories, each of which is replaced by the following one, in accordance with the principles of 'dogmatic falsificationism'. This latter seems indeed to ignore that the same reasons, in light of which it is not possible to use empirical evidence to verify theories, also cast serious doubts on the very possibility of making use of empirical proof to refute a theory at all. The Duhem–Quine thesis (See above, p. 250) represents one of the main obstacles to this dogmatic use, in a falsificationistic sense, of empirical proof. Which is the assertion, among those that are included in the set of axioms of a theory, that the 'empirical proof' falsifies? Furthermore, given the probabilistic nature of every scientific theory, a failed experiment alone cannot falsify anything, i.e., it is not the refuting 'empirical proof' we were looking for. How many and how structured must the empirical tests be so as to provide the falsifying 'empirical proof' we were looking for? As it clearly emerges, the same arguments that can be brought against the verifying use of the empirical test, can also be brought against its falsifying use.

Fallibilism, or the methodological, non-dogmatic use of falsificationism at the basis of the development of the 'research programs' in science

Hence, we arrive at the idea that, instead of making use of dogmatic falsificationism, we should rather draw on a *methodological falsificationism*, or *fallibilism*, in the belief that this latter also represents the correct way to understand Popperian falsificationism. Such a falsificationism considers, as the core of the development of science, not a sequence of single theories, but rather the definition of *research programs* that include a number of scientific theories that are unified by the use of common – both logical and empirical – methodological rules, and by a shared commitment to fallibilism. Within these programs, theories are not created and replaced in accordance with the strict rules of dogmatic falsificationism, but according to the fundamental criterion of Popperian fallibilism, which states that other things being equal — Lakatos also asks what are the criteria to make a judgement on this condition, in order to avoid the dogmatic use of falsificationism — we should prefer the theory that is capable of making sense, in the simplest way, of the greatest amount of empirical content. This is the key-idea of the 'research programs' in or-

der to understand science as a *rational construction* of theories, and not as the mere effect of psycho-social or social-cultural influences, as Kuhn suggested.

P.K. Feyerabend's Methodological Anarchism

The extreme, but consistent, consequences of conventionalism and falsificationism in P. K. Feyerabend's methodological anarchism

Among the three most famous followers of Popperian falsificationism, the most straightforward and honest one, in his drawing the most extreme consequences of ontological conventionalism and methodological falsificationism, was the Austrian philosopher and historian of science Paul K. Feyerabend (1924-1994), who was Professor at Berkeley in California, where he was celebrated as one of the 'prophets' of the *new frontier* in the '60s and '70s.

In accordance with the acceptance of the principle of *incommensurability of scientific theories*, that was also endorsed by Kuhn, in his most famous work, *Against Method* (Feyerabend 1973), Feyerabend denies the existence of a *rigorous scientific method*, neither verificationism (in light of the reasons suggested by Popper), nor falsificationism (in view of the reasons proposed by Kuhn and Lakatos) can be the ultimate criteria of distinction between what is *science* and what is *non-science*. On what basis, then, do we continue to contribute, in our time, a superior status to *scientific knowledge and tradition* in comparison with other forms of knowledge, such as art, religion, or even magic?

Methodological anarchism is a necessary condition of falsificationism and forerunner of 'weak thought'

The power of Feyerabend's argument, despite its being paradoxically and deliberately provocative, is in the claim according to which there are no ultimate arguments to refute methodological anarchism, both in light of the *falsificationist* (or fallibilistic) *method* and of a *conventionalist ontology* that derives from the metaphysics and the empiricist epistemology of modern transcendentalism: an ontology that is shared both by Neopositivistic philosophy of science and by Popperian as well as Neo-Popperian 'falsificationism'. In such a light, Feyerabend can be defined as one of the first and most celebrated representatives of that 'weak thought' that has recently become very popular, especially in Italy, also in view of the financial support received by certain centers of economic power (where anarchy reigns it is only the law of the strongest – i.e., of the 'strongest powers' – that rules).

From the logic of justification of the truth of hypothetical propositions to the logic of discovery or constitution of true propositions

As we have already suggested, in order to hope in something better and to characterize the conviction of the realism of scientific enterprises not as an *act of faith*, but as the result of a *rational argumentation*, we need to operate a shift from a *logic of justification*, or of the proof of hypothesis (axiomatic method) – in which the empirical test may essentially have a function of falsification of the hypothesis – to a logic of the *constitution of hypothesis* (analytical method), in which the reference to experience and,

accordingly, to reality is not used as an instrument to test already constituted hypothesis, but rather as a means *to constitute the hypothetical propositions themselves* (Cellucci 1998). This may be done opportunely integrating:

- ◆ The *verificative function* of experience, at the *constitutive level* of hypothesis, according to the procedures of the *analytical method*, and
- ◆ The *falsificative function* of experience, at the *level of the proof* of the hypothesis that have already been constituted, according to the procedures of the *hypothetical-deductive method*.

The falsification of a hypothesis could thus indicate that the time to change such hypotheses has come and, consequently, suggest the need to go back to the constitutive phase, and so on and so forth, following a sort of recursive meta-logical procedure for the foundation of scientific hypotheses that aspires to become *more and more true*, more and more adequate, always within the boundaries of certain given contexts.

To do this, we need to extend our logical analysis to the very propositions themselves, i.e., to the *logic of predicates* and no longer simply to that *of propositions*, as we have done so far with the reference to the hypothetical deductive method. This will have to be done in light of an understanding of the logic of predicates in *semantic terms*, and not only in syntactic ones, as a *content-based* logic (i.e., intensional logic, or logic of properties) and not only as *formal* logic (i.e., extensional logic, or logic of classes; See footnote 83, p. 239).

However, before tackling this issue, it is necessary to clarify another aspect which is crucial for understanding the limits of the hypothetical-deductive method from another viewpoint, that is very relevant to us. We need to analyse, in other words, whether the limits of the hypothetical-deductive method in its relation to the real – limits that we have already acknowledged when dealing with Popper – are only related to falsificationism, or are they intrinsic to this very approach and to its empiricist epistemological background.

4.3 Ontological limits of the hypothetical-deductive method

Ontological limits of the hypothetical-deductive method and the ontological limits of modern empiricism

We have learned from the history of philosophy that the representational theory of knowledge, both in its empiricist (Locke, Berkeley, Hume) and in its rationalistic form (Descartes, Leibniz) as much as in its Kantian transcendental form – as a synthesis of the former two – was well aware that the object of its investigation is not the *reality* (i.e., Kant's

‘thing in itself’), but its *representations* (i.e., sensations for the empiricist, concepts for the rationalist, and phenomena for Kant) within the mind. This is not different for the ontology of the hypothetical-deductive method: there, it is always very problematic to deal with the reference to the extra-mental objects of the names that constitute the extension of predicates, if not the predicates themselves.

Formalist notion of
existence in the
deductive methods

More precisely, within the hypothetical-deductive method, within the axiomatic method and, more generally, within all deductive methods, when the existence of something is affirmed, this does not mean anything else than posing a limit, by way of appropriate quantifiers, on the free variables of the related propositional functions of the logical calculation of predicates. In the terms of classical, scholastic logic, we cannot go beyond the nominalism supported by the classical logicians of the late Middle Ages, from Ockham onwards. This is the fundamental thesis supported by the most famous and appreciated representative of contemporary logical empiricism, i.e., Harvard’s Willard Van Orman Quine.

The ontology of
logical empiricism in
one of Quine’s
works

In one of his works, entirely devoted to the ontology underlying the hypothetical-deductive method (i.e., *Sticks and Stones: or the Ins and Outs of Existence*, Quine 1984), Quine summarizes his ontology according to this scheme:

Ontology of Logical Empiricism			
First Level: Objective Reference		Second Level: Verbal Processing	
<u>Ordinary</u> <u>Ontology:</u> Conditional Expectations	<u>Scientific</u> <u>Ontology:</u> Values of the Vari- ables of Functional Propositions	Logic	Grammar
BODIES 'Lassie' 'Chair' 'Stick'	INDIVIDUALS 'observable' 'unobservable'	TERMINAL PREDICATES single-argumental dual-argumental tri-argumental	WORDS Nouns, particles, conjunctions, rela- tive pronouns
SUBSTANCES 'Milk' 'Dog' 'Stone'	PHYSICAL OB- JECTS 'Organism' 'Electron' 'Particle' 'Field'	PROPOSITIONAL PREDICATES denial conjunction implication	PROPOSITIONAL CONTEXTS Subjects Predicates
ATTRIBUTES 'White' 'Thunder' 'Lightning'	ABSTRACT OB- JECTS 'Number' 'Property' 'Class'	VARIABLES, QUANTIFIERS 'For all x' 'For some x' 'an x such that'	ASSENT DENIAL 'Yes, No' 'Always, Everywhere' 'Everything, Something'
NON-EXISTENT 'Pegasus' 'Unicorn'	REDUCTIVE IN- TERPRETATION 'Class succession' 'Space-time slices'	PROXY FUNC- TIONS 'The f of a...'	TRANSLATIONS 'Number' 'Numero' 'Nombre' 'Zahl'
'The inscrutability of reference'			

Table II: 'The Ontology of Logical Empiricism according to Quine (See Wallace 1996, 315).

Ordinary ontology
and scientific
ontology

According to the principles of logical empiricism, there is no theory about the way symbols *are constituted* in our universe of discourse in relation to the entities that they *denote* (see *denotation**) and/or *connote** (see

*connotation**). On this basis, the only possible approach to ontology is to carry out a logical analysis of our discourse on the world, from a *two-fold* perspective:

- ◆ From the perspective of ordinary language, which produces the world of common sense, i.e., ordinary ontology.
- ◆ From the perspective of scientific language, which produces a more complex universe (i.e., the universe that is studied by science), in other words, scientific ontology.

Since it is *systematically impossible* for the Neo-positivistic approach to produce a theory on the way in which *linguistic symbols* are constituted, in relation to the entities, that – once they are constituted – they make reference to, in both of these cases, the problem of the reference to extra-linguistic objects becomes an *impenetrable problem*, as it is shown in the last row (“The inscrutability of reference”) of the two left-hand columns in Table II.

Definition of the term ‘ontology’ in relation to the term ‘metaphysics’

First of all, let me define what it is meant by the term *ontology*. In general terms, it can be quite hard to distinguish it from the more classical term ‘*metaphysics*’. Both of these two philosophical subject areas have, as a matter of fact, as their object of study a systematic and *demonstrative* (therefore also *scientific*, in the sense specified in Definition 4 at p. 235) of being. The fundamental difference between the modern term ‘ontology’ and the classical term ‘metaphysics’ is usually understood as related to the fact that, while by metaphysics we mean the ‘science of being *qua* being’, by ontology we mean the ‘science of being *in its being knowable*’. Ontology has, accordingly, a stronger epistemological character, which is ultimately related to that distinction between the modern notion of ‘transcendental’ and the classical notion, that we will address shortly (See § 5.6).



Definition 6

By ‘ontology’ we generally mean philosophical investigation concerning the fundamental structures of the being of the entity, which is strictly related to epistemology. The possibility of conceiving of ontology as a science is based on the presumption that, beyond the specific attributes every entity has and beyond each class of entities – on which the

investigations of particular sciences draw – there are shared necessary determinations, which are constitutive of every entity as such.

In particular, within a Neo-positivistic framework, it is possible to recognize a fundamental, or at least preliminary function for ontology, in the sense that the construction of every linguistic system presupposes the univocal determination of the meaning that one wants to attribute to the assertions of existence. It is in accordance with this line of argument that Quine's position needs to be interpreted.

Ordinary ontology and constitutive function of the rules of grammar of ordinary language according to Quine

As to *ordinary ontology*, Quine's position can be summarized taking into consideration the first and fourth columns of Table II. Briefly, Quine argues that during the evolutive phase, thanks to the learning of the grammatical structure of their languages, children learn, in the first place, to name objects, i.e., they are stimulated to attribute a bodily individual nature to such objects of their ordinary experience as 'Lassie', 'the chair', 'the stick'. At a second stage, through the articulation of propositions and thanks to the essential distinction between subject and predicate, they learn to recognize different natures of bodily substances to such objects of their ordinary experience as 'milk', 'dog', 'stone' (e.g., 'Lassie is a dog', 'the chair is made of stone', etc.). The passage to what is the core of the following rational – both scientific and philosophical – discourse happens when, through the learning of grammatical structures that make them possible to produce statements, children gain knowledge of the fundamental metaphysical and meta-logical distinction between substance and accident and, accordingly, between subject and attribute. Children, in other words, acquire the capacity to state the assertion (or the negation) of quantified statements such as 'milk is (always) white', 'some chairs are made of stone', etc. In other words, children learn to distinguish what is essential and *necessary*, from what is *accidental* and, accordingly, *contingent* about the properties of the objects of their ordinary experience. Science and rational discourse, generally speaking, start from here.

Formalist spirit of Quine's ordinary ontology

As can be noticed, within the ontology of logical Neo-positivism, a significant role is played by that Kantian transcendental epistemology that we noticed when we covered the analysis of Wittgenstein's work from a strictly linguistic perspective. What is at work here is the principle according to which the structures of the objects, and even their own metaphysical properties, can be derived from the *a priori* structure of the knowing subjects, and in this case from the linguistic and grammatical *a priori* we learned from the environment. It is not then the *real* (causal)

relations between the objects that ground the logical relations among linguistic symbols, but quite the opposite. If possible, such an issue becomes even more important in the analysis that Quine devotes to *scientific ontology*.

Principles of
Quine's scientific
ontology

Quine's scientific ontology is described by the two internal columns of Table II. At the core of the ontological Neo-positivistic analysis is the test of the semantic value one attributes to the assertions of existence (in this case, to the assertions of existence of scientific language). It is therefore not surprising that Quine's scientific ontology is entirely reduced to the famous maxim: *To be is to be the value of a variable*.

Scientific ontology is thus reduced to the individuation of those logical conditions that give consistency, case by case, to the imposition of a limit, by means of the appropriate universal ('for all x the property $P(x)$ is valid') or existential ('there is an x such that the property $P(x)$ is valid') quantifier, on the variable (x), or on the free variables in a given propositional function. On the basis of such principles, within the domain of scientific ontology, we can distinguish:

- ◆ between different types of *individual* objects, both observable and non-observable (if the related statements are to be individually quantified 'for one x such that ...');
- ◆ between the different kinds of *collective* objects, that are common to several individuals, such as 'organism', 'electron', etc. (if the related statements are to be quantified as collections 'for some x such that...');
- ◆ between the different kinds of *abstract* objects, such as 'number', 'property', 'class', etc. (if the related statements are to be universally quantified 'for every x such that...').

By means of the related 'connectives' or 'propositional predicates' (such as 'not', 'and', 'implies', etc.), the single assertions, thus constituted, become articulated within more complex discourses and even within scientific theories. Through such theories, science builds its interpretations of reality that are normally rather reductive, in the sense that they try to explain what is complicated using simpler notions (e.g., explain 'number' through the notion of class and of 'class of classes', or a physical object as something that occupies a certain 'slice of space-time', etc.).

The capital role of
'proxy functions' in
the scientific
ontology according
to Quine

The last row of the 'logical' column of Table II is particularly interesting. It is devoted to what Quine expresses in terms of *proxy functions*, that are defined as 'the f of...', e.g., 'the f of a dog', 'the f of a first number', etc.. Thanks to these functions, isomorphisms (i.e., bi-univocal correspon-

dences) can be established between objects of ordinary language and objects of one or more scientific languages. For instance, every physical object of the ordinary language can be conceived of as the material content of a certain space-time location. In this case, the proxy function becomes 'the place of'. In this way, every instance of a certain predicate 'the x of P ' (= ' x is a stick') can be replaced by its proxy function ' x is the space-time location of a stick'.

This allows us to replace the ordinary language term, denoting the object 'stick', with scientific terms, related to other connotations, such as 'molecular aggregate', or 'atomic aggregate', or, with even more abstract scientific terms, such as 'field of forces'. All these connotations are in any case characterized by the same proxy function that determines the same space-time position and, accordingly, allows the inter-changeability between ordinary and scientific terms. By means of different proxy functions, characterized by particular n -ples of arguments, we can consequently define more abstract objects, such as, for instance, 'sets', 'classes', 'numbers', etc.

The formalist ontology underpinning the scientific ontology of logical empiricism

As we can see, we are endorsing here a purely *extensional, formalistic* approach to ontology. Looking carefully, enquiring about the objects one person is talking about means, according to Quine, simply to translate his/her terms into ours, so that *the structure of the logical relations* in both of the languages is preserved, without however any possibility of grasping their *extra-linguistical semantic contents*. What matters here is to preserve the *discourse structure*; objects are merely functional to the preservation of these purely formal correspondences. In other words, it is like, facing the crucial question 'what do I make reference to by the term 'stick?', one could only answer, from the perspective, say, of a chemical physicist 'What you make reference to by the term 'stick' is what I make reference to by the term 'that given molecular aggregate'. Accordingly, if a similar question were to be asked by the chemical physicist to, say, an atomical physicist, this latter could answer with something along these lines: 'What you make reference to as 'that given molecular aggregate' is what I make reference to by the term 'that given atomic aggregate', and so on and so forth without anyone being able to overcome that linguistic barrier represented by 'names' in order to investigate what they actually denote from an extra-linguistical viewpoint. This is the impenetrable problem that Quine defined as the problem of the 'inscrutability of reference'.

Neo-positivism can accordingly be seen as inserted within the domain of a *formalist ontology*⁹⁰ of being, i.e., as a Kantian ontology, as we will argue more extensively below. In Frege's terms, within such ontology, claiming that 'x exists' is the same as claiming that 'some x belongs to y'. In other words, a claim about the existence of an object is reduced to the assertion of the belonging of that object to a class of objects and, at most, to a sequence of equivalent classes that are defined in different languages, without ever having the possibility of stepping outside this web of equivalences (See § 4.5.1.). To say it with Quine:

Objects are needed as the mere 'knots' of the structure, and this is as much true for sticks and stones, as for electrons, quarks, numbers and classes (Quine 1984, 24).

Science has indeed only one thing to carry out: i.e., its discourse, its own statements,

That are true statements, we hope; i.e., truths concerning nature. The objects, or the values of variables, only constitute reference points on our way, and we can exchange and replace them as we like, *as long as the statement-by-statement structure is preserved* (Quine 1984, 54).

The problem of the real reference to extra-linguistic objects can only be solved at the level of the creation of the 'true' symbol, and not of the justification of the truth of an already created symbol

The description of Quine's ontology is thus useful to us in view of a precise goal: i.e., to highlight, as we have anticipated at the beginning of this section, that the problem related to the extra-linguistic reference of our assertions – and, accordingly, the problem of 'being', as the object of metaphysics – becomes impenetrable, thus leading to formalistic outcomes both in ontology and in metaphysics, and to nominalistic results both in logic and semantics. This will be the case as long as we follow the modern inclination to tackle the issue of reference as a particular relation between an *already constituted* linguistic symbol and its either intra- or extra-linguistic object (= denotation). Following such a route we cannot but end up with the formalism of an ontology that is reduced to the pure comparison between different speech structures and their correspondences, without any possibility of 'jumping out of the magic circle' represented by the web of languages, going towards the extra-linguistic reality.

⁹⁰ Formalist ontology needs to be distinguished from contemporary *formal ontology*, which pays a great deal of attention to the *content-based* aspects of knowledge and languages, and to the related use of intensional logic, thanks to the influence that Husserl's philosophy and phenomenology in general played on the development of such a subject area, besides a careful consideration of the contributions of medieval logic and of the theory of *suppositiones*. See www.formalontology.it.

Analogy with representational epistemologies at the beginning of the modern era

This is the very same mistake as that made by modern representational epistemology: if we place the *mind* as the foundation of *knowledge*, since we take *evidence* as the criterion of truth, we will never be able to ‘jump out the magic circle’ of mind and directly touch the real. Ultimately, as Pedro Calderón de la Barca acutely noticed, it would not be possible to distinguish between ‘reality’ and ‘illusion’, between ‘reality’ and ‘dream’.

The problem of reference can only be solved if the being of the entity causally enters the constitution of a logical symbol

Vice-versa, the reference problem can produce results that go towards a well-grounded *metaphysical realism*, if and only if the *extra-linguistic object* is inserted within the constitutive process of the symbols of language itself. Only in this way, can we attribute a certain ontological value to the study of the formal correspondences between different kinds of language. If a given symbol is constituted within a certain language in relation to a certain object, and if we can single out a formal correspondence with another symbol within another language, then we truly can constitutively think that they have the same extra-linguistic *referent**, even though the processes of definition of the two different symbols, and of their connotations in their particular languages, are very different.

We shall see in § 5.5.2 how an analogous problem in the ontological foundation of the being of a entity, within Aristotelian metaphysics, drove Aquinas to formulate his metaphysical notion of being as act, and to explain its *participation* in individual beings (= foundation of the being – essence + existence – of the *natural* entity as *ontological* participation of the entity to subsistent being). Such a metaphysical doctrine thus represents the ontological basis Aquinas proposed to solve the same problem that was also addressed by Quine, i.e., the very problem that has been encountered by all nominalistic logics since the Middle Ages: this is the problem of the *meta-logical foundation of the reference* of ‘true’ propositions (= foundation of the truth of the *logical* entity, as *logical* participation of the semantic content – both connotative and denotative – of the proposition to the being of the extra-linguistic referent. Cf. § 5.5.3).

Psychological justification by abstraction vs. meta-logical justification by constitutive induction of truth

Unfortunately, modern Scholastic philosophy, instead of providing a *metaphysical* and *meta-logical* foundation for this process of constitution of symbols in their relation to extra-linguistic referents, suggests a purely *psychological* justification for it. In other words, in order to justify the *reference* and *truth* of linguistic symbols, the *abstract* capacity of the human mind is used. This is an approach that can have either a reassuring or a polemical value with respect to the formalistic ontology that reigns over modern thought. But it cannot go further than this. We cannot expect to ground either an epistemology, or a logics, or not even an ontology and metaphysics on psychological notions. At least, modern transcendentalism – both Kantian and Husserlian – has been very careful not to give in to psychologism, through a sharp distinction between individual

subject and transcendental subjectivity, which is necessarily *meta-individual!*

Wallace himself, despite his pointing the reader and the scholar in the right direction, is not capable of suggesting another justification for metaphysical realism, in terms of both ordinary language as well as scientific language, apart from reference to the *theory of abstraction*. Nonetheless, we can still follow him for a while. With the aim of addressing and solving our problem of the *truth* and *reference* (being) of both scientific and non-scientific languages, Wallace invites us to abandon the hypothetical-deductive method and the logic of propositions, so as to focus on the *analytical method* and on the logic of predicates. This is done in order to study how it is possible to build true propositions, that are *necessarily*, and not only *probabilistically*, true – although we are always faced with *partial, incomplete and perfectionable* truths – on the basis of the *necessity* of the *causal relations* that these very same propositions express in a linguistic form. “That which is necessary is much greater than that which is demonstrable” (*An. Pr.*, 47a, 33-35), as Aristotle enjoyed saying. Logical necessity is grounded on ontological necessity, and not the other way around, as Kant and the moderns have claimed.

4.4 Foundation of necessity

4.4.1 Logical necessity and truth

Formalist
foundation of
necessity and truth
within empiricism
and logical neo-
positivism

As we have already seen, it is typical of the Neo-positivistic approach to the hypothetical-deductive method of modern science to ground the *necessity* of scientific explanations on the *tautology* of the logical-mathematical forms of scientific discourse (logical laws), through which the primitives of formal language (conceived of as simple propositions – linguistic atoms) are logically connected to build complex propositions (linguistic molecules) and, consequently, demonstrative chains of propositions (*corpus* of a theory). Through such chains, it is possible to deduce new propositions (theorems) from the axioms of a theory, and/or — in the case of natural sciences — it is possible to set logical formally correct procedures for the *probable* empirical *validation* (either for the falsification, or for the *verification*) of propositions that are included within the scientific theory.

Independence from
the semantic
content of the
proposition

In this sense, the logical analysis of scientific discourse is limited to the *justification* alone of formally correct procedures of construction, demonstration, and empirical legitimating of propositions, in a way that is completely independent of the *semantic content* of the propositions themselves. This content is envisioned in the two-fold sense of an *intended meaning*, or *connotation*, of the linguistic symbol (i.e., what is meant by a

given term⁹¹ for the relations it has with other terms of language) and of the referent, or *denotation*, of the symbol, through the aforementioned connotation (i.e., the object, being it either a natural or a logical entity). In the terms of classical logic, these considerations are situated within the domain of *formal* (i.e., without content, without any reference to the object), not *material* (i.e., content-based, objective) logic.

Limits to this position for the scientist who does not feel represented by this logical formalism

This poses precise limits on the problem of the *truth* of scientific languages, in its twofold component of *formal* and *material* (or *objective*) truth.

- ◆ *On the formal truth of scientific languages.* The validity of the arguments, their *formal truth* or *consistency*, is absolutely independent of the *material* or *objective truth* (adjustment to reality) of propositions, given the *hypothetical* nature of the demonstrations. The formal truth of an argument, within an hypothetical demonstration, is indeed absolutely independent of the truth of its premise, and even less of its semantic content. In light of this, the formal verification of arguments can be replaced by a mere *decision-making* procedure, e.g., a procedure for the recursive enumeration (that is a binary one, in the case of a logic with only two values; that is tertiary in the case of a three-values logic, and so on) of the different alternatives, on the basis of the rules of the so-called 'truth table' of the different propositional (connective) predicates, for different combinations of values (true/false, 1/0) that are attributed to the different variables. For instance, the formal verification, by means of a decision-making procedure, for the *modus tollendo tollens* of the hypothetical argumentation (2) is the following:

$((p \supset q) \cdot \sim q) \supset \sim p$			
$p/1, q/1$	$p/1, q/0$	$p/0, q/1$	$p/0, q/0$
$((1 \supset 1) \cdot \sim 1) \supset \sim 1$ $(1 \cdot 0) \supset 0$ $0 \supset 0$ 1	$((1 \supset 0) \cdot \sim 0) \supset \sim 1$ $(0 \cdot 1) \supset 0$ $0 \supset 0$ 1	$((0 \supset 1) \cdot \sim 1) \supset \sim 0$ $(1 \cdot 0) \supset 1$ $0 \supset 1$ 1	$((0 \supset 0) \cdot \sim 0) \supset \sim 0$ $(1 \cdot 1) \supset 1$ $1 \supset 1$ 1

⁹¹ 'Term' is understood here, in the sense of formal logic, as 'any expression (sign, word, sentence...) which has a meaning within a given language, whose truth or falsity cannot be affirmed'. By 'proposition' (and therefore by 'statement', in the sense that a single proposition can be stated in different ways, according to different contexts) we mean, instead, 'a meaningful expression that can be asserted as either true or false'.

- ◆ *On the objective truth of scientific languages.* The only way for the hypothetical-deductive method to approximate the problem of the *objective truth* of scientific hypotheses (e.g., of its laws) is the partial use of methods of *empirical corroboration*, which make it possible for the objective truth of scientific hypothesis to be affirmed only in a *probabilistic* sense, and never in a *demonstrative* one. A correspondence is thus established here between the formal necessity of its assertion and a mere probability as to its content, to the specific meaning to be attributed to assertions themselves. Moreover, such a meaning would be limited to the pure *connotative* contents, since a purely *empiricist* approach in epistemology and a hypothetical-deductive one in logic are precluded *a priori* from the solution of the problem of the extralinguistic reference of scientific statements, i.e., the problem of what they really *denote*, as we noticed in §4.3. Significantly, as we saw in Table II at p. 260, Quine speaks about the ‘inscrutability of reference’ in the ontology of logical empiricism.

To consistently deal with the truth of propositions as to their object, we need to move on to the logic of predicates of the analytical method

A fundamental theoretical reason, underlying the serious difficulties experienced by the hypothetical-deductive method in dealing with the – *necessary and not only probable* – objective truth of its assertions, lies in its being concerned exclusively with *propositional logic*. The necessity of its argumentations cannot accordingly affect anything but the relations through which propositions are linked one to the others. If we are interested, instead, in understanding how certain hypotheses are to be considered either true or false in their relation to a given object — i.e., if they are ‘true’ within a certain *context*, so as to constitute the fundamental ingredient for the definition of a ‘true’ *model** for that given formal system, within that context —, it is necessary to take logical analysis within the constitution of the hypothesis itself. In other words, we need to move from the *hypothetical deductive method* of falsification/verification, to the *analytical method* for the invention/constitution of hypothetical propositions that correspond to their object, i.e., that are true, within a certain context. As Cellucci underscored in his work devoted to the analysis of 20th century logic, we need to bravely ask ourselves:



What is logic? Is it only a formal and abstract science [= formal logic], or has it something to do with reality [= material logic]? Is it of some use only to justify ideas that have already been discovered [= axiomatic, hypothetical-deductive method], or can it also be used to create new ones [=analytical method]? (Cellucci 1998, presentation).

After Gödel,
according to Quine,
logical necessity
cannot be kept
apart from an
Aristotelian kind of
essentialism

We need to examine, in other words, the necessity of the logical connection not only between propositions, but also, and above all, *between subject and predicate within the same proposition*. In particular, truth is a property of the assertions, and not of the theories. For this reason, we need to move on from propositional logic to the logic of predicates, and to syllogistics, in particular. We need similarly to abandon a purely extensive approach (formal logic) to logic, and to adopt an intensive, but also content-based one (material logic). To look carefully, W. V. O. Quine himself, in one of his famous collections of essays, was forced, despite himself, to maintain that after Gödel and its theorems of incompleteness, *analyticity* (i.e., the truth of a proposition that can be reduced to the meaning of its constituent terms) and *logical necessity* are no longer the same thing as they used to be in Leibniz's and Kant's times.

Thus, in Quine's view, if we still want to speak of *logical necessity* we cannot avoid making reference to intensional logic — i.e., to be precise, to modal logic — and, accordingly, to the semantic content of certain expressions. Moreover, and most significantly, we cannot prevent ourselves from adopting a somewhat Aristotelian 'essentialism'. In other words, we cannot avoid assuming that certain predicates (i.e., those that denote the 'essential' properties of certain objects) are 'more necessarily' appropriate than others to certain subjects (that denote those objects that are characterized by those properties). As Quine correctly notes, this is due to the fact that, even though certain properties are analytically derived from the way of determining (i.e., connotating) a certain object — just like some others are analytically derived from other ways — only some of them *necessarily apply* to the object that is considered. For instance, according to Quine, we could connote number '9' by means of two equivalent expressions, that are nevertheless very different in terms of their contexts of connotation:

$$x = \sqrt{x} + \sqrt{x} + \sqrt{x} \neq \sqrt{x}$$

'There are exactly x planets in the solar system'

Only the former has as its necessary consequence that ' $x > 7$ ' since such a property necessarily applies only to numbers. Conversely, this property does not apply at all to the second way of connoting '9'. 'Number of planets' does not certainly imply that such a number is bigger than 7, while the first way to determine '9', as a number coming out of a certain arithmetical operation, *necessarily implies* the possession of such a property.

However, instead of taking such an Aristotelian turn, or being tempted to endorse intentional logic, Quine prefers to renounce logical necessity and limit himself to analyticity (See also Quine 1986, 20ff.).



The causal explanation of essences is the *proprium* of Aristotelian philosophy of nature, at the basis of the foundation of logical necessity resting on causal necessity

The defence of Aristotelian essentialism (...) is not among my goals. Such a philosophy is so unreasonable to my eyes, as it is to Carnap's and Lewis's. But, unlike Carnap and Lewis, my conclusion, is: so much the worse for modal logic (...). As a matter of fact, if we do not aim at quantifying by means of the necessity operator, I cannot see what the advantages are of that operator compared to simply affirming that a certain proposition is an analytical one. (Quine 1986, 145).

It is, instead, essential to us to provide a foundation for *logical necessity*, even if after Gödel — but consistently with pre-modern logical thought — we no longer believe in the myths concerning the foundation of ‘absolute’ necessities, or truths. Accordingly, we are not negatively struck in learning that a renowned logician as Quine, who is certainly not an Aristotelian scholar, has arrived to the same conclusion (i.e., without making reference to Aristotelian essentialism, we cannot found logical necessity in our time) as we have; despite the fact that we do not really look at Aristotelian essentialism as ‘unreasonable’. As we will see in greater details in the next chapters (See § 5.4.4), Aristotle defends a *causal foundation* of essences in ontology and of logical necessity. Properties and qualities do not denote, accordingly, objects, but rather the *actions* of the agents they are referred to (*forma ut actus*). The ‘whiteness’ of milk is thus an action: i.e., the ‘whitening’ action, or, in this specific case, the action of provoking in the sensory apparatus of both human and animal brains the sensation of white⁹². In turn, the necessary appropriateness of certain qualities for certain objects - i.e., their being ‘essential’ properties for that object - from which the *immediate necessary belonging* of certain predicates to certain subjects derives — and hence the *analyticity* of the consequent propositions is grounded — is based on the fact that such properties are ontologically the result of the same causal action, through which the object that possesses such properties passes to existence. Without such actions, there would be no object and, more precisely, there would not be ‘such an object’. As far as a property is related to the effect of a causal action that is *necessary to the existence* of that object, the resulting property will *necessarily* be *bound* to that object.

⁹² On this point, see the valuable analysis suggested by (Henry 1972, 10 and 88-95). See also (Basti 1997, 89ff.).

Existence of the logical entity as constructed by the mind, freely and only restricted by logical laws in mathematics, but also restricted by the reference to an extra-linguistic referential object in physics and metaphysics

Incidentally, this also perfectly explains Quine's example: the property ' $x > 7$ ' necessarily applies to number '9', since it is connoted by means of the arithmetical operation that is the cause of it, i.e., that makes it exist. This interpretation is perfectly consistent with that suggested by the philosophy of mathematics, according to which numbers — and, in general, the *logical entities of mathematics* — only exist as the result of the operations performed to calculate and to logically constitute them; all this with the only constraint of not violating the first principles of logic, the p.o.n.c. in the first place. This is precisely the Aristotelian and Thomistic (radically anti-Platonic and implicitly *constructivist*) sense — that is incidentally very close to the philosophy of mathematics underlying A. Church's λ -calculus — in which the existence of the logical entity in mathematics is to be conceived (See § 5.3 and § 5.4). This is to be distinguished from what happens in the definition of logical entities within *physical and metaphysical subject matters*, where the logical entity itself (i.e., a concept or a statement) is constituted by the mind, but *cum fundamento in re*, i.e., it is grounded on the properties of *extra-mental objects*.

If for mathematical entities the reference to 'causal explanation' can be understood only in an analogical sense, this is not at all the case for the essences of natural and of physical entities that are studied by philosophy of nature. For instance, it is evident that a given genetic set that is composed of 46 chromosomes, and that is different for each individual, is an essential, characterizing property for every human being. Now, the set of causal actions that we define by the collective term 'act of conception' is what determines both the human individual in its concrete corporality and his/her genetic set.

Analytical method, constitutive induction and syllogistic demonstration of true propositions

The analytical method, accordingly, is the core of logic and of Aristotelian epistemology, and it represents the largest contribution that this latter can offer to the current post-modern discussion on epistemology and on the logic of physical science. In particular, within the analytic method, a central role is played by the procedure of *constitutive induction* of *analytically true* propositions. This is the so-called *inventio medii*, i.e., 'search for the middle term' of syllogistic procedures, which are able to constitute the last and decisive *middle term* for procedures of a syllogistic kind. In the natural sciences, such syllogistic procedures will at first be *inductive* (i.e., non-demonstrative) ones: they will be built on *singular* terms and their conclusion will be an analytical statement of a *causal kind*. In Aristotelian epistemology, following an appropriate generalization in terms of a causal law, it will represent the major premise for deductive (i.e., demonstrative) syllogistic procedures. However, in the epistemology of modern sciences, such a statement, if interpreted in a rigorous extensional sense (e.g., if it is translated into '*operational*'*, mathematical-

quantitative terms), will constitute in turn the axiomatic proposition of a formal hypothetical-deductive system – belonging to a scientific theory in the modern sense of the term – according to the law of *predicate calculus** (Bochenski 1995,110):

$$(\exists x P(x) \supset p) \equiv \forall x (P(x) \supset p)$$

Complementarity of
induction and
deduction

Aquinas highlighted the complementary character of *induction* and *deduction* as the two main forms to acquire science. He also underscored that, according to the Aristotelian-Thomistic conception in opposition to the modern formalist approach, deduction can increase knowledge, thus confirming from this perspective the ‘open’ nature of the logical systems within such logical theory.



The way to acquire science is twofold: one way is represented by *demonstration*, and the other by *induction*, as it has been stated since the beginning of this book. These two ways are nonetheless different, since demonstration proceeds from universals, while induction starts from particulars. Accordingly, if the universals from which demonstration proceeds could be known without induction, it would follow that man could directly acquire science of what cannot be known through sensations. *However, it is impossible to speculate on universals without induction.* (Thomas Aquinas, *In Post. An.*, I, xxx, 252).

Inventio medii and
inductive syllogism
in Aristotle

Let us consider now what Aristotle meant by ‘inductive syllogism’. Technically, if we define by S, P and M, respectively, the major and minor terms and the middle term of a syllogistic procedure, the non-demonstrative inductive syllogism, according to Aristotle, consists of ‘deriving through one of the extremes the belonging of the other extreme to the middle’ (*An. Pr.*, II,23,68b,15), in a way that the resulting proposition can be considered as the *major premise* of a deductive, demonstrative syllogism, of a *causal* kind, that is typical of physical and metaphysical sciences. A causal syllogism is not limited to the justification of the ‘that’ of a certain conclusion — as it is the case for the syllogisms that are used to construct definitions —, but it aims at justifying its ‘why’, i.e., the cause of the event that is denoted in the conclusion, which is thus ‘explained’. It is clear that, since Aristotle provides a causal justification of essences, all the definitions of the essence (*quidditates*) presuppose an explanation and, hence, a causal syllogism. Going back to our inductive syllogism, by means of symbols, taking ‘&’ as the symbol of the logical product, and ‘ $\rightarrow$ ’ as the symbol of implication, the form for the inductive syllogism is the following:

$$“S \text{ is } P \ \& \ S \text{ is } M \rightarrow M \text{ is } P” \quad (1)$$

Condition for the convertibility of the middle term: non-equivalence with one of the extremes, yet inclusion

For instance — to use the purely fictional example suggested by Aristotle — ‘man, horse, mule, ..., i.e., a certain collection of animals (S) has no bile (P), but the (S) are also long-lived (M), hence the long-lived (M) have no bile (P)’. The conclusion of such syllogism, which should show in the terms of a law that the condition of ‘having no bile’ is one of the causes of longevity, is valid if and only if we can suppose that the collections related to the predicates S and M are somehow equivalent: $S \equiv M$. This is a rather odd equivalence for the modern logicians, considering that Aristotle poses, as a condition of convertibility, a relation of inclusion between **M** and **S**: $S \supseteq M$. In other words, to say it in Aristotle’s terms, if and only if we can suppose that

S is converted with M and the middle (M) is *not more extended than* S, in this case it will be necessary that P be inherent to M. As we have indeed proved earlier, if two predicates belong to the same object, and if such an object, that is one of the extremes, is converted with one of the two predicates, the other predicate, then, will also belong to the predicate that has been converted. Moreover, we need to take S as constituted by single objects: induction is indeed constituted through the totality of single objects (*An. Pr.*, II, 23, 68b, 23-29).

It means that the classes are not conceived of as closed but as augmentable

In other words, the conclusion of the inductive syllogism (1) will be analytically necessary if and only if we can suppose that ‘being long-lived’ (M) is a typical characteristic of the collection of animals that are listed as (S), regardless of its range (see the condition according to which M must not be more extended than S), so as to transform the form (1) of our syllogism — which is inherently incapable of justifying the analyticity of the conclusion and that, in the way it is written, can only justify that ‘*some* M is P’ (Mignucci 1969, 702; See Basti 1997, 79ff.) —, in another one, i.e.,

“S is P & M is S $\rightarrow$ M is P” (1a)

which is necessary to guarantee the analyticity of the conclusion and, accordingly, the possibility of a generalization in terms of a universal law: ‘*all those who are long-lived have no bile*’.

In this case, the conclusion, that is properly generalized in terms of a universal statement, can be the premise of a deductive causal syllogism of a demonstrative kind, which will accordingly assume the ordinary formulation of the one *in Barbara*:

“M is P & S is M $\rightarrow$ S is P” (2).

where the belonging of one extreme to the others, through the middle term, is shown. As we can notice from the comparison of the two con-

sidered syllogistic forms – on the one hand (2), and on the other (1)/(1a) – the formal key of the movement from one to the other one is the subject-predicate *convertibility* between S and M. Such a relation can be extensionally justified through the supposition of equivalence between the class **M** and the class **S**. In this case the minor premise would not only be analytical, but also tautological. However, the inductive syllogism would be valid only through the supposition of the complete enumeration of the two classes, which would make the constitutive induction equivalent to the demonstrative one, thus making the analytical method a subset of the axiomatic one. This is the *extensional* interpretation of the inductive syllogism, as it is usually presented by modern logicians.

The interpretation given by Thomas Aquinas: intensional justification of convertibility through the adaptation S-M

Aquinas' suggestion is instead — as we will see in greater depth in § 4.4.3, pp.281ff. — to *intensionally* justify, from a content-based point of view, the aforementioned relation of double implication, through a mechanism of specification, of *reciprocal subject-predicate adaptation*. In order to understand the core of this alternative interpretation, let us have a closer look at the issue.

The convertibility of the minor term of the inductive syllogism shows that the *inventio medii* procedure has reached its apex

Let us take into consideration the two syllogistic forms, i.e., the inductive one (1a) and the causal–deductive one (2): both of them share the same minor premise, but in inverted terms, i.e., respectively (M is S) and (S is M). The subject predicate convertibility lets us know that we are facing an analytical statement that expresses *an essential property (quidditas)* of the subject S in terms of the conclusion. More precisely, the minor premise of the two syllogisms is an analytical – non-tautological – statement that is capable of univocally connoting a species of entities (or, ‘universal one-to-one’), just like when, for instance, ‘mammals’ are connoted like ‘viviparous’, or ‘reptiles’ like ‘cold-blooded animals’. It is exactly such a convertibility that guarantees that M is the middle term sought by the *inventio medii* procedure; M is, in other words, the predicate that belongs to the subject S without any further mediations.

In Aquinas's interpretation, induction is based on analogy

According to Aquinas' interpretations, then, the convertibility of M with S, that is at the basis of the validity of the syllogism, is grounded on an attribution to the individuals S of the predicate M, which is not *univocal*, but rather *analogous* (See § 4.4.3, pp. 281ff.). For instance, in our case, this would mean that the longevity we are talking about is *exclusively* the one that is related to the animal species we have taken into consideration⁹³ and, *vice-versa*, the animal species are considered *exclusively* in relation to the characteristic of longevity (See the condition according to which M, i.e., ‘being long-lived’, is not more extended than S, i.e., than the consid-

⁹³ The “longevity-of-the-horse” is not the “longevity-of-the-mule” and vice-versa.

ered animal species), just like in the major premise we considered them in relation to their 'having no bile'. Such an organic characteristic ('having no bile'), since associated to the totality of the finite collection of individuals we have taken into account, can be taken as the condition (and, hence, as the 'cause' for the other), as it is affirmed in the conclusion of the inductive syllogism.

In sum, in the inductive constitution of the proposition 'M is P', that is the conclusion of the related inductive syllogism (1*a*), it is as if the bi-univocal correspondence (i.e., convertibility) between the elements of the two considered classes, **M** and **S**, in relation to which the aforementioned conclusion is to be validated, would be redefined every time, in relation to each element of S, since each of them has its own *analogous* way of being M, i.e., of 'being long-lived', as they all have their own *analogous* way of having P, i.e., of 'having no bile'. In a word, S and M convert thanks to the non-convertibility of S and P. Therefore, because of this *proportionality* between M and P through the elements of S, we can arrive at the generic conclusion: 'every M has P', that is the causal law according to which 'every long-lived animal has no bile'. This can be the major premise of an indefinite series of 'scientific' demonstrations that are based on the form (2) of syllogism⁹⁴.

Complementarity
between inductive
and deductive
syllogism

Such an interpretation can explain, far better than the modern interpretation, the conclusion drawn by Aristotle at the end of his illustration of the inductive syllogism :

On the one hand, the syllogism that is constituted through the middle term [i.e., deductive syllogism] comes first by its nature and it is more evident. On the other hand, the syllogism that is developed through induction is to us the richest in knowledge (*Post. An.*, II, 23, 68b, 35s.).

Finally, let me make one last point, which we will investigate better in Ch. 6 (Cf. § 6.3.2.2), that links the Aristotelian approach to inductive syllogisms to contemporary epistemology. According to Aquinas, demonstrations within physical theories — unlike metaphysical and mathematical ones — are, generally speaking, of a *hypothetical* non-apodictic kind. Such an epistemological doctrine is deeply linked to the Aristotelian theory of the four causes, within his ontology of physical entities. The emphasis that modern Scholastics have placed on the syllogistic and on the apodictical nature of its demonstrations, have forgotten that such an apodictic character, in Aquinas's view, was recognisable only in meta-

⁹⁴ For further explanations, see (Basti 1997, 80ff.). See also (Cellucci 1998, 360ff.), who underscores the centrality of an induction based on the analogy between the *analytical methods* of discovery, which can be, and actually are, fruitfully used in scientific practice.

physical and mathematical demonstrations, never in the natural sciences. This represents, in broad terms, a contribution to post-modern epistemology and logic, that follows from the more specifically ontological one on the relevance of the Aristotelian scheme to the justification of the causal determination in the study of complex physical systems (See § 2.7.2).

From enumerative
to constitutive
induction

Recalling the terms we have previously used in this chapter, we can say that we need to move from the tyranny of the 'logic of proof', which is typical of the axiomatic and hypothetical-deductive method, to the 'logic of the invention /discovery', which is instead typical of the analytical method. More precisely, we need to abandon the *demonstrative use of induction* – or *enumerative induction* – of empirical evidence that is produced as a justification of already constituted hypothetical propositions, since this can only lead us to reach hypothetical conclusions, of a probabilistic kind, about the truth of the propositions themselves. We should, instead, make a *constitutive use of induction*, or better make use of the *constitutive induction of propositions*, which can work, within certain contexts (recall here the experimental nature of laws in natural sciences), as the *necessarily* – and not merely probabilistically – *true* premise for deductive procedures of a syllogistic kind (i.e., categorical, or demonstrative – not merely hypothetical – *syllogism**). Such propositions, once they are interpreted in a rigorously extensional sense (e.g., once they are defined in a rigorously quantitative way by means of suitable operations of measurement)⁹⁵, can in turn constitute the *hypothetical* premises for the deductive formal systems of the logic of propositions, within certain *scientific theories*.

From causal to
logical necessity

In this sense, in natural sciences, the necessity of the *logical* connection between the terms subject and predicate of the considered true proposition is grounded on the necessity of a particular *causal* connection (See *infra* § 4.5.3, pp. 299 ff.). Let me now make an intuitive example of the difference between an argument of a strengthening kind for a true proposition, within a prepositional calculus and within the hypothetical-deductive method, and its syllogistic counterpart (demonstrative syllogism) within the logic of predicates and within its analytical-deductive method.

⁹⁵ In the epistemology of modern sciences, this is what is referred to as the *operationalization** of a theoretical concept (such as 'color') in terms of an operational concept, that can be measured and/or calculated (e.g., 'electromagnetic radiation wave-length').

4.4.2 Causal connections and truth

The consequences
of constitutive
induction based on
causality for
propositional logic

The importance of the study of causal connection in the foundation of the truth of propositions is linked to the fact that, in specific cases, they allow the transformation of a hypothetical reasoning that is based on material implication (*if...then*) into one that is based on double material implication (*if and only if...then*). This latter, from the point of view of propositional logic, allows the foundation of hypothetical procedures that are in need of verification, and not only of falsification. (See the arguments (6)-(8) at pp. 250 ff.). Let us see, in detail, what this all means following an example presented in Wallace's book (Wallace 1996, p. 281ff.).

Wallace's example deals with a quantitative (i.e., dimensional, geometrical, and not numerical, or arithmetical) property of the earth: i.e., its being (approximately) spherical.

Exemplification of
the use of
experimental
evidence in a
hypothesis-
strengthening
procedure

The hypothetical-deductive formulation of the process of probabilistic corroboration of the truth of a hypothetical proposition like 'the earth is spherical' would thus have the form of the following argumentation, which we have already presented at p. 252.

If (p) the earth is a sphere,

then (q_1), such observations as the shape of a ship that goes towards the horizon, and the fact that certain constellations can be seen only from certain points on the surface of the earth, besides the existence of precise geodetic measurements, would reveal that they have been made from a spherical convex surface; **moreover** (q_2), the earth would project a circular shadow on the surface of the moon during a lunar eclipse; **moreover** (q_3), bodies will gravitate perpendicularly to the earth at any point of its surface.

But (q_1), the abovementioned observations *actually reveal* that they have been made from a spherical convex surface; **and** (q_2) the earth *does really project* a circular shadow on the moon during lunar eclipses; **and** (q_3) bodies *actually gravitate* perpendicularly to the earth along its entire surface.

Thus (q_3) the earth is *probably* a sphere.

Shift to a
constitutive use of
experimental
evidence in the
logic of predicates

As we have already noticed, despite the fact that the majority of scientific argumentations cannot guarantee their truthfulness except in a probabilistic sense, in such cases as this one, we can get to the reformulation of our initial proposition p so that its truth would appear to be demonstratively *necessary* and not only *probable*. In other words, its truth would not seem to be the conclusion of a probabilistic corroborative argumentation of a given hypothesis, but rather the conclusion of a de-

The syllogistic
scheme of
demonstration in
the logic of
predicates

monstrative syllogistic argumentation, which accordingly *needs* a logic of predicates. Let me recall here that a demonstrative syllogistic argumentation (categorical syllogism) is bound to the demonstration of true propositions, i.e., to the demonstration of the *necessary inherence* of a certain predicate (P) to a given object (S) through a middle term (M).

Within the ordinary logic of predicates, a categorical syllogism (e.g., *In Barbara*, or of ‘first mode of the first figure’) is characterized by the fact that the necessary inherence of P to S in the conclusion is demonstrated building on the necessary inherence of P to M, in the major premise, and of M to S, in the minor premise. The categorical syllogism according to the *In Barbara* model assumes such a prescribed form:

Every M is P
every S is M
thus, every S is P

More concisely:

‘S that is M is also P’

That is:

‘S is M is P’

In our case, we will end up having a three-party syllogism:

Every M_1 is P
every S is M_1 ,
Every M_2 is P
every S is M_2 ,
Every M_3 is P
every S is M_3 ,
thus every S is P.

More concisely,

S is M_1, M_2, M_3 is P

That is to say, as we can see in the following table:

S	M	P
The earth	M₁ Is a body from which the observations conducted, both on the earth and in the sky, reveal that they are made from a spherical, convex surface	
	M₂ Is a body that projects a circular shadow on the surface of the moon during eclipses.	
	M₃ Is a body around which all the other bodies gravitate perpendicularly at any point of its surface.	is a sphere.

Table III. Scheme for the demonstrative (categorical) syllogism as the attribution of a predicate P to a subject S, through one or more middle terms M.

The best-prepared readers would have certainly noticed that Wallace's example is both effective and weak at the very same time:

Effectiveness of the intuitive example

- ◆ It is effective since it shows how using the same experimental evidence – within a scheme that is constitutive of the proposition in the domain of the logic of predicates – we can arrive at the justification of a given statement in a necessary form, also when we make use of a finite number and, even, only one experimental piece of evidence. The necessity of the link between subject and predicate is not founded, indeed, on an infinite – and hence always only probable on a finite scale – enumeration of different experimental proofs. It is instead grounded on necessary mathematical laws that are bound, in this specific case, to the theorems of projective geometry and/or to the transmission along a straight line of rays of light on relatively short distances in geometric optics. In this case, even one single experimental proof of a cause-effect kind – that has, that is to say, in the roundness of the earth its cause and that can be thus related to the aforementioned mathematical laws (theorems) – is sufficient to justify the necessity of the attribution of 'be-

ing a sphere' to the earth and, accordingly, the necessary truth of the statement 'the earth is spherical'.

Weakness of the intuitive example, since it does not make explicit the constitutive induction scheme, which lies beneath any physical and mathematical law.

- ◆ It is weak since, at least in the formulation offered by Wallace, it does not say anything about the fundamental problem that would attribute a real demonstrative value to the procedure that is outlined here. This is the problem of how a formal law of a hypothetical-deductive kind – like, for instance, one or more theorems of Euclidean geometry – are applicable to the singularity of individual physical entities/events, which are objects of experimental control. From a formal perspective, indeed, the scheme of the syllogistic argumentation that is proposed by Wallace follows the pattern of the first figure of the first mode syllogism, *In Barbara*, which assumes that all terms are universally quantified. In fact, according to Wallace's application – as it generally happens with any other experimental application of a physical kind of a mathematical formal law (= physical-mathematical law) – this scheme no longer follows the traditional Barbara's model that is characterized by universal quantification. It is, instead, extended to such terms that are *singularly* – and not only particularly, as it is with the *Darii* mode – quantified, as experimental physical entities or events. Such an inductive extension of the Aristotelian syllogism was studied, in particular, by Medieval Scholastic logic (Bochenski, 1956, vol. 1).

Let us have a closer look at a question whose analysis will lead us back to what we have already characterized above as the Tomasian, intensional interpretation that is based on the analogy principle of the Aristotelian inductive syllogism (See § 4.4.1, especially pp. 274 ff.).

4.4.3 Analytical method and constitutive induction

Inductive extension of the universal syllogistic laws to classes of single individuals

The syllogistic scheme that was actually used by Wallace in the previous example is not, in fact, the one he presented:

Every M is P
every S is M
thus, every S is P

as in the well-known case:

All men are mortal
all Greeks are men
thus, all Greeks are mortal

Its inductive extension applies to a potentially infinite domain of *single* cases, among which one is chosen:

Every M is P
a Greek-individual S is M
 thus, *a Greek-individual* S is P

As in the well-know case:

All men are mortal
 Socrates is a man
 thus, Socrates is mortal

As a matter of fact, in all of Wallace's examples on the foundation of the necessity of a subject-predicate link for a *true* proposition that has as its object the property of a physical entity, its causal syllogism is based on an extension to the single physical entities of a universal mathematical law (= physical-mathematical law). Consequently, the scheme that is actually used is not a traditional *Barbara* one, which is characterised by universal quantification, but rather an inductively extended *Barbara* model.

In each of the three syllogisms related to the three middle terms that were used, we end up having a syllogism with:

- ◆ A major premise in the form of a universal mathematical law, since it is implied as a theorem by the hypothesis of Euclidean geometry;
- ◆ A minor premise given by the inductive extension on an experimental basis of such a law, to a possible infinity of physical bodies and, in particular, to the single case of that physical celestial body that is the planet 'Earth'.

For instance, in the case of the middle term M₂, we will have:

Every solid body that projects a circular form on a surface is spherical
 The earth projects, during an eclipse, a circular shadow on the moon
 Thus, the earth is a sphere

In a non-formalist or 'content-based' ontology of being, the inductive extension of the syllogistic laws does not require a full induction

Now, like the Moderns argue following Hume, if to obtain such an inductive extension of the *Barbara* we need a *complete enumeration* of the infinite class of elements that share the property M, it is obvious that the same problems we have in the justification of a procedure of verification within the hypothetical-deductive method of propositional logic, would also emerge within the analytical method of the logic of predicates (See *supra*, pp. 274ff.).

But this is exactly what the elaboration of such a problem, suggested by Medieval Scholastic logic, helps us avoid. This is particularly true for the

elaboration that was developed by Thomas Aquinas, thanks to his *intensional* – and not only *extensional* – conception of ‘being’, i.e., *being as act* (*esse ut actus or actus essendi*), and to its application in logic to solve problems concerning reference (See §§ 5.5.2 – 5.5.3). Thanks to this notion, it is possible to give content to the idea of *existence* in terms of the being of an *entity* — i.e., the being of material logic, or of the ‘logic of contents’ (*Inhaltlogik*), as it is characterized by the formalist notion of existence of modern formal logic. Being allows the inductive extension of formal predicates (e.g., mathematical predicates, since mathematics is the ‘formal science of essences’), making them progressively richer in content and, accordingly, in individual instances that actualize them concretely (e.g., the physical-mathematical arguments of those predicates. See Basti 1997; Perrone 1997). As Aquinas states:



The power of the intellect to comprehend is *somehow* (*quodammodo*) infinite. It indeed infinitely comprehends types of numbers, through incrementation, and similarly types of figures and proportions. It also knows the universal that is *virtually* infinite *according to its domain* [i.e., not in an absolute sense]: it contains, indeed, individuals that are *potentially infinite* [i.e., undefined] (*S. c. Gent.*, II, 49, 1250).

Virtual existence of individuals in the generic unending domain of a universal predicate

The fact of *virtually* containing individuals means that a universal cannot actually contain an infinite number, in accordance with a *domain* that is *a priori* defined. Such a domain can contain infinite *generic* elements, but not individuals: i.e., it contains infinite men, but not Socrates, Alcibiades, Jack, etc. The use of the universal quantifier, that was well known also by Aristotle, means exactly this. The virtual knowledge of an infinite number of individuals means that in order to *actually* include a new individual as such in the domain of a generic predicate, we need to apply step by step some kind of intensional inductive rule for the redefinition of the generic predicate to its individual occurrences (analogy).

The inductive extension to the individuals of universal premises of syllogisms is the integration between analytical procedures of discovery and axiomatic procedures of proof

Accordingly, if we repeatedly make use of an intelligent alternation of axiomatic (extensional) methods on the *generic* domains, and analytical (intensional) methods to define, in a more and more precise way, the fields of *actual* (i.e., calculable, and thus constructively *true*) application of the universal, we can integrate universal *necessity*, which is only possible over infinite but generic domains, with the *actuality* of the individual application that is possible, by definition, only on finite, at most unitary, sub-domains of the generic domain. To demonstrate (and/or to calculate) something in universal terms over generic domains can be both useful and interesting for *abstract* mathematics and logic, but it has no use for *concrete* applications. Keeping these two dimensions separated only leads to logical and meta-logical anarchy in the applied sciences — i.e., those

that have an influence of the real life of individuals and societies — and to the sterility of logical and mathematical *theoretical* disciplines.

This integration is known as the analytical procedure of the *inventio medii* within the axiomatic procedure of syllogistic demonstration

This integration between analytical and axiomatic methods, as is well known, is what Aristotelian Scholastic logic meant by the procedure of the *inventio medii*, i.e., of the search for the middle term within the deductive syllogistic procedure. Let me recall here Aquinas' definition of the analytical procedure of the *inventio medii*, which was conceived of as a procedure that ends by an *analytical* proposition in which subject and predicate immediately belong to each other. This cannot be but a proposition in which, through the predicate, the *quidditas* of an entity, identified by the subject of that proposition, reveals itself (which is obviously different from its being defined through it). In the appropriate terms of classical logic, the universal by which the procedure ends is a 'universal *one to one*', i.e., the univocal connotation of an object. Only these kinds of propositions, according to Aristotle, can be *analytically true*, without being tautological — we should remember here that we are inserted within a context of material – not formal – logic.⁹⁶



In order to infer a given conclusion, we need two propositions, i.e., the *major* and *minor* terms, since once we know the major proposition we still know nothing about the conclusion. The *major* proposition needs to be known in advance, not only by its own nature, but also from a temporal point of view. Subsequently, if, in the *minor* proposition, we *induct* and/or *assume* [note here the difference between the induction and the assumption of a specific particular within a generic universal] something that is included in the universal that is the major proposition – but that is not patently included in such universal – we do not have any knowledge yet of the *conclusion*, since the truth of the minor proposition is not certain yet [in other words, from the proposition 'all men are mortal' we cannot deduce that 'Socrates is mortal' until when, in the minor premise, we have determined that 'Socrates is a man'. But in order to affirm this on the sole basis of the major proposition, I should have had the *whole* class of men enumerated]. Conversely, if in the *minor* proposition we assume a term that is patently included in the universal of the *major* proposition, then the *truth* of the *minor* proposition becomes suddenly clear [i.e., we have found a solution to the problem]. What is included in the universal is indeed knowable, hence we also have an

⁹⁶ Obviously, 'analytical proposition' is conceived here in its proper sense, which is not synonymous with tautology. It is rather the sense that was also used by Carnap, as a 'proposition whose meaning is reduced to that of the (primitive) terms that make up such a proposition' (Cf. the passage commented on in footnote 98).

immediate knowledge of the conclusion. (...) Suggesting an example of those things that are known even *temporarily* earlier than the conclusion [i.e., the major propositions of every syllogism], Aristotle affirms: let us suppose that someone, following the knowledge of a conclusion from a previous demonstration, knows beforehand the following proposition: every triangle has three angles that are equal to the sum of two right angles. Now, through the *induction* of this further assumption, i.e., that this [single] object that is inserted in a semicircle is a triangle, he/she would also simultaneously know the conclusion, *since this inducted object possesses, on its own, the universality in which it can be inserted so that there is no need to look for an additional middle term* [as a basis for its belonging]. (...) It is indeed the term of a procedure of resolution [= *analytical* procedure] since the mediated propositions are always reduced to the immediate ones. This can also be read in the following way: the last *term* [i.e., the subject of the minor premise in a demonstrative syllogism] that is included in the universal middle term [i.e., the predicate of the minor premise] does not imply that it is known to be included in that universal, through some other middle term. At this point, [Aristotle] *says what are those things that imply the immediate knowledge of their universal: things of this kind are only the singular predicates* ['universal one to one', or 'singular connotations'] *that cannot be attributed to any other subject, because there cannot be any other interposed term between the singulars and their species* (*In Post. An.*, I, ii, 21. Emphasis mine)⁹⁷.

Universal one-to-one as the term of the *inventio medii*

Let me now quote another passage that shows what Aquinas means by referring to those 'singular' terms to which universals 'one to one' can be attributed. They are individuals or essences that are connoted as subjects of assertions that express *quidditates*. In other words, they are exactly what we would need to justify an inductive, non-demonstrative syllogism, where the crucial point of the *inventio medii* seems to be the expression of the *quidditas* of the so-called middle terms, through an appropriate analytical proposition.

We need to consider that something is said to be *single* (*individuum*) on the basis of the fact that it cannot be multiple by its own nature. Now, something cannot be multiple by its own nature in two ways:

a) In one way, because something is determined in relation to something unique in which it exists, as 'whiteness' that by definition of its species must exist (by nature) as multiplied in many, even though such whiteness that is received in a certain subject [e.g., the whiteness of snow] can not help but being in it [e.g., this

⁹⁷ One can surely notice here the proximity of this Aristotelian example, discussed by Aquinas, to the example suggested by Wallace for the inductive extension of the properties of a generic spherical object to the roundness of the earth.

is not the same whiteness as the one that characterizes milk]. According to this reading, though, it is not possible to go on infinitely, since we cannot infinitely proceed with material and formal causes, as Aristotle showed in the second book of *Metaphysics* (2, 994a, 20ff.; 994b, 17 ff.).

b) It is hence necessary to reach something unique that cannot be received, by its own nature, in something from which it receives *indivualization*, like when something is received in prime matter (materia prima) that is the principle of individuality (*singularitatis*) for bodily things. Therefore, it is necessary that everything that, by its own nature, cannot be in something is for this very reason an *irreducible single* (*individuum*).

And this is the *second way* in which something cannot be multiple by its own nature: *this is because it cannot be, by its own nature, in anything else*. It is like whiteness would be (considered as) separated, without an existing subject, and would be for this viewed as a singularity (*individual*). It is according to this mode, that singularity (*individuatio*) is to be conceived of also in separated substances [the ‘angels’] that are forms having their being in the First Cause, that is the subsistent being itself (Thomas Aquinas, *In de Caus.*, IX, ix, 235).

Constitutive induction as a procedure of assumption of the individual within the genre

Now, what Aquinas defined in the previous quote from the *Posterior Analytics* as ‘induction’, i.e., the procedure of attribution of a generic predicate (e.g., ‘being a triangle’) to an existing individual object (e.g., *this* polygon inserted in a semicircle that is drawn on this piece of paper), is simply the procedure of re-definition of the generic predicate on the singularity of an individual, whose existence can be *independently* proved and/or supposed, so as to avoid any risk of impredicativity. In other words, we are not faced here with an individual that exists as a *generic* element of a domain that is characterized by universal quantification, according to the well-known law of the logic of predicates, $\forall xP(x) \supset \exists xP(x)$: i.e., ‘if a property *P* is valid for a totality of generic elements, it is also valid for at least one generic element of that totality’.

Metaphysical key of the principle: the proper being of the subject is independent of the being of the essence, of the belonging to one class.

This kind of existence reduces the being to its mere belonging to a certain class: it is, that is to say, the being of a Kantian-type copula. *Vice-versa*, the being to which Aquinas makes reference is independent of the essence, of the belonging to a given class. It is, instead, the *proper* content-based, intensive (and not extensive) being; it is not bound to the extension of predicates, of the Aristotelian *tode ti*, of the subjectivity of the first substance (see § 5.4.2). It is the being that, within Thomistic metaphysics, will become *being as act*, which is dependent on the causal action of the First Cause, as distinct from, despite its being the basis of, the being of the essence. This is the core of Thomistic metaphysics,

which we will study in Chapter Five, when we will present it as the ultimate evolution of Aristotelian metaphysics.

This is how Aquinas presented the application to our logical problem of constitutive induction as the assumption of an individual, that exists independently of the class of belonging, to the class itself. In the following passage, we can clearly see the logical significance of the metaphysical distinction between ‘being of the existence’ and ‘being of the essence’, or ‘being of the property’.



Constitutive induction as the mutual and reciprocal redefinition between the subject and the predicate

The subject has both its *connotation* (*definitio*) and its *being*, which does not depend on the property. Its *proper being* comes before the intellect in comparison to the being of the property within it. Therefore, we do not simply need to know the *quod quid est* of the subject, i.e., the ‘what it is’ [as it is with the property], but also its *an sit*, i.e., the ‘that exists’. This is essentially because it is from the connotation of the subject and of the property that the demonstration of the middle term depends. (*In Post. An.*, I, ii, 15)⁹⁸.

Let me introduce here another quote from Aquinas’s *Commentary on Posterior Analytics*, where he explicitly affirms that this procedure of ‘induction’ of an individual, in the domain of a universal predication, is simply a procedure of re-definition of the generic universal predicate (e.g. ‘being a man’) in relation to some specific difference (e.g. ‘being Greek’) of the singular subject (e.g. ‘Socrates’), to which the predicate applies. Thanks to this mutual re-definition, subject and predicate immediately belong to each other. Accordingly, the considered predicate becomes the middle term that we were looking for. In Aquinas’s words,

we need to know that here universal is not understood as what is predicated of several subjects, but it is conceived of according to some adaptation or conformity (*adaptationem vel adaequationem*) of the predicate to the subject, in accordance with which neither is the predicate conceivable without the subject, nor is the subject without the predicate (*In Post. Anal.*, I, xi, 91).

Subject and predicate thus *necessarily* belong to each other, without any further mediations (i.e., *immediately*). The considered proposition will therefore be an *analytical* proposition, where the reciprocal belonging be-

⁹⁸ This passage confirms that an analytically true proposition is, according to Aristotle, one whose meaning is reduced to that of its constitutive primitive terms, and through which a single existing entity is indicated (See footnote 96). Undoubtedly, as we have seen, it is possible that this single object is, instead of a physical entity (e.g., Socrates-as-a-man), an abstract logical entity (e.g. humanity), which exists as such only within the human mind and in its real foundation in the common causal concurrence, though which all men come to life (= the ‘ecological niche’ of human species).

tween subject and predicate is not justified by syllogistic rules of construction — such as, for instance, the use of a middle term.

The independence of the existence of the subject from the predicate avoids the impredicativity of the mutual redefinition

Thanks to the condition according to which the existence of the subject does not depend on the definition of the domain of the generic predicate, *the mutual definition of subject and predicate, in which the analytical procedure for the search of the middle term is resolved, does not turn into an impredicative definition, which would be trapped in a vicious circle.* The definition of an object (i.e., typically, the definition of a — at most a single unit, as in our case — subset of a set) is impredicative, *if and only if* the existence of the subset, whose belonging to a given set needs to be proved, is already assumed in the existence of the considered set. If the existence of ‘Socrates’ is, instead, independent of the existence of the generic class of ‘men’ (to which we want to prove Socrates belongs, as one of its specific elements), it is perfectly legitimate to suppose a recursive procedure of progressive (re-) definition (= specification) of the generic domain over the typical existence, i.e., over any typical property (e.g. Socrates’ ‘being Greek’) of the individual that is to be ‘inducted’ (on the plain understanding of it as *inducere*, i.e., ‘to take in’) in the domain itself. It is indeed only through that typical property that we can make sure that we are connoting that individual. This is despite the fact that, since we are dealing with an inductive procedure that can be repeated within different contexts, there is no need that that property be the *heccitas* (to say it with Duns Scotus), i.e., the property that *absolutely* makes that individual distinguishable from the infinity of all possible objects — i.e., Socrates’ ‘Socrates-ness’, in our case. The repeatability of the constitutive induction that is typical of the open, and accordingly contextual, nature of logical systems, which are being constructed here, prevents us from losing our way. This is how Aquinas defined such a procedure:

Generalization to many of a predicate referred to one, end result of the inductive procedure of *inventio medii*

It seems that here Aristotle says that the connotation (*definitio*) of the property [e.g. ‘being a man’] is the middle term in the demonstration. But we need to add that the connotation of the property in the middle term can only be completed through the connotation of the subject. It is evident indeed, that the premises that contain the connotation of the subject are premises concerning properties. Therefore, the demonstration can be solved in its first foundation (*first cause*), only by taking as its middle term the connotation of the subject. Accordingly, it is necessary to find a conclusion about the property connoting the subject [e.g. ‘being Socrates’]⁹⁹, through the middle term of the connotation of that

⁹⁹ It is evident that the property of ‘Socrates-ness’ cannot be used as such: this would be Scotus’ *heccitas*, i.e., what characterizes the single as such throughout all its possible contexts. But there is no real need of this: we can obtain the same effect, by using for each

property about which we want to predicate [e.g. 'being a man'], and *furthermore* (*et ulterius*), it is necessary to find a conclusion about the connotation of the property that is predicated in relation to the subject [e.g. 'being man' in relation to Socrates], by means of the connotation of the subject [e.g. 'being Socrates'] (*In Post. An.*, II, i, 415).

Constitutive
induction and
analogy

What is evident here is the presence of a close relation – that Cellucci has correctly highlighted in his book (Cellucci 1998, pp. 360-379) – between induction, analogy and generalization as methods of discovery, within the domain of the analytical method and, specifically, of the syllogistic procedure of the *inventio medii*. It is clear, indeed, that the redefinition of the predicate on the singularity of an individual, so as to include it in the class that is singled out by the predicate, is simply a form of (not univocal) *analogical attribution* of a generic predicate to a given individual. And it is equally clear that repeating the procedure for different individuals (e.g. 'Socrates', 'Plato', 'Aristotle', 'Alcibiades'...) that share the typical property, which was used to connote 'Socrates' in a given context (e.g. his 'being Greek'. See footnote 99), and that can, accordingly, be connoted in turn through it, it is possible to inductively construct a minor premise that is characterized by universal quantification, i.e., a *generic* premise, for a classical (categorical) demonstrative syllogism, such as 'all Greeks are men'.

This ends the process: we made our first move beginning with the problem of how to adapt a generic predicate to a single individual. This is a typical problem for applied mathematical sciences: in particular, in our example about the roundness of the earth, this was a problem for physical-mathematical sciences. Now, through the explanation of the way in which a single proposition can be generalized, we arrive back at the starting point: i.e., the consideration of the demonstrative syllogism with necessarily generic premises. In broad terms, through a procedure of generalization, it is possible to construct a proposition that works as the axiom for other demonstrative propositions, in accordance with the aforementioned law of predicate calculus (Bochenski 1995, 110):

$$(\exists x P(x) \supset p) \equiv \forall x (P(x) \supset p)$$

Integration of
syllogism within the
analytical method

This analysis confirms what Cellucci underscores in his book (See Cellucci 1998, 14-16). If, against one of the dominating attitudes within modern Scholastics, we highlight the centrality in the syllogistic procedure of the *inventio medii* within the *analytical method*, it is obvious that the

context, one of the properties that characterize Socrates not in absolute terms, but rather in relation to a specific context, e.g. the context of humanity in its 'Greekness'.

sylogism, instead of being a *method of proof* for already constituted true propositions, appears to be a procedure for the constitution of true propositions within well defined contexts. According to Aristotle, as a matter of fact, the aim of the syllogistic method is to define a method

that shows us how we will always manage to find syllogisms to solve any problem and how we will manage to assume the appropriate premises for each problem (Aristotle *An. Pr.*, I, 27, 43a20-22).

This would place the syllogism within the *logic of discovery*, instead of merely within the logic of proof; and this would confer upon it a well defined role within the *hypothetical-deductive* epistemology of modern science, as a set of rules for the constitution of true propositions, through the redefinition of the generic hypothetical axioms over singular contexts, with the aim to build actual *models* of formal systems for well-defined contexts of application.

The integration of the *inventio medii* as an analytical method of discovery within the hypothetical-deductive method for the constitution of true propositions and, therefore, of models that make reference to existing objects

As a matter of fact, from a *formal* perspective, it is true that a hypothetical demonstration can be valid (or not valid) independently of the truth-value of its premises. But when we use *semantically* a hypothetical demonstration, building a *model** in the inferential procedure of a science that is applied to objects — or, also, in the use at the level of inferences that are applied to the objects of ordinary knowledge —, and not abstractly, in the construction of a *formal system**, in those cases the truth-value of its premises becomes essential. Being able to validly decide on the conditions according to which every premise is either true or false becomes a fundamental requisite to judge the validity of a scientific theory in the solution of a given problem. As we have already seen, a procedure of this kind is provided by the use of the syllogistic rules of the *inventio medii* in the procedure of constitutive induction for the search of a *true* analytical proposition, which can represent the middle term of a subsequent syllogistic demonstration. Its result will be the construction of a true proposition that, if we want it to, will work as a premise for a hypothetical demonstration within a certain scientific theory, or within common-sense reasoning. The hypothetical demonstration will nevertheless remain as such, since the truth of the premise is not an absolute one: i.e., it is not a self-evident truth, but rather a proposition that is shown to be true within a given context, in which the demonstration of the syllogistic *inventio medii* has validated it.

Solution to the problem of the inscrutability of reference of the hypothetical-deductive method

The reference to a notion of the existence of a subject (the *proper being* of an individual) that is *independent* of the belonging to the domain of some predicate, shows that, with the logic of the *inventio mediū*, we are faced with a content-based, or intensional (and not only extensional) logic; an *Inhaltlogik* that is opposed to formalism in ontology. Moreover, the fact that the intensional reference is used to construct domains for ‘actual’ universal predicates emphasizes that such an opposition is considered in a constructive way, not merely to distinguish between two logics that are incommensurable. In particular, the reference to extra-linguistic objects is placed at the level of the constitution of the proposition, thus solving the problem of the inscrutability of reference, which was correctly denounced by Quine for all axiomatic languages, i.e., for those languages that work building on already constituted axiomatic propositions (See *supra* § 4.3).

Difference from Husserl’s intensional method

In this sense, such a use of the opposition between intensional and extensional logic is significantly different from the one made by Husserl’s phenomenology. In Husserl’s view, despite his original aim to use intensional logic as the founding theory for extensional formal logic, the *Inhaltlogik* is considered in a way that is epistemologically and ontologically limited to the modern principle of evidence. Husserlian intensional logic is, accordingly, inserted within the ‘magic circle’ of the mind, which is understood as the *transcendental foundation* of truth. Consequently, it does not seem to be usable — as Husserl himself maintains by means of his famous principle of the *epoché* taken from the naturalistic thesis — to ground either a logic and a mathematics, which are capable of demonstrating and/or calculating something useful and applicable to the field of natural sciences, or generic propositions, that are related to a mythical ‘mathematical intuition’, or a realistic ontology in the field of metaphysics¹⁰⁰.

¹⁰⁰ The transcendental logic of the phenomenological approach identifies the intensional (with an “s”) content of concepts, as the foundation of the same logical-formal necessity in the intentional (with a “t”) relation between subject and object — that constitutes every act of knowledge — against the formalism (i.e., absence of content) of Cartesian and Kantian transcendental logic. According to this scheme, the intentional relation is the one that is given within each knowing act of logical thought (and/or of empirical perception) between the fact of being aware of something (object), and the self-knowledge of this being aware of something (subject), which constitutes the two focal points of every act of knowledge. What the analysis of awareness brings to light is therefore that there is no such thing as the Cartesian pure *cogito*, or as the pure ‘I think’ introduced by Kantian transcendentalism: if we think, we are always thinking of ‘something’, just like we always perceive, or desire something. In this sense, according to Husserl, there cannot be a formalist foundation for formal logic, but rather a content-based or intensional one: here is the key to understand the sense of Husserl’s main work on the fundamentals of logic,

Thomistic
metaphysics of the
act of being as non-
formalist
metaphysics of
being

The use, which we have roughly outlined, of the notion of existence for a procedure of constitutive induction of universal propositions shows that it is exactly the notion of 'existence' that is taken in an *intensive*, and not purely *extensive*, sense, as it is, instead, in all formalist ontologies. The constitutive use of induction as a procedure for the constitution of new axioms, and not merely as a corroborative procedure, is inserted within a non-formalist metaphysics of being. In other words, it is adequately inserted only within a '*content-based*' metaphysics of being, just like modern *formal ontology*, and, more specifically, within the domain of the so-called not (yet) formalized descriptive ontologies, within Thomistic metaphysics of being as *act* (intensive being), and not only as *existence* (extensive being). Such metaphysics is, for this reason, well distinguished from all – both old and modern – 'formalist' metaphysics of being as mere existence.

Such a rediscovery of Thomistic metaphysics as the metaphysics of the *actus essendi* is the main result of 20th century Neo-Scholastic speculation, in particular through the work of Maritain, Gilson, and, above all, Cornelio Fabro (Fabro 1961). Regrettably, attention has been so far devoted mainly to its metaphysical and theological consequences, neglecting, instead, its meta-logical and epistemological ones. This work of ours, as some others (Basti & Perrone 1996; Basti 1997), aims to fill in this gap.

Constitutive
induction as meta-
logical equivalent of
the psychological
act of abstraction

Returning to the point, *constitutive induction* – understood as a method for the discovery of the conditions under which certain propositions are shown to be *necessarily* true, even though in an always *partial* and *perfectionable* form – seems to be that logical and, more precisely, meta-logical correspondent of what, in knowledge theory, is defined by the term *abstraction* (See what we suggested at the end of § 4.3). The psychological process of the Aristotelian *epagwgh* (induction) of logical universals is presented in the conclusion of the *Posterior Analytics* (*Post. An.*, II, 19 100a, 1-9 and 100b,4), just like in the *Prior Analytics* Aristotle gave us the logical correspondent of the *epagwgh* in the course of his investigation of the inductive syllogism (See *supra*, pp. 274ff.). We cannot explore further here the distinction between *constitutive induction* (that grounds assertions) – having a necessary value as long as it is integrated by means of axiomatic deductive methods – and *enumerative induction* (that corroborates already constituted assertions) – having a mere probable value – which we have extensively tackled elsewhere (Basti 1997). We shall return in the two next chapters to the ontological and metaphysical foundations of constitutive induction. To prepare ourselves for this task, let

i.e., *Formal and Transcendental Logic*, and, at the same time, to understand the unusableness of Husserl's content-based logic as the logic of a metaphysics of being, in a realist sense.

me hint at the different notion of ‘being’ and of ‘existence’ that characterizes Thomistic meta-logic, being, as we have suggested several times, its ontological basis.

4.5 Conclusion: from philosophy of science to metaphysics

4.5.1 Extensive notion of ‘being’ by modern thinkers.

The analytical approach to Thomistic philosophy requires, first of all, a formalization of the Thomistic notion of being as act

Positive interest of many analytical philosophers in Aquinas

At the end of Chapter Three we claimed that the demise of scientism, the birth of philosophy of science and the rebirth of philosophy of nature impose new tasks upon 21st century philosophy. It is necessary for philosophy and metaphysics to retrieve expositive and demonstrative rigour, so as to combine the formal accuracy of modern science and logic with the contents of classical metaphysics. In general, this task has been endorsed by *analytical philosophy*¹⁰¹, deriving from Wittgenstein’s thought, even if this is outside the epistemological and metaphysical reductionism of the original Neo-positivist movement.

Accordingly, we can notice the increasing interest of contemporary *analytical philosophy* in a strictly formal study of Thomistic philosophy and metaphysics. In a recent work, Brian J. Shanley underscores such a phenomenon:

The analytical interest in Aquinas is nowadays so strong that some thinkers even consider themselves as ‘analytical Thomists’. A recent issue of the journal *The Monist* (vol.80, n.4, 1997) has been entirely devoted to Analytical Thomism, and one of the next volumes of the ‘Oxford Companion to Philosophy’ will be focused on the same topic. While every Thomist should rejoice facing such an analytical interest in Aquinas, many traditional scholars would certainly wonder whether ‘Analytical Thomism’ is an

¹⁰¹ Analytical philosophy is a philosophical movement, widely developed within the Anglo-saxon context, which has followed Wittgenstein’s teaching of philosophy as ‘linguistic therapy’, as a rigorous analysis of different kinds of languages, beyond the limits of a mere philosophy of science. The most significant results have been obtained by analytical philosophy in the two fundamental fields of ontology and ethics. There, as a matter of fact, the use of rigorous languages is particularly important, against ideological and rhetorical temptations, in which other kinds of philosophical analysis have often fallen (See Strawson 1992 and www.formalontology.it).

oxymoron as ‘Transcendental Thomism’¹⁰² seemed to be. Can one really be both a Thomist and an analytical philosopher? Is the philosophical basis of the former compatible with those of the latter? (Shanley 1999, 79f)¹⁰³.

But the use of the analytical method needs to pay attention to the *proprium* of Thomistic ontology

Shanley answers that the dialogue between Analytics and Thomists is certainly fruitful. From a Thomistic perspective, it is worthwhile so as to regain a demonstrative rigour that has often been weakened by superficial expressions of Aquinas’ thought. From an analytical perspective, it can be fruitful if analytical philosophy is capable of appreciating the novelty – and not the reducibility – of Thomistic ontology, in comparison to the dominating ontology in analytical philosophy, that indirectly derives from Kant’s ontology and, more directly, from G. Frege’s.

Systematic mistake made by many Analytics of using the Kantian–Fregean notion of being (extensive being) with reference to Aquinas (intensive being or being-as-act)

The systematic mistake that Shanley recognizes in several Analytical Thomists is that analytical philosophers, in studying Aquinas’ ontological and metaphysical thought, have too often missed the target, since they unconsciously reduce his notion of *being* to the mere formalistic idea of *existence* that was introduced by modern philosophy, in particular in its Fregean version. In fact, as we will see in the next two chapters, the *proprium* of his metaphysics in the domain of Western thought is the notion of *being as act*, of *intensive being*, having, that is to say, a *content* that expresses and shows the inner *nature* of an entity, i.e., its *concrete essence*, that comes out of a *causal process* and is, consistently, actualized within a given *existent*. Let me accordingly develop further what we have already seen in a preliminary form. Let me focus on how all this applies to the issue of the *constitutive induction* of true propositions, which are to be inserted as axioms for the demonstrative procedures of natural sciences. In this way, we can connect what we have said here to what we will say in the next two chapters.

Aquinas describes being not as the mere factuality of existence, of the belonging to one class, or of Kantian being as copula

When many contemporary Thomists, beginning with C. Fabro (Fabro 1961), denounce the formalist deviation of modern metaphysics, which reduces the richness of the notion of *being* to the mere factuality of *existence*, they mainly have in mind that Kantian claim that denies *existence* the dignity of a predicate. We all know well Kant’s critique of the ontological argument on the existence of God suggested by Anselm of Aosta,

¹⁰² The neo-Thomistic school, which widely spread on continental Europe especially after World War II, has tried to re-interpret Thomistic metaphysics within modern transcendental philosophy and that had, among its main representatives, such philosophers as Joseph Maréchal, S.J., Johannes B. Lotz, S.J., and such theologians as Karl Rahner.

¹⁰³ The paper is included in a special issue of the journal *Divus Thomas* (vol. 3, 1999) — edited by Alberto Strumia, Claudio Testi and myself —, devoted to the debate fueled by the recent Encyclical *Fides et ratio*, from the perspective of the relation between ‘analytical’, ‘continental’ and ‘Thomistic’ philosophers. See (Basti *et al.*, 1999).

which was also endorsed by Descartes, according to which God necessarily exists since his essence is part of the Very Perfect Being (*id quo maius cogitari nequit*, 'something greater than which cannot be thought'). Therefore, if the Very Perfect Being did not exist in reality, we would be able to think of another more perfect being, that would add to the perfection of the former also the perfection of its existence. Kant's critique, which was partly shared by Aquinas himself, is that by such a claim we witness an improper shift from the logical level to the ontological one. 'One hundred possible thalers' (common coins in Kant's time), which in fact do not exist, do not certainly become 'one hundred plus one' if we consider them as existing: to exist is not an 'additional property', which can be added to the essence of an entity. According to Kant — and here the agreement with Aquinas ends — to exist is to become in act within the existent of a given essence, which is purely possible. Being is thus reduced, in Kant's view, to the *copula* between a subject and a predicate, i.e., to the 'is' of a declarative judgement, like 'this *is* a cat', through which a certain phenomenical datum is 'assumed' within an *a priori* category of the intellect

'Existing' is reduced
 to 'being
 something', to the
 extension (domain)
 of a predicate

In the terms of Fregean-Russellian logic of classes, to exist simply means that a certain class **P**, defined by a specific predicate $P(x)$, is *not-empty*. Therefore, when we say, 'a certain thing (e.g., a cat) *is*', meaning, 'a certain thing *exists*', we are not properly predicating something of an individual. We are rather simply suggesting some predicative statement, like, for instance, 'something belongs to the class of cats'. In the terms of Fregean ontology, in relation to his logic of classes (See § 1.6), by 'existence' we merely express the *belonging* ' $\in$ ' of a certain element x to the domain of a given predicate $P(x)$, so that class **P** is not-empty, i.e.:

$$x \in \mathbf{P}$$

In other words, for every non-empty class of elements, the truth that every element x of the domain satisfies the predicate $P(x)$ implies that there exists at least one element x that satisfies the predicate, i.e.,

$$\forall x P(x) \supset \exists x P(x)$$

According to Frege, to exist is to attribute a number different from zero to each generic element of a domain

In this sense, according to Frege, affirming the existence of something, or, better, of one thing with the quality or the property that determines it, is simply to attribute a number that is different than zero to one object, which is understood as a generic element of the domain of the predicate that expresses, or defines, that property. The statements of existence are simply numerical statements — ‘1’ for the existence, ‘0’ for non-existence¹⁰⁴ — and all numbers can be constructed building on these two figures, according to a binary system.

To affirm the existence of an object is the same as to enumerate it within an appropriate class

Now, just as the attribution of a number, or the enumeration of an object within a given collection, does not attribute any further property to one object – the only property is the one that determines the collection to which the object belongs, and that is expressed by the related predicate – so it is in the case of existential statements. Saying ‘this cat *exists*’ does not add any property to this object other than that (or other than the set of those) in accordance with which we were able to say ‘this *is* a cat’, i.e., ‘this is an element of the class of cats’, or ‘it belongs to the class of cats’. In other words, affirming that ‘*x exists*’ means simply affirming that ‘some *x is y*’. Saying that ‘some jolly Welsh exists’, as Shanley exemplifies, in the logic of classes, is equal to saying that ‘some Welsh is jolly’.

If the being is reduced to this form of existence, it becomes purely formalistic, extensive, independent of its content. Impossibility to solve the problem of reference

According to this logic, ‘it exists’ does not make any reference to any content. It only talks about the belonging of the name of an element *x* to the *extension* (or domain) of a certain predicate *y*. Therefore, we shall define the being of existence in this formalist, non-content-based, modern sense as ‘*extensive being*’. We have already suggested, talking about the limits of Neo-positivistic ontology – for instance, in relation to Quine’s approach (See § 4.3) – that such a formalist ontology makes it impossible to solve the *problem of reference*, reducing the question about the referential object of a given language to a procedure for the search of predicates having an equivalent extension. In other words, it reduces the problem related to discovering what certain propositions really make reference to,

¹⁰⁴ Such a discovery, that is at the basis of the possibility of constructing a mathematical logic, was not made by Frege, but rather by Ernst Schröder (1841-1902), who first theorized that ‘zero’ denotes the ‘empty class’, which is a class that is contained in all other classes, meaning that every well-structured class must exclude all the elements that do not share that property (predicate) that defines the class itself. Along these lines, every class must contain as an empty class the one that contains all the elements that do *not* belong to it, that *do not exist* within it. Conversely, ‘one’ denotes the universal class *V*, i.e., the class that contains all other classes, including itself (Bochenski 1956, II, 475 ff). More precisely, every class needs, so as to exist, to contain at least *one* element (otherwise it would be an empty class, which denotes non-existence). It was due to the reflection on foundations, which followed the discovery of the Cantor-Russell antinomies, that we concluded that strictly speaking *V* cannot be a class (or a set), characterized by a well-defined graph of the relations between all its infinite elements, but only a ‘collection’, i.e., the collection of all the entities that satisfy the predicate, ‘it exists’.

to the search for equivalent ways of stating the same proposition, without ever having the possibility of knowing what is concealed ‘beyond the mirror’¹⁰⁵ of the language’s syntactic web. This result is also shared by Kantian ontology, according to which the ‘thing in itself’ is nothing but a *noumenon*, i.e., a thinkable object, a hypothetical referent of judgements and, accordingly, of declarative propositions.

This ontological theory of Fregean logic of the classes is intimately linked to Russell’s antinomy

Going back in the historical reconstruction of modern science that we have begun in the previous chapters, we have already seen that this Fregean ontological theory of existence, which is bound to his logic of predicates as a purely extensional logic of classes, leads to an antinomy, i.e., the so-called ‘Russell’s antinomy’ (See § 1.5.2.2). As a matter of fact, in order to justify the existence of an object as simply belonging to a class, we need to justify the existence of that class as the non-empty domain of the related predicate, i.e.,

$$\forall x A(x) \supset \exists y \forall x (x \in y) \cdot (x \neq y)$$

If being is reduced to the belonging to one class, the result is necessity of the total class

To give an ultimate consistency to the construction, we would thus need to admit that the *total class* of those classes that do not belong to themselves exists. However, we have seen that this leads to an unsolvable contradiction, as soon as we would want to justify in this way the existence of such a total class: does it belong to itself, or does it not?

Hence Russell’s suggestion to avoid a constructive approach through axioms of existence

In this case, the previous formula should be rewritten as,

$$\forall x A(x) \supset \exists y \forall x (x \in y) \cdot (x \notin y)$$

This is clearly contradictory. Hence Russell’s suggestion not to justify the existence of predicate domains, in Frege’s theory, by means of constructive, syllogistic procedures of implication among different predicates and, accordingly, of inclusion within more and more extensive classes. On the contrary, he suggested doing so through appropriate *existence axioms*, such as the famous ‘comprehension axiom’ of the naïve theory of classes. According to this axiom, through the affirmation of a predicate $A(x)$, we also constitute the domain of the elements that satisfy it:

$$\exists y \forall x x \in y \equiv A(x)$$

¹⁰⁵ The reference is here, obviously, to the ‘mirror’ of *Alice in Wonderland*, the famous novel of that ‘oblivion of being’, which characterizes modern philosophy, having its origin in the bewilderedness of the ‘being in itself’ within Kantian representationism.

Impossibility of a justification of the constitutive induction in Frege's extensional logic of classes

In addition, Frege's formalist theory will never be able to justify that procedure of *constitutive induction* we have made reference to as the essential requisite to confer to certain inferences of experimental science that necessary, demonstrative – though always perfectionable – value, which we have mentioned earlier. Besides judgements of consistency, indeed, what Fregean theory of classes can grant is the existence of the *generic elements* of a class. To make it possible for it to grant the existence of a concrete individual within it — e.g., not simply that 'some Welsh is jolly', but rather that 'John Smith is jolly', or, not that 'some Greek is a man', but rather that 'Socrates is a man' — it would be necessary that the enumerative procedure including the elements within a class could be complete, and actually extended to the whole infinite domain of a universal predicate. Which, again, drives us back to the limits of enumerative induction taken as a procedure of proof.

4.5.2 Intensive notion of 'being' by Aquinas

In Thomistic constitutive induction, the assertion of existence is not an enumeration of generic elements of already constituted domains

Conversely, when, within the Thomistic logic of constitutive induction, we apply a certain generic predicate to a subject, which exists by its *own being* – that is clearly intensive, and not extensive –, we are not merely enumerating an element within a specific and already constituted class. We are rather doing something far more radical: we are, that is to say, *conforming* the generic predicate to the unique *particularity* of the subject of that predicate, in view of some *specific difference* characterizing the individual we are dealing with. The 'it exists', in Aquinas' view, ultimately corresponds to the affirmation of the belonging of an individual to a class; but the 'it exists' is not a mere calculation, that is extrinsic to the concrete essence, to the entity of the individual. The 'it exists', that is denoted by the symbol '∃' in predicate calculus, is not a mere 'quantifier' within Thomistic logic, it is not a mere 'accountant' that limits herself to tick a list, marking 'it is there/it is not there', '1/0'. It is, instead, an *operator* that exercises its function over the domain of the predicate, and over the element that should belong to it, *conforming* one to the other. (Basti & Perrone 1996).

Re-adaptation of the predicate to the being of the individual, and predicamental participation

In order to be able to validly say 'it exists', meaning that 'it belongs to that class', we need to re-conform, to 'contract', the predicate that determines the class to some specific property of the individual we are taking into account. This is what Cornelio Fabro defines, in his famous work on the notion of participation of being in Aquinas, *predicamental participation*, meaning the participation of the individual to the essence that is typical of its species.

Predicamental participation supposes transcendental participation of being as act

The predicamental participation of the individual to the species, since it supposes that the individual exists by its own, supposes in turn, and hence it is grounded on, the *transcendental participation* of the being of every entity — that is characterized by a finite and limited essence — in the Subsistent Being, in which essence and being coincide (Fabro 1961). Such a ‘participation’, in the two senses we have now described, is a particular form of causality that, respectively, leads back to the existence of *universal causal agents* in the physical order for the causal explanation of concrete essences, of the entities of single individuals (i.e., physical entities), belonging to the same species, in accordance with Aristotle’s clever intuition of the form as act (See § 5.4.4); and to the existence of a *Universal Absolute Agent* – or *Pure Act* – in the metaphysical order for the causal explanation of all of being, i.e., essence and existence, of every entity (See § 5.5.2).

4.5.3 Intensive being and the foundation of induction

In-depth analysis of the notion of predicamental participation

Let me analyse further the notion of ‘predicamental participation’, which is what interests us most for a causal foundation of logical necessity within inductive scientific syllogism. For instance, in the case of the proposition ‘Socrates is a man’, in order to be able to validly insert the individual ‘Socrates’ within the class of human beings, we need to *conform* the generic predicate ‘to be a man’ to some specificity of Socrates’ humanity, of Socrates’ being a man, i.e., of his own *entity* of man.

To insert a specific – and not a generic – individual within a class, there is no need to enumerate the whole class, it is enough to redefine the generic predicate on a specific property

For this reason, we do not need to have enumerated in full the class of human beings to be sure of the *truth* of the proposition ‘Socrates is a man’. Neither do we need to possess the ultimate specific difference – that is both *necessary* and *sufficient* to individuate Socrates in the possible infinity of *all* men, i.e., Duns Scotus’ *heceitas* — to be sure of the truth of that proposition. It is enough to know a difference that is *necessary* to characterize him as *that specific* individual (e.g., ‘If he is Socrates, then he is a Greek philosopher who drank hemlock’), which will be also *sufficient* as long as we do not find any contrasting evidence (i.e., ‘If he is a Greek philosopher who drank hemlock, then he is Socrates’ is also a sufficient condition to individuate Socrates, as long as I have not found another Greek philosopher who died in the same way). In other words, as long as further discoveries, within the finite domain of inductive experimental knowledge, will not compel us to repeat the procedure to find out, in Socrates’ entity, another difference that is *necessary* — and, again, sufficient as long as contrasting evidence is discovered — to grant that it is

really Socrates, i.e., the Socrates that actually exists, i.e., who belongs to the class of human beings.

Through a synthesis of what we will analyze in greater detail in Chapter Six, so as to apply it immediately to the problems of foundation for the demonstrative procedures we have discussed in this Chapter, let me anticipate the essential meta-logical conclusion that we will draw in § 6.1.

'It exists' according to an intensive sense is not a predicate, but a meta-predicate

It is true that 'it exists' is not a predicate. But this is in the sense of intensive being that we have just mentioned. It is not even the simple existential 'quantifier' of class logic. Nor is it that 'accountant' that limits himself to 'take notice of', to enumerate a single element within generic domains. In other words, it is true that being is not a generic predicate, or better, it is not the most generic predicate that can be attributed to a subject (i.e., the most general sense of being). Being is not a genre, even when one would want to indicate by it *common being*, i.e., being that is common to every entity, i.e., existence: that being, that is to say, in accordance with which every x satisfies the universal formula ' x is an entity'. In fact, 'it exists' is not a predicate, but, in the intensive sense we have just mentioned, it is a *meta-predicate*, i.e., a predicate that deals with:

- ◆ a given predicative, categorical proposition, such as 'Ralph is a cat'
- ◆ and the relation to the extra-mental object, 'Ralph-cat', which, making the proposition a *designator of an individual* (a 'universal one to one') through the procedure of re-definition (predicamental participation), makes the predicative proposition 'Ralph is a cat' *true*.

Intensive being – being as act – is being that intensively (and not extensively) includes any other formal act, any other property and, consequently, any other predicate

Indeed, *if and only if* 'Ralph-cat' *exists*, 'Ralph is a cat' — i.e., the categorical proposition that expresses the *entity*, i.e., the *kind* of existence which that entity *exercises* as its most fundamental act, which intensively includes any other act, any other form and presence — is true. Being is indeed the fundamental, transcendental act (*esse ut actus*, 'being as act'), which every entity exercises. It is the act of all categorical acts, that are formally distinguished and that can be expressed by other corresponding both *generic* (e.g. to eat, which is common to all animals) and *specific* (e.g. to meow, that is specific to cats) predicates that an entity exercises (e.g., 'Ralph is a cat' includes 'Ralph is an entity that eats and that meows' or 'a meowing animal'). Because 'being entity' is the transcendental act that is common to all entities (*esse commune*), and that contains all the other categorical, formal, distinct acts that make every entity different, we can say that the being as an act 'contains' in itself *all* the formal acts of *all* entities. Indeed, 'being an entity' 'includes' in itself the 'being a dog', but also the 'being a cat', 'the being a man', and 'the being red', etc. (See also footnote 104, p. 296).

Since the 'being as act' of an entity includes only some formal actuality, every entity takes part in being, according to its essence

But, since every single entity has, as its own properties, only *some* of these formal acts (e.g. a cat meows, but it does not bark, or think, and, accordingly, Ralph meows, but in a unique way in comparison with other cats), it is clear that every entity takes part of being as act in accordance with its concrete essence, i.e., its specific entity, *qua* outcome of a *causal, unique, not repeatable, both physical and metaphysical, process, which had led it to exist, and to exist in this particular way*. For the same reason, it is clear that there can be only one entity which, for its own essence, takes part of the *whole* being. This entity could be the universe of the entire specific and differentiated entities, as, for instance, Severino maintains.

There is only one subsistent Being, in which the essence corresponds to the whole being and that will intensively 'contain' *qua* Agent the being of the universe of all entities

But this notion of a unique entity that contains in itself, as simultaneously defined diversities, the complete graph of the relations between the infinite entities that make up the 'whole being' – as Severino defines it, according to a Neo-Hegelian fashion – is an antinomical notion, as both Aquinas in the Middle Ages, and the Moderns after Cantor, Russell and Gödel know well. Consistently, this Being, which contains the whole richness of being as act, can have it not as the 'universe of the distinct entities, *qua* all *currently* and *equally* existing' — i.e., the *universitas rerum* of the Medievalists, or Severino's 'whole being' — but rather as the Universal Agent of the whole universe of the distinct entities that currently exist, even if they are not *equally* existing, i.e., existing *in the same way* (e.g. they exist in different times, with different degrees of necessity, with diverse properties, etc., with, in a word, distinct *entities*).

Being as act is a content-based being and the content of the being of each entity is its presence

As we can see, and as we will explain better in the next chapter, existence, according to a Thomistic understanding, has a *content*. Unlike what the Moderns claim, it does not correspond to an essence that purely becomes 'act'. 'To be' says something about an entity: it says something fundamental, i.e., 'being an entity', as the content of the meta-predicate 'it exists', which contains any other thing that can be validly predicated of that entity. Therefore, Aquinas says that the presence of a thing is the *first transcendental of being*, the one that contains in itself all other predication, starting with the more universal categories or predicates (See § 5.6.2 and § 6.2.2.).

'It exists' does not only convey the factuality of 'being-here' within a given class, yet its also points at the source of all properties and hence of all predicates: the presence of the entity

'It exists' conveys not only the *factuality* of the being-*there* of an entity within a certain class, as it was for Kant, Frege, Quine and for all the Moderns; it also and inevitably conveys its *presence as a thing*. It also conveys, in other words, the being of its *way of being there*, i.e., the being in its concrete essence – and not of the abstract essence of the genre, of the concept, but rather the being of the concrete essence of an existent, *qua*, from a metaphysical perspective, the result of a causal process. More precisely, existence also conveys, together with the 'fact' of being *there*, the *transcendental* or *pre-predicative source* – i.e., the *presence* – of every predicative 'something', which can be said in a *true* way of a certain individual entity. These are ideas that we will develop further in the next chapters (See, in particular, § 5.5).

Being as act in relation to the limiting potentiality of essence, analogous to the presence of the form in relation to the limiting potentiality of matter

Through the notion of 'being as act' of all the formal acts exercised by the entity and that constitute its essence, we can make the core of Thomistic metaphysics explicit, i.e., the distinction between *act and potency*, extended from the categorical constitution *form–matter* of the *essence* of the material being (which is typical of Aristotelian philosophy of nature), to the transcendental constitution *being–essence* of the entity itself, whatever it may be, whether material or spiritual, natural or artificial (i.e., logical) (See § 5.5). Such a distinction applies to every being except Absolute Being. In Shanley's words,

[I]n Aquinas's view, every entity (*ens*) is made up of two separate principles: *esse*, understood as the fundamental act that makes the *ens* exist, and *essentia*, conceived of as potency in relation to *esse*, which formally determines the kind of existence that the *ens* has (Shanley 1999, 83).

The determination of potency on act is, in general terms, the limitation of a potentiality in relation to an active capacity

In order to avoid any confusion with the Moderns, in particular with what Hegel suggested by his theory of the indeterminate being in his *Logic*, the 'determination' that the essence imposes upon being cannot be that of act towards potency, in the sense that we all know, for instance, of form in relation to matter – like the sculptor, who actualizes from the marble a shape that is already there in its potentiality, or like in physics, where from the instability of a certain dynamic process the structure of a final state of affairs emerges, in a totally unpredictable way (chaotic attractor: See § 2.7.1.). On the contrary, here we are making reference to the determination that the potentiality of the essence (= *potency*) imposes on the act of being, in the sense of a *limitation of its active capacity* (= *power*). It is like when, always making reference to the well-known dichotomy between form and matter, we assert that no material and individual realization of a form can express all its richness. For instance, like when one says that no man (or group of men) can express all the richness of *humanity* (against racism, or the idolatry of a given per-

sonality). In its being passive, potency is, therefore, a sort of ‘resistance’ that disperses ‘the energetic potential’, i.e., the active potentiality of a source of activity. In the case of essence, this is a passive power that ‘dispersed’, limiting it to the proper presence of another entity, the – most fundamental – primary active potentiality of being as act.

Being as act is limited by the essence of an entity to the presence of that very entity

Accordingly, the *common being* for all entities, i.e., the evidence that they all *exist* – that is the metaphysical foundation of the p.o.n.c. – is limited, by the essence of every entity, to the proper *presence* of each of them, i.e., to the way of existence that is typical of each of them. Consistently, the *presence*–horse limits common being to the specific way of being of the horse, together with its typical formal acts, i.e., with its typical *properties*, which are *different* from those of cows, or of men. In this way, being as act is seen as a *source*, in the sense of the *power* of the separate *presence* of entities. On the other hand, the *properties* of the entities are defined by Aquinas in terms of *passiones*, i.e., of *passive* potentialities.

Speaking of being as act sends us back to a causal scheme of justification of the proper being, like in Aristotle the form as act sent us back to a causal explanation of the essences

This terminology, that is typical of *Thomistic* metaphysics and meta-logic, recalls an implicit *causal scheme*. To affirm that the common being for all entities is *act* sends us back to a *Unique Agent Entity*, just like in Aristotelian philosophy of nature; to speak of forms as *acts* – and not as subsistent individual substances belonging to an immaterial world like in Platonism – links back to the action of physical-entities-universal-*agents* (i.e., the celestial bodies) on the material basis from which such forms are ‘educated’, i.e., actualized. In other words, the web of causal relations between ‘causing and caused’ determines, at the level of the entities, the properties (*passiones*) of each of them. As to physical entities, it determines that it is a part of their essence, or ‘nature’, that the specific presence of a thing can only exist in certain time-space slices of the universe of the physical entities, and nowhere else.

An application to evolutive biology of the causal scheme of essences: the extinction of species

For instance, it determines that dinosaurs can exist, at least from a natural point of view, only under certain conditions of the environment that, according to the current paleontological theory, took place in the late Mesozoic era (about 210 million years ago). Such conditions are thought to have disappeared, in a relatively sudden way (i.e., in about a million years) more than 60 millions years ago, because of some catastrophic climatic change, maybe deriving from the fall of an enormous asteroid on Earth. This happened after a period in which dinosaurs dominated the biological scene on the Earth — always according to what palaeontologists say — for 150 million years!

The causal justification of the being and the essence of entities results in the contingency of everything in the physical universe

Leaving dinosaurs aside, the existence of each entity, with its own presence, requires a precise combination of physical causes that, because of their *contingency* — due to their being caused, in turn, by causes that can be prevented from happening in view of other causes — cannot exist ‘always and everywhere’. The metaphysical foundation of the common being for such an immanent causal web of both caused and causing entities, by whose order the single *natures* or *essences* of physical entities are determined, requires its ‘actualization’ by a unique *Metaphysical Causal Agent*. This latter is viewed as simultaneous, universal and transcendent in relation to any space-time location within the physical universe, which can grant metaphysical consistency — i.e., which can guarantee the metaphysical composition of essence and act of being — to the entire web of relations, and to every ‘notch’ (entity) in it. We will investigate in the next chapter this topic of the metaphysical foundation of the causality of the natural order, as the last step in the theoretical synthesis of Thomistic metaphysics (See mainly § 5.5.2).

4.6 Summary of Chapter Four



In this chapter, after having proposed two definitions respectively of *philosophy of nature* and *philosophy of science* (§ 4.1), we have analyzed in depth the issue of the definition of a scientific method (§ 4.2). In the first place, we have seen that what characterizes scientific methods, in natural, mathematical, (meta)logical and metaphysical sciences, is the *explicative*, and not merely descriptive, nature of their propositions, and the demonstrative – and hence universal and necessary – character of their statements. Universality and necessity should not be understood in an *absolute* sense, but rather always in one that is *relative* to the acceptance of the hypothesis (or starting axioms), to the degree in which they are adequate (even if not exhaustive) for their objects.

Scientific language is never *exhaustive* of its own object of investigation; this is because of its *incompleteness* that is the necessary ingredient of its *consistency* (i.e., non-‘contradictory’-ness of its propositions). Finally, we also have recalled that, in the Modern age, we have shifted from the assumption of *apodicticity*, and accordingly of absoluteness, of mathematical demonstrations in modern sciences, — a supposition that is based on the presumption of self-evidence of the postulates of Euclidean geometry and Newtonian physics — to the *hypothetical-deductive* method of contemporary science. Since it is typical of a hypothetical premise of an ar-

gumentation that it can be either true or false without invalidating the necessity of the implication, it becomes essential to study the foundation of the axioms, or better, the conditions under which the axioms are either true or false, so as to define the domains of applicability of a formal system, i.e., its giving birth to different models. However, in order to do so, as we have seen, we need in the first place to operate a shift from the *logic of propositions* – that characterizes the hypothetical-deductive method – to the *logic of predicates* – that, for instance, is typical of the *sylogistic* method of demonstration.

In line with this, in § 4.2.2, we devoted our analysis to the *hypothetical-deductive* method, to its potentialities and its limits. We have studied its *empiricist* origins (§ 4.2.2.1), in relation to the seminal work by Wittgenstein and to his redefinition of philosophy and epistemology in the terms of an *analysis of language*. We have then focused on the influence Wittgenstein's work exercised on the foundation of the Vienna Circle and on logical Neo-positivism (empiricism) (§ 4.2.2.2). We have then taken into consideration (§ 4.2.2.3) Popper's correct critique of the Neo-positivistic claim according to which the empirical control of hypothesis, within the hypothetical-deductive method, could be of use to logically (i.e., necessarily) justify the *verification* of the hypotheses themselves. Within this approach, empirical control can in fact justify the *falsification* of the hypothesis and, at most, empirical evidence can *strengthen* a hypothesis, and, accordingly, the probability of its being true – without ever being able to guarantee its *certainty* – which remains nonetheless always relative to something, and that is never exhaustive of a given object. Conversely, what turned out to be unacceptable (because simply being *false*), is Popper's claim according to which there cannot be a logical methodology that justifies the discovery of hypotheses and the foundation of their truth as the *conformity to an entity*, to the *referent* of a certain proposition. The reference to an object cannot be grounded by means of a *procedure of justification* for an already-constituted proposition; it has rather to enter *the same constitutive procedure for that proposition*, depending on the object to which it will *make reference*. Therefore, when we are faced with such problems, we have to step out of the domain of the *hypothetical-deductive method* and of its *logic of justification* within the *calculation of propositions**; we are rather to step in to the domain of the *analytical method* and of its *logic of discovery*, within predicate calculus and of both *inductive* (i.e., non-demonstrative) and *deductive* (i.e., demonstrative) *sylogisms*. This very same necessity has been reinforced in the analysis of the *ontological limits of the hypothetical-deductive method* (§ 4.3). Following Quine's analysis we have seen indeed that, moving within the domain of Neo-positivistic principles, *the problem of the reference of a proposition to the extra-linguistical object* becomes logically unsolvable. We can at most justify that what a certain

language makes reference to is equivalent to what another language makes reference to, so as to make them interchangeable, but nothing more than this, i.e., than indefinitely postponing the solution of the problem.

The *necessity*, and not the mere *probability*, of the *truth* of a proposition can be proved, only within the *logic of predicates* and the *sylogistic method* of demonstration (§ 4.4), as the *logical necessity of the inherence of subject and predicate*, which is founded on a necessity of an *underlying causal relation* that is expressed through the so-called inherence in the *subject* of the conclusion of the *middle term*, which is present in the premises (§ 4.4.1). In this case, we do not need an always increasing number of empirical verifications (= *enumerative induction*) to corroborate the truth of a proposition. One single piece of evidence can be enough (= *constitutive induction*). We have suggested an *intuitive example* of such a procedure (§ 4.4.2), which has been particularly effective in showing the efficacy of this method when the major premise of the considered 'causal' syllogism is a universal statement of a mathematical kind (*mathematical* and/or *physical-mathematical law*). Despite the pedagogical usefulness of such an example, it is better not to forget that the core of the problem resides, within experimental science, in the way in which to pass from a universal major premise (mathematical law) to a *singular* minor term, that is such because it is related to (a) experimental evidence(s), on the basis of which we can ground the necessity of the inherence of subject and predicate in the conclusion. This can only be justified through the procedure of *constitutive induction* that we have presented, staying within the Aristotelian syllogistic method, by making use of Thomistic meta-logic. Thomistic meta-logic of constitutive induction is based on the Thomistic ontology of intensive being, or 'being as act', thanks to which we can assert, without falling into impredicativity, the existence (proper being) of the individual (= part), independently of the existence of a class (= totality) within which it is to be inducted (i.e., assumed) (§ 4.4.3).

In § 4.5, to conclude, we have focused our attention on the importance that a formalization of Aquinas' metaphysics would have within an analytical philosophical approach that is really adequate for his thought. This would be an approach, this is to say, that is capable of operating the distinction between the notion of being — reduced to pure *extensive existence* (i.e., class belonging), which equates the assertion of the existence / non-existence of an object with the act of its enumeration 1/0 — that is typical of Kantian and Fregean thought and, in general, of modern thought (§ 4.5.1), and (*intensive*) being as an act, that is typical of Thomistic thought, together with its distinction between the *being of the essence* (presence of a thing) and the *being of the existence* (common being),

based on the real difference existing between *being* (as act) and *essence* (as potency) in the metaphysical constitution of every entity (§ 4.5.2). We will offer an historical presentation of such a Thomistic theory, within classical or pre-modern thought, in Chapter Five.

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Chapter Four

5. Classical metaphysics and modern thought

From the origins of the reflections on being in Greek thought, with pre-Socratic philosophers and particularly Parmenides, to the solution of the problems posed by Parmenidean metaphysics in Democritus, Plato and Aristotle. Up to the thought of Thomas Aquinas, one of the most important philosophers of pre-modern classical thought. Investigation into the doctrine of transcendentials as an illustrative turning point between classical and modern thought, as developed in the Thomasian and Kantian interpretations.

5.1 Being: from the origins to Parmenides

5.1.1 The meaning of this account

Proprium of Thomist metaphysics of being-as-act

The question of being and of the fundamental distinction between the *entity* (being of an essence) and *existence* (common being) of ordinary things led us to investigate the intimate relationship that exists between *Man* and *Being*. Indeed, this is the basis of a theory of *being* (= metaphysics) and of *truth* (=meta-logic) that avoids the insubstantiality of a formalist theory of both. This relationship represents the *leit-motiv* of all western metaphysical thought, which we will go through with the aim of showing, in a preliminary and schematic way, the *proprium* of Thomas Aquinas' metaphysics and meta-logic.

If we follow the Ariadne's thread of the relationship between Man and Being, this relationship is reversed in modern thought compared to classical thought

If we take Aquinas' philosophy as the highest point of classical pre-modern metaphysical thought, both Greek and Medieval, the fundamental difference between classical and modern thought appears clear. At the same time, the difference between classical and modern thought on the one hand, and post-modern contemporary thought on the other—that was called to perform a creative synthesis between the two—also emerges. Following the «Ariadne's thread» of the relationship between Man and Being, we will see that this relationship is reversed in modern thought compared to classical thought. In classical thought, Man had to adjust the *a-priori* of his individual mind to the essence of things. It is the being of things, particularly their *entity*, which has a constitutive, *transcend-*

dental role on logical universals. In modern thought, after the «Copernican revolution» carried out by Kant, the being of things must adjust to the *a-priori* of the mind, which has risen to the meta-individual role of constitution of logical universals (transcendentalism).¹⁰⁶ The basis of this revolution is linked to the need to ground causal *necessity* in the ontological order through the logical *a-priori*. Hence, causal necessity in the ontological order does not ground the necessity of law in the logical order; rather the *reverse* is true. (Cf. § 2.7.2).

The endpoint of this brief historical and theoretical account will be the Thomistic doctrine of the *transcendental determinations* of being. In particular, we will focus on the doctrine of the first of these, *being*, intended as the ante-predicative («transcendental») foundation of any true *predication* that can be attributed to the subject of a proposition denoting a given *existing*. The Thomistic meta-logical doctrine is intimately connected to the Thomistic metaphysical doctrine of the real distinction between *essence and act of being*, therefore between the *entity*, or «being of an essence», and *existence*, or «common being». According to the authors, this constitutes the apex of *pre-modern* classical metaphysical thought. This Thomistic doctrine will be contrasted with the doctrine of the *modern transcendental of self-awareness*, the (purely formal and devoid of content) «I think», as understood by Descartes and Kant, which places in *self-awareness*, hence in *evidence*, the foundation of the truth and necessity of *predication*.

Grounding truth on awareness rather than on being = basis of modern thought

The grounding of logical truth and necessity on the self-aware thinking of a meta-individual, or «transcendental», subjectivity —rather than on the essence of the known entity and of its real (causal) relations— is the metaphysical and meta-logical turning point between classical and modern thought. In Kant's words, it is the core of the modern *Copernican revolution* in philosophical thought.

Constitutive relation between thought and being: man = metaphysical animal

In any case, one thing is established, and therefore constitutes the Ariadne's thread of both classical and modern Western thought. This relates to the constitutive relation — of thought for classical thinkers (being grounds thought), of being for modern ones (thought grounds being)— between thought and being. In this sense, not only is Man one *entity* among other entities; he is the only entity with the potential *to relate* to the being of any other entity. In fact, two among the transcendental determinations of any entity, *truth* and *value*, depend on the relation between the being of every entity and human intellect and will. Man is, therefore, a *metaphysical animal par excellence* or, to use Martin Heidegger's

¹⁰⁶ One should recall Kant's idea, expressed in the *Introduction* of the *Critique of Judgement*, that the «transcendental T» is *nature's legislator*, a role that in classical thought was attributed to God...

expression, the «shepherd of being», who leads existence from the darkness of the unknown to the light of knowledge.

5.1.2 The school of Miletus

In pre-Socratic philosophy the relation between being and thought as a rational rather than mythical-religious search for the beginning of being

Attention to the intimate relationship between human intellect and being dates back to the origins of Western thought, in particular to the apex of the pre-Socratic search on the *principle* of all things (**archē**), the *primary source* of being in all existing things. This search was no longer of a *mythical-religious* nature as in early Greek thought — with Hesiod's cosmogony, for example. It was rather of a *rational*, theoretical nature, or «scientific» in the original sense of a rational search for an *explanation* of what manifests itself to empirical evidence.

The first philosophical school, the so-called *School of Miletus* (Asia Minor) was characterized by this search for the *principle* of all things:

Thales: principle = water

Anaximander: the indefinite

Anaximenes: mist

- ◆ For *Thales* (640-560 B.C.): **archē** = *water*, vital principle of all things.
- ◆ For *Anaximander* (610-547): things are as such since they are *definite*, hence **archē** = *the indefinite* (**apeiron**).
- ◆ For *Anaximenes* (585-528): the materialization of Anaximander's **apeiron** led him to argue that **archē** = *mist*, source of all things through a mechanism of *rarefaction-condensation*.

5.1.3 The Pythagorean School

Pythagoras: numbers (points) given that all things are geometric entities

The second important philosophical school, the *Pythagorean School*, developed in Greater Greece from the teachings of its founder, Pythagoras (Croton, ca. 570 B.C.). The latter developed Anaximander's and Anaximenes' arguments by giving them a *mathematical* underpinning, for the first time in the history of western thought. For Pythagoras all things derive from the synthesis between *definite-indefinite* and *limited-unlimited*. Hence, being *geometric figures* is the essence of all things. Ultimately, these are composed of *points*, or indivisible units; therefore, for Pythagoras *points* = *numbers*.

All things are measurable: given that they are extended; and countable: given that they are composed of indivisible units

All things are defined as *measurable* —since they are extended entities (geometric figures)— and *countable* —since they are composed of indivisible units. Reality, then, originates from the *harmony of opposites*. The first and most important juxtaposition is that between *limited* and *unlimited* —that is, respectively, between *odd* (one, limited, form) and *even* (two, unlimited, matter)— given that all numbers, and therefore all geometric figures, can be constructed starting from one and two. The other oppo-

sitions, which derive from the first, are those of *straight-curved*, *stillness-motion*, etc.

Modern science is in some way related to the Pythagorean dream

As mentioned in Chapter One, the birth of modern science, with its attempt to reduce physical entities and their variations (bodies and their motions as mathematically definable) to geometric objects and relations is a sort of fulfilment of the Pythagorean metaphysical dream. In fact Copernicus, at the dawn of modernity, referred to the Pythagorean theory of harmonies to justify metaphysically his theory on planets' motion.

5.1.4 Parmenides of Elea

Parmenides' distinction between the two «ways»: first metaphysical and anthropological correlation. Beginning = being

Another Greater Greece philosopher —Parmenides from Elea (presently Velia in Cilento: 520-440 B.C.), who introduced the concept of *being* in Western thought— established the first link between metaphysical and anthropological issues. In so doing, Parmenides was able to distinguish two categories of men: the first is the category of those that «sleep» in the so-called *way of belief* (*doka*), accepting as true the fusion of being and non-being that is linked to quantitative *multiplicity*, qualitative *diversity* and *change*. The second is the category of those who «awoke» to the awareness of «being», thus walking the *path of truth*. *Being* is the beginning of all things. It is absolutely *positive* and in no way intermingles with *non-being*, which is understood as absolutely *negative*.

It is necessary to say and think that being is: indeed *being is and non-being is not*. These things I urge you to consider (Parmenides, *Fragm.* 6).

Identity between thought and being: beginning of rationalism

Being, therefore, is the only thing that can be thought and expressed. Hence Parmenides' position claiming the *identity between thought and being*, which created the *basis of rationalism* as well as the systematic confusion, in *metaphysics*, between *entity* and *existence* and, in *ontology* and *epistemology*, between the being of a *thing* (material entity) and the being of the *concept* that refers to it (logical entity): "... Indeed, *thinking and being are the same*" (*Fragm.* 3). In other words, Parmenides' mistake lay in conceiving of being *univocally*, as a «most general genus», a unique notion, the most universal of all, rather than in multiple ways as Plato, Aristotle and all subsequent classical metaphysical thinkers will do.

First, confused formulation of the p.n.c.

On the other hand, we owe to Parmenides the formulation of the *fundamental logical principle*, the so-called «principle of non-contradiction» (p.n.c.) that, for the philosopher, due to his rationalism, was also the *fundamental metaphysical principle*. In this sense, Parmenides did not distinguish between *meta-logical* and *metaphysical* and/or *ontological* use(s) and formula-

tion(s) of this principle, as instead is necessary in order to avoid mistakes.

Apparent nature of: Given these premises, Parmenides asserted the purely *apparent* nature (= way of belief) of:

1. Quantitative multiplicity ♦ *Multiplicity*: in order to quantitatively divide one entity from another, I must divide it through non-being, but non-being is not, hence being is one.
2. Qualitative diversity ♦ *Diversity*: in order to qualitatively distinguish one entity from another, I should allow the reality of non-being (if one thing «is this» then necessarily it «is not that»); but non-being is not, hence being is undifferentiated.
3. Change in its different forms ♦ *Change*: in order to claim any form of change, even simple local motion, I must accept the passage from being to non-being and the reverse (e.g., in the passage $A \rightarrow B$, A becomes non-A and non-B becomes B). But non-being is not hence being is immovable.

Being = sphere
Contradictory
notion, once we ask
whether the sphere
is limited/unlimited
(Melissus of Samo)

An entity, that which exists must therefore be *one, undifferentiated, immovable*. From this derives the notion of being as *sphere*, as a single entity (*panentheism*: all is one single entity) that should be able *to contain itself*. However, here lies the problem, a theoretical difficulty that one of Parmenides' disciples, Melissus of Samo (ab. 444/439 B.C.), identified and treated. If being is one single entity, it cannot be *limited*, because unlimited denotes a non-being, a negativity. Therefore, it will be limited, that is, a «sphere». But who *limits* the sphere, if being is limited and, by definition, it is all contained in the sphere? This limit should be «outside» the sphere. But who will be able to limit being, other than non-being, if all being is in the sphere? Non-being however, is not... Hence, whether being is considered limited or unlimited, it will have to deal with non-being: here lies the antinomy. Clearly Parmenidean metaphysics, just as any rationalist metaphysics—as well as, in modern terms, any formalist meta-logic—is prone to antinomy.

5.2 Democritus' atomism

5.2.1 The first answer to Parmenides

Problem: 'saving' phenomena without undermining the p.n.c., basis of any logical rationality of discourse and theory

Beyond the criticism illustrated above, which is applicable to any Parmenidean and/or rationalist interpretation of the metaphysical identity between essence-existence —or the epistemological identity between being and ideas— later philosophers aimed at solving the so-called «problem of Parmenides». This problem can be subsumed under the following question: how can we reconcile the notion of being with the «evidence» of numerical multiplicity, qualitative difference and temporal change of entities? According to Aristotle's famous expression, the problem is one of *saving phenomena* (= what appears evident) without undermining the laws of thought, first and foremost the p.n.c., to which Aristotle devoted memorable pages in his *Metaphysics*.

First answer to Parmenides by Democritus: non-contradiction in the notion of quantitative multiplicity

Democritus of Abdera (Thrace, 460-370), who demonstrated the *non-inconsistency of multiplicity*, gave a first answer to Parmenides. Zeno of Elea (one of Parmenides' disciples) had highlighted the contradiction in the notion of *infinite divisibility* of extended material entities. For this reason, in order to allow the existence of ultimate indivisible parts, or *atoms*, of extended reality, it was necessary to overcome Parmenides' and Zeno's idea that the notion of *numerical multiplicity* is inconsistent.

To justify multiplicity, the existence of *nothing* (= absolute non-being) is not necessary, since the notion of void (= non-being or absence of matter) is sufficient

Democritus' idea was simple and powerful, and it represented a great step forward for the history of Western thought, even if it is Plato who developed it to its fullest. To be able to divide atoms (the ultimate parts of extended reality) it is not necessary that *non-being exists*, which would be contradictory. Rather, it is necessary that the *Void*, intended as *absence of matter*, does. The Void's nature is not absolute non-being, but the simple *lack or absence* of something, in this case of matter. As Plato later explained in his dialogue *Parmenides*, the contradiction lies with the *absolute* juxtaposition between being and non-being. This, however, does not prevent one thing from *being* in relation to something and *not being* in relation to something else. The Void is absence of matter and not the absurd «existence of non-being». The Void is not nothing; rather it is the «non-being of something». Similarly —even if mathematical thought will come later to this conclusion, during the Middle Ages with Arabic mathematics— the «0», with which I refer to a certain discrete entity, or «atom», does not denote «nothing» compared to «1»; rather it denotes the absence of that entity, hence the Void.

5.2.2 Metaphysical atomism

Atomist
reductionism:
reducing qualitative
differences
(diversity) to
quantitative
differences

Democritus' idea was therefore to reduce all differences among entities, including *qualitative* ones (= *diversity*), to *quantitative differences* (full-empty, one-many) among atoms. Indeed, according to Democritus all atoms are different:

- ◆ *In themselves*, for their shape: according to Democritus each atom has a specific geometric shape, understood as a synthesis between full and empty.
- ◆ *Compared to other atoms*, for their *ordering*, hence *position*. Therefore each body composed of atoms differs from others for the number and/or shape and/or order and/or position of the atoms it is made of.

Reduction of all
forms of change to
one: the becoming
extrinsic to bodies
typical of local
motion

Democritus was then able to justify also a particular *form of change: local motion*. Local motion does not imply a passage from being to non-being, but simply the *change of position* of an atom (= full) from an *empty* space to another *empty* space. Hence we have the idea that atoms form *different possible worlds*, by moving locally along absolutely deterministic trajectories, and by colliding with one another.

Reduction of every
body to a collection
of atoms without the
intrinsic unity of the
substance that
implies actuality-
potentiality

Democritus' cosmology was the first and most fundamental instance of *metaphysical atomism*. Every body is nothing but a sum or aggregation of *atoms*, which possesses no *intrinsic unity*. In opposition to Aristotle, Democritus did not make a distinction between *being-in-actuality* (the «whole»), *being-together-of-parts* (*σύνολον*, *composite*) or «substance», and *being-in-potentiality* (material constituent parts), hence he did not seize Aristotle's notion of bodies as «composite» of *matter and form* (Cf. § 5.4).

The metaphysical
reductionism of
atomism also
touches on
anthropology
(atomist explanation
of the mind)

Given its metaphysical nature and consequent tendency to give an ultimate explanation to reality, such an atomism also touched on *anthropology*. According to Democritus, the human soul is composed of *lighter atoms* (air and fire) distributed in the sense organs, in the nerves and limbs.¹⁰⁷ Mental states are different from physical, bodily states, because bodies are primarily made of heavier atoms. In this sense, all present *materialist* theories that equate mental states with electro-chemical modifica-

¹⁰⁷ In ancient neurophysiology –according to a hypothesis that remained valid up to Luigi Galvani's (1737-1798) electricity experiments on the freshly skinned bodies of frogs– nervous impulses were transmitted at a distance to nerves through *pneumatic* impulses (compressed air). All ancient physiology, including the Bible, considered nerves as small capillaries containing *mist* (hence “air, water and fire”, the famous “bodily spirits” that Descartes will also treat) for the transmission of sensory-motor impulses at a distance between body parts.

tions of the Central Nervous System are renewed versions of Democritus' classical metaphysical anthropology.

As well as on gnoseology: sensualism as the theory of identity, in a sensation, between an organ's physical state and the related mental state

Similarly, Democritus' theory of knowledge (= *gnoseology*) was historically the first and most fundamental form of *sensualism*, which implied the ultimate *identity* between physical states (of the sense organs) and mental states (of the sensory faculty). Naïvely, Democritus claimed that sensations occur because atoms, while parting with things, penetrate through the pores of the sense organs that have the same shape (e.g., triangular atoms enter triangular pores, etc.). In this way they touch and stimulate the atoms of the soul, thus provoking a sensation.

All theories that are monistic in gnoseology are monistic in psychology. This is not applicable to Democritus, since the distinction between matter and spirit is Plato's

This validates a principle that will be relevant through all our illustration of the various Western philosophical anthropologies: all theories that are *monistic* in gnoseology (= *identity* between the form of a physical state in an organ and the form of the related conscious state in the mind) are also *monistic* in psychology (= *reduction* of the soul to a bodily function: *materialist* monism). Democritus never explicitly spoke of materialism, unlike his ancient and modern successors (e.g., the Epicureans and Sceptics with Pirrone and Sextus Empiricus among the first, D. Hume, the Positivists and Neo-Positivists of the last two centuries among the second). This, however, is only because Plato had not yet made the distinction between the *spiritual* and the *material* world —between the *hyperuranium* (= «beyond the sky») of incorporeal forms and self-sustaining essences, and the *material* world (= «beneath the sky») of physical entities.

5.3 Plato's idealism

5.3.1 The discovery of universals

The discovery of logical universals

When discussing the ontological consequences of Gödel's theorems of incompleteness we could point out how the discovery of logical universals was the fundamental, indeed *primary* discovery of Western science, in its distinction between logical-mathematical and physical sciences.

Notion of the spiritual nature of the human soul linked to the discovery of the necessarily immaterial nature of logical universals

Starting from the origins of philosophical anthropology with Parmenides, questions of *gnoseology* have been strictly connected to questions of *metaphysics*. This appears evident with Plato, who formulated the first *dualist theory* in cosmology and anthropology. According to the philosopher, in the universe there are two worlds. One is «material», made of entities in continuous change; the other is «immaterial», made of entities that are immovable in their fixity without change. Similarly, the human

microcosm is not one single substance, a psychophysical unit or «person», as it will be defined later in the Middle Ages. Rather, it is the sum of two substances, one spiritual (the soul) and the other material (the body). This dualist theory of the *spiritual* or *immaterial* nature of the soul is linked to the fundamental discovery made by Platonic gnoseology: the necessarily *immaterial* nature of conceptual universals —the immediate objects of the act of thinking that are expressed in (elementary and logically reversible) analytical propositions.

After Democritus, unity/multiplicity of quantity linked to matter. The uniqueness of universals is non-numerable

After Democritus, it became clear to Greek thought that *quantity*, understood as both extension and numerability, is linked to matter. What becomes also clear, thanks to Plato, is that concepts are characterized by *universality* —the *unrepeatable and non-multiple uniqueness of ideas* (εἰδῶς). As non-multipliable and therefore non-repeatable in a countable sequence, ideas will necessarily have a *non-quantitative*, hence *non-material* nature. A universal, an intelligible concept of the mind —that to which each definition of «what is» ultimately refers, the entity of something, or its essence— is neither one nor many: it is a *unique, unrepeatable* one.

A quantitative unit is typically repeatable, unlike a universal: it only refers to itself; it is *self-referential*

Numerical quantification is characterised by the possibility of repetition, in the sense that one same unit can be extrinsically attributed to many, even different entities. To the contrary, when we define something by assigning it a *quality* that makes it distinct from any other thing, we indicate something *intrinsic* to that thing. This belongs to it in such an intimate way that it constitutes its *unrepeatable uniqueness compared to anything else*, its being «universal», *unum versus alia*, «unique-compared-to-others», the principle of its being *identical to itself* and therefore *different from anything other than itself*. The term «universal» precisely refers to this capacity of an idea to be unique and able to refer only to itself (self-reference) in order to be characterized.

This property is defined as the *being-for-itself* of every idea, or essence

In Western philosophical thought, starting from Plato, this property is defined as the «being-for-itself» of every idea or universal in the logical order, but also of every essence in the metaphysical order.¹⁰⁸ For example, the only way in which I can define the *unrepeatable uniqueness* (i.e. universality) of the idea of «red» is to say that «red is red»: self-reference is the core of the immaterial formality of Plato's εἰδῶς.

¹⁰⁸ Due to this characteristic *being-for-itself* (self-reference), ideas and essences have often been equated in Western thought, starting from Plato. The clear distinction between logical and ontological levels, hence between ideas and essences, is due to Aristotle.

Ideas are never definable: they are the self-referring *primum* to which any speech ultimately refers

An *essence* in the Platonic sense, or «idea», is never definable, but is only that to which every definition ultimately refers, while it can refer only to itself. As such, therefore, it is *immaterial*, un-extended. Stated in modern terms, a Platonic essence, or idea, is neither a predicate nor a symbol of class, since no element or set of elements in a predicative domain can exhaust it. It is rather the source of the predicates that refer to it, each with its own domain and related classes that more or less coincide. For example, the predicates «being water» and «being H₂O» ultimately refer to «water-ness» or idea-water in order to acquire meaning.

Eidos is not a predicate, neither a symbol of class, but a *totalitas ante partes* that determines the related *totalitas ex partibus* (class)

In Aquinas' words, a universal is a *totalitas ante partes*, an ante-predicative «totality without parts», distinct from the *totalitas ex partibus*, the «totality made of parts» (classes) that, as a predicate, it determines. In classical logic of predicates, the extensional component originates from the intensional one. The attribute of self-reference that characterizes Platonic ideas is not of a predicative kind. Also from this point of view one can understand the accuracy of Gödel's second theorem, and its strong connection with the Platonic roots of Western thought (Cf. above § 3.1.3.)

A universal is not the collection of all infinite elements that can be described with the same predicate

For example, when I connote a given material entity as «a cat», I implicitly state that that connotation can be common to many material entities that are *similar* to it. Clearly, I can try to define the «being cat» by referring to other properties that are common to all cats and only to them (e.g., meowing), as well as to other properties that are common to other animals different from cats (e.g., being a feline, a mammal, etc.). That to which all these definitions and connotations ultimately refer, however, is a *unicum* —«cat-ness»— that, as such, is no particular «cat», neither is it the sum of all possible (infinite) materially existing cats, which can be perceived through the senses. This *unicum* of the universal-cat is closely linked:

Universals are linked: 1. Ontologically, to essence; 2. Logically, to logical universality

- ◆ *Ontologically*, to a specific essence (= «secondary» substance), that is, to the existence of a (possibly endless) multiplicity of entities that are all «cats».
- ◆ *Logically*, to the fact that «always and anywhere», even in different languages, all men call «cats» these entities taken as a whole, and «cat» each one of them.¹⁰⁹

¹⁰⁹ Wittgenstein spoke of one *universal grammar*—that cannot be expressed linguistically in itself—to which all logical grammars of the different languages—that can be expressed linguistically—refer, in order to be able to justify the possibility of their translation and the universality of their comprehension.

Objects of intellectual, rather than sensorial knowledge



«Cat-ness», the self-referring «being-cat of cats», is the intelligible core, unique-compared-to-others, *unum versus alia*, hence «uni-vers-al», to which each connotation and/or definition of «cats» that materially exist as *individuals* ultimately refers. As such, it can only be *immaterial*, and therefore it can only be apprehended intellectually, not sensually, contrary to Parmenides' being. Here is how Plato explains this point in *Cratylus* (386a):

If, therefore, things are not always the same for all (that is, there are real differences among things, *N.o.A.*), nor is every thing in its own way for each one (that is, there is some knowledge that is universal, common to all, against the opinion of Protagoras and the Sophists, *N.o.A.*), it is clear that *things have a particular and immutable essence in themselves, they do not depend on us*, neither do they fluctuate according to our imagination, but *they exist in themselves*, related only to their own essence, as prescribed by nature.

Plato's cosmological and metaphysical dualism derives, as in Parmenides, from the confusion between the ontological and logical realms, between essences and ideas.

The core of *Platonic metaphysical dualism* lay precisely in this discovery of the *immaterial* character of *forms*, or *essences*, or ο *ουσιᾶι*—*ousiē* (literally, «entities»), even if intended differently from Aquinas. These are unique, «universal» essences, which are actualized in different *material* individuals of the same kind, and which one can apprehend only intellectually. This is the basis of Plato's division of the universe of existing things in two worlds: the *world «under the sky» of material entities*, which are multiple, individual, changeable, contingent, objects of sensorial knowledge; and *the world «beyond the sky» (hyperuraniūm) of intelligible entities*, which are unique, immaterial, immutable, necessary and not knowable through the senses. These entities exist independently from, *prior to*, both their possible, material realizations in the different individuals of the same physical species, and the human intellectual knowledge of them.

Differences with Aristotle and Aquinas

Naturally, if we apply the Aristotelian and Thomasian distinction between *quidditas* (intelligible) and *essence* (Cf. § 6.2.1) on the one hand, and *entity* and *existence* (Cf. § 6.2.2) on the other, Plato's discovery of the *being-for-itself* or *self-reference* of intelligible universals (of their being *for themselves*) becomes distinct from the *being-in-itself* of substances (their being *in themselves*, as *existing* things). Similarly, the *necessary immaterial nature of conceptual universals* becomes distinct from the presumed necessity that they exist as such, as spiritual substances, «outside» the mind, in an immaterial world. That which exists «outside» the mind is the anything but immaterial *essence* of a physical entity, which is immanent to the latter as its intrinsic «nature». It is at once the result of a precise causal concurrence and the

principle of the specific operations that are characteristic of that entity.¹¹⁰ It is a nature from which universals are abstracted through an act of the mind.

Universals, because they are immaterial, only exist in the intellect and are not the essences of beings.

Therefore, universal logical entities, characterized by immateriality, or by the pure formality of self-reference, and absolute *being-for-themselves*, only exist in the mind that thinks them, they are *beings-for-themselves* immanent to the mind. However, they have an extra-mental foundation to their universality,¹¹¹ which is anything but immaterial (at least concerning physical entities) and to be found in the *being-for-itself* of the being of an essence, or the entity of existing things (e.g., the natural entities that exist outside the human mind, but in a way that is not immaterial).¹¹²

Foundation of logical realism and primarily individual character of entities

Following Aristotle, it is therefore possible to be a realist and non-nominalist in logical ontology (logical universals, with their immaterial character, have a real extra-mental foundation), without sharing Plato's dualism in metaphysics (universals are neither entities nor immaterial substances that exist «outside» the mind, in some hyperuranium). «Outside the mind» there are only individual material substances, bodies, each of them with its own immanent essence. The essence, or «nature» of things is «inside» things themselves, not «outside» them, in the world of ideas. If spiritual substances also exist, they are not «ideas», but «intelligences», personal entities, even if not «individuals», given that they are immaterial. These are the fundamental developments that Aristotle first, Aquinas later, derived from Plato's metaphysical reflections.

¹¹⁰ For example, in the case of the nature of an animal species, the so-called «ecological niche», which is essential for a species to survive or, in atomic and sub-atomic physics, in the case of the specific nature of a given kind of sub-atomic particles, their characteristic «Feynman's diagram», etc. We will come back later, in the second volume of this work, to this notion of «nature» of a given kind of physical entities, both living and inanimate, intended as a «passive principle» that is the effect of a stable causal concurrence of multiple factors.

¹¹¹ Philosophers of nature cannot discard in principle the existence of purely spiritual entities (angels, immortal souls, God...), even if these are not the objects of their metaphysical investigation. Clearly, these entities would have a purely formal or immaterial essence.

¹¹² The *being-for-itself* of a material substance, or body, can only be relative. Because of its material character, a material substance cannot completely refer to itself, hence act completely for itself (each moving thing is in turn moved). Similarly, its self-identity is not absolute, because it is always subject to change (self-identity, the being-as-such of a body, relates to its formal component, or essence, not to the material parts that make it up, which constantly change through time). These notions, however, are proper of Aristotelianism, particularly in its Thomistic interpretation.

Metaphysical theories, even in their most developed form, are incomplete and partial.

Plato still partly bound by Parmenidean rationalism

These developments, that are only an example and should not be considered as exhaustive, are not surprising, if one thinks that metaphysics is a demonstrative science, and that any specific metaphysical theory, because scientific, is necessarily incomplete and partial.

We can conclude this brief illustration of Plato's fundamental discovery—without detracting from its importance, given that, without it, logical science, metaphysical science and perhaps science *tout-court*, in the Western meaning of the word, would not exist—by pointing out that the philosopher's limit was in the strong legacy of Parmenidean rationalism. Because of this, he was unable to distinguish between entities of nature and entities of reason, hence between the ontological and the logical realms.

5.3.2 Knowledge as recollection

Gnoseological theory of intellectual ascent as a consequence of metaphysical theory

Plato's gnoseological doctrine directly derived from his dualist metaphysics. From the gnoseological point of view, Plato's philosophy was an attempt to give an *objective*, metaphysical *foundation* to Socrates' (Plato's master) theory of knowledge, which was grounded on the *maieutic principle*. According to this principle, knowledge consists of bringing to light, through one's *self-knowledge*, an intelligible set of eternal, *pre-existing* «ideas», which are the basis both of the knowledge of the human mind and of the single thing or physical entity that is the object of experience, and that exists «outside» this mind.

Universals cannot derive from (material) sensation: pre-existence of universals

- ◆ Clearly «ideas», because immaterial and universal in nature, cannot be induced from experience, which is material and particular. If they are apprehended by men—who are indeed capable of universal knowledge—they must already «lie» deeply in the unconscious of their experience. Experience does not build them, or abstract them, as Aristotle will later argue; it simply brings them to awareness, makes them evident, re-cognizes them in the objects of experience.

Identification, in the doctrine of ideas, between logical universals and essences of things

- ◆ Other than being immaterial and universal, these «ideas» are the «essences» (ουσιαι, ousiê), or profound entities of multiple material things (objects of experience). These, in turn, are like individual, partial copies of these models, these universal, unique and unrepeatable paradigms.

Platonic ideas, therefore, are the foundation of both *knowing* and *being*, as well as of their *correspondence*.

Knowledge as
recollection

Plato's doctrine of knowledge is found primarily in the famous dialogue *Meno* (82b-86c). How is it possible — asks Plato — that an illiterate slave, who has never studied geometry, was made able to demonstrate Pythagoras' theorem, through a simple maieutic procedure? If nobody has taught it to him, then it means that, unconsciously, he already possessed it. To be able to know it, he simply had to *remember* it (knowledge = *anamnēsis*, *recollection*, *memory*). Every *knowledge*, for Plato, is therefore a form of *recollection*, an instance of «taking from oneself» a universal knowledge that has always been there. Contrary to what Aristotle will argue later, universal ideas are not *abstracted* from the particular, but simply recognized in coincidence with a certain experience, because universal ideas have always been *formed in themselves*. For Plato, therefore, universals are *substances*, because they exist on their own, independently from the mind that thinks them or from the single entity in which the particular form exists as a *copy of likeness*.

Objective,
substantive nature
of ideal realities,
even if they are
invisible

Plato justifies this position in another dialogue, entitled *Phaedo* (73c-ss). There is a gap between sensory knowledge (which is imperfect) and the related intelligible universal notion (which is perfect: for example, there is no perfect square or circle in the sensory world). The intelligible, therefore, cannot derive from the sensorial; rather it has to precede it. In practice, Plato identifies the problem of the justification of *inductive inference* — which Aristotle will formalize in his treatment of *inductive syllogisms* (*An. Pr.*, II, 23) — as a process that constitutes, rather than demonstrates, universal, predicative propositions. Plato's solution to this problem is the following: there is an *objectivity* of ideal realities, notwithstanding their invisibility (= they belong to the «Hyperuranium», they are therefore «beyond» the sensorial), which is the foundation of both the physical *reality* of material entities that have that form, and of the *knowledge* that refers to them.

The line segment
analogy and the
allegory of the cave

At the end of Book VI of the *Republic* (509c-ss), Plato identifies *four levels* of human cognition: *imagination*, *belief*, *understanding* and *intuition*. The four levels, as well as an individual's process of cognition, are explained in Book VII of the same work, through the famous *allegory of the cave*.

- ◆ Sensory knowledge, or *opinion* (*dōxa*, *doxa*):
- Knowledge of the images of senses, or *imagination* (*eikasía*, *ei-kasia*);
- Knowledge of sensible objects, or *belief* (*pistis*);
- ◆ Intelligible knowledge, or *science* (*epistēmē*):

- Knowledge of mathematical objects, or discursive knowledge, *understanding* (*diahoia*)
- Knowledge of intelligible objects (Ideas) up to the absolute Unity of the Idea of Good, or *intuition* (*nohsij*).

5.3.3 The doctrine of participation

Logical ascent: from the multiple to the One versus ontological descent from the One to the multiple

From the point of view of *logic*, this ascent is a passage, through varying degrees, from the *multiple to the one-ness of universals* (e.g., from images to objects; from objects to ideas; from ideas to the One) which corresponds, *ontologically*, to the reverse passage from the *unique to the multiple* (from the One to ideas; from ideas to objects; from objects to images). Plato's world is then conceived of as a kind of *deductive system*. In a deductive sequence the *multiplicity* of consequences *participates in* the *truth* and *uniqueness* of *premises*; similarly in the Platonic world. Hence it is always possible to go up or down the different levels. At the intelligible level Ideas, in their hierarchy, participate in the *One*; at the sensory level, the *multiplicity* of sensory objects (of the same kind) participate in the *uniqueness* of the Idea.

Plato's metaphysics: metaphysics of formal participation of the many in the one

Platonic participation is therefore the *formal participation of the many in the one*. As in geometry, starting from a unique object with *n* dimensions (e.g., three-dimensional) one can create different projections with (*n*-1) dimensions (e.g., two bi-dimensional projections). These are only simulacra; they only participate in the wholeness of the original model. Platonic participation, in other words, is not Aquinas' *participation of being*. The latter assumes that the entity of one thing is not in the hyperuranium, but in the metaphysical constitution of each existing thing—in the network of causal relations that make that thing exist and make it exist as such, that is, its *essence*—thus determining the *modality* with which such a thing *exists* and participates of the unlimited *wholeness* of *being-as-act*, according to the degree of its *essence* or *limited actuality*.

5.3.4 Dualist anthropology

Metaphysical dualism → anthropological dualism; loss of Man's unity and individuality

From this theory of knowledge derives a direct *anthropological consequence*: the necessity of anamnesis implies that the soul is *pre-existent* to the body and therefore its relation to it is not intrinsic. Stated otherwise, the soul for Plato *is not individual*. Successive reincarnations in many individual bodies are possible, according to the moral improvement reached with each preceding incarnation. Plato's notions of immortality and spiritual-

ity of the soul are different from the Christian notion of *individual* immortality, which rejects dualism and the identification of the body as the «prison» of the soul (*Phaedo*).

Doctrine of the three human souls

Plato divides the human mind in three parts, or «souls»: the *rational*, the *inascible*, and the *appetitive* (Rep. IV, 438d-440a. In *Timaeus* he adds a fourth soul, the *sexual* one, while he calls the third *nutritious* 70d-72d), each of which is located in a part of the body (Ivi, 69c-77c). Only the rational soul can be *immortal* (Rep., X, 611b-612a; *Tim.*, 42a-44a, 89d-90d).

Doctrine of the soul of the world

The analogy between the «soul of the world» and the «rational soul» (*Tim.*, 41d; 43a; 47b-d; 90d; *Leg.*, X, 891b-893d) is closely connected to this doctrine of the soul. This analogy considers the universe as one single living being (*panpsychism*), and the soul as the «movement that moves itself», therefore the principle of any other movement, both in the world and in the human body (*Ivi*, 894a-898d; Cf. *Tim.*, as above).

Definition of the soul as «capable of self-movement». Aristotle extended this definition to all living things, spiritual and material

Contrary to this, Aristotle will attribute the ability of «self-movement» to the whole of living things (soul + body), where the soul is the formal-final cause of motion, but not the moving cause. In organisms, only the action of one (organic) part over another (organic) part is the moving cause. The animate body moves itself, because the soul, as the *form of the body*, gives unity to the whole body in its different organs, so that the action of each organ on the others is *immanent* to the body itself. These metaphysical notions of Aristotle's biology will be dealt with in Part Four. Also, contrary to the «*panpsychism*», or «*vitalism*» (= all is animate, or living) inherent to the Platonic doctrine of the soul of the world as presented in the *Timaeus*, Aristotle attributes to «separate intelligences» the function of simple final causes of the perfectly symmetrical motion, with no beginning or end, of the «celestial spheres» that make up the universe. Given the natural perpetuity of their movement, these spheres are the ultimate moving causes of the motion of all bodies that make up the physical universe.

For Aristotle moving causality exclusive of material beings: the soul does not, and cannot «move» particles

For Aristotle, *local motion* and *moving causality* are properties *specific* to the physical or material universe, which make it different from the spiritual universe. Thanks to this distinction it becomes clear, with Aristotle, that the causality exerted by forms or spiritual substances on matter can never be of an *efficient (moving)* kind —that is, able to provoke local motion in physical bodies.

Forms, whether «material» (= forms of physical entities) or «spiritual» (= rational souls) *never act*, they never impart *initial*, moving action on material bodies. We will come back to this principle, which is fundamental to

understand the *proprium* of Aristotle's hylomorphism in comparison to ancient and modern dualism.

5.3.5 The second answer to Parmenides

Qualitative difference does not imply the absolute non-being of an entity (=nothingness), but only of one of its parts, in this case form and not matter, as with quantity. The problem of change still unsolved

As Plato himself explained in his dialogue *Parmenides*, the definition of *difference* —of quality as not reducible to quantity— through the doctrine of ideas entailed overcoming the allegation of inconsistency advanced by Parmenides (Cf. § 5.1.4). For example, the «difference» between A and B, while implying that A means *non-B* and that B means *non-A*, does not entail the notion of *absolute* non-being. It does not negate «all A» or «all B», but something specific of A or B —the form «a» of A and the form «b» of B. «Difference», therefore, implies *relative*, not absolute *non-being*. «Different entities» are not different by opposition of *contradiction* (A/*non-A*) but by opposition of *contrariness* (A/*non-a*).

Difference is an *opposition of form*, not of *being*. By affirming B, we do not negate «all» A, but only its form. Clearly, then, A or B are not composed only of «form», but also of «matter» —physical or intelligible, according to whether they are physical or logical entities. Aristotle developed this brilliant intuition by Plato with his theory of *potency* and *act*, which allowed solving the third part of the «problem of Parmenides», relating to the justification of change.

5.4 Aristotle's actualism

5.4.1 Essences and logical universals

Criticism to formal participation: the diversified multiplicity of material entities cannot derive from unique forms

As explained, we owe to Plato the matter-form distinction, which allowed solving Parmenides' problem of the metaphysical justification of *difference*. However, Plato's metaphysics gave an insufficient justification to these notions of «form» and «matter», since it derived both the pair «identity-difference» and «form-matter» (respectively, Pythagoras' «one» and «two») from the ideal One, source of all forms and entities.

Formal participation is insufficient

Aristotle's main criticism to Plato was that the notion of *formal participation* (one-many) is insufficient to justify the passage:

- ◆ From identity to difference
- ◆ From discrete unity (form) to spatial continuity (matter)

Inadequacy of Platonic formal participation to ground the passage from identity to difference

Inadequacy of participation to derive difference from identity

According to Plato, any *difference* in forms derives, through participation, from the absolute identity *in itself* and *for itself* of the One. In the so-called «non-written doctrines» of later Plato (= Plato's oral teachings, which were not contained in the *Dialogues*), of which Aristotle was one of the main witnesses and sources, the philosopher tried to «explain» formal participation in a neo-Pythagorean sense, through the theory of ideal One-ness and Dual-ness, of the *One* and the *Dyad*.

Even in the dialogues, for example in *Philebus* (25a-26b), Plato called *Identical* the *One*, and *Different* the *Dyad*. Again in *Philebus* (56d-57a) he distinguished between two types of arithmetic:

- ◆ *Material arithmetic* (= applied arithmetic) is made of numbers defined according to different metric «units», on the basis of the unity in itself of the different material objects that must be measured/counted (e.g., the unit to count horses is different from that to count ants);
- ◆ *Formal arithmetic* (= used by philosophers and mathematicians) is made of *ideal numbers*, probably the Pythagorean *decade*, which is intended here according to the doctrine of Ideas, as a *paradigm*, an immaterial *model* of the material numbers of applied arithmetic.

Aristotle's metaphysics based on the allegation of inconsistency of Plato's thought

Hence we have Plato's idea, witnessed by Aristotle, to conceive the ideal *One* and *Dyad* as, respectively, the formal principle (= principle of determination) and the material principle, a kind of «intelligible matter» (= principle of indeterminacy and multiplicity: generic relation «big/small»), from which all ideas originate just as, in arithmetic, all numbers derive from 1 and 2 (Cf. also dialogues *Teet.* 191c; *Tim.* 50c). «Forms are like numbers», *formae sunt sicut numeri*, as medieval philosophers, including Aquinas, will argue, following Aristotle and developing Plato's legacy. The following is an excerpt from Aristotle dealing with this fundamental development of Platonic thought.



Given that Forms or Ideas are the cause of all other things, Plato concluded that the constitutive elements of Forms or Ideas are the constitutive elements of all things. And he took the *large* and the *small* as the material element of Forms or Ideas and the *One* as the formal cause: indeed, he thought that Forms and Numbers derive from the *participation* (*kata/meqekin*) of large and small in the *One*. (...)

From this, we see that Plato only considered two causes: the *formal* and the *material*. Indeed, ideas are the formal causes of other things, and the *One* is the formal cause of Ideas. And to the question of what the matter that acts as substratum is, of which we predicate Ideas—in the sensible realm—or the *One*—in the

Ideas realm¹¹³— he replies that it is the *Dyad*, that is, the large and the small. Furthermore, he attributes separately, to each of those two principles, the cause for good and bad, just as, in our opinion, some preceding philosophers, such as Empedocles and Anaxagoras, had tried to argue (Aristotle, *Metaph.* I,6,987b,18-22; 988a,7-17).

Problem pointed out by Aristotle: the Dyad cannot derive from the One

Aristotle brilliantly identified the main problem in Plato's system, which is typical of any formalistic system in metaphysics and meta-logic that strives to be self-consistent, therefore to ground itself. Having 1 and 2, the principle of formal unity and the principle of material multiplicity of the only form—that is, the *limit* and the *unlimited*, to use Pythagoras' words—I can create any *difference*. But how can I distinguish between 1 and 2? As such, 2 cannot be derived, formally «deduced», from 1. Indeed if 1 is made of a single quantitative unit («|»), two is made of two quantitative units («||»). However, it is also made of the *formal unity* of these quantitative units («0»), of the «two-ness of multiple units» («(| |)»). So, if the «form-two», the principle of difference between the «unit of 1» and the «units of 2», does not *already* exist, one will never be able to distinguish 1 from 2. Without the form «two», the units that make it up will never be «one 2», but the simple multiplicity of units «one-one», «1-1». Naturally, the same reasoning can be applied to any other whole number.

This is how Aristotle summarized his criticism:



Each of the units that compose the dyad will have to presume a prior dyad, but this is absurd. In other words, by virtue of what is the number, which is compound, something unitary? (Aristotle, *Metaph.*, I,9,991b31-992a,1).

(...) The Dyad itself will derive from the sum between another 'one' and the One in itself. But, if this is so, it is not possible that

¹¹³ In the realm of physical entities, matter defines a genus, for which forms, or Platonic ideas, define the specific difference. For example, we can predicate the «being-cat» or the «being-dog» of the organic matter that defines the genus «animals». In this sense, Aristotle argues that, of matter, in the realm of physical (sensory) entities, one predicates ideas. Of ideas, instead, one predicates the «being-one». For example, to speak in a Platonic sense of the «idea-cat» means to speak of the «unique-cat-model» of which all «materially existing» cats are «copies», or «participation». It is as if one stated the ultimate (formal) unity of the (material) multiplicity of cats. Similarly, in arithmetic, the «ideal five» is the paradigm or ideal model of the multiple «fives» that one can define on different numerical scales, that is, in relation to different units of scale. The unit of scale that I use to count «five» ants is different from the unit of scale that I use to count «five» elephants. These different types of «five», however, are all representations, participations, of the unique form-paradigm of the ideal «five».

one of the two principles from which the number originates is the *indefinite* Dyad: a single unit (*moḥaj*) engenders a numerical unit (*miān*) but, in this way, the dyad will never be derived (*Ivi*, XIII, 7, 1081b, 24. Cf. also: XIV, 1090b, 22).

Dilemmas of
Platonic formal
participation

Stated otherwise, with his theory of ideal forms and/or ideal numbers, Plato tried to explain those mental *universals* that reduce a multiplicity of (physical or mathematical) objects of the same kind *ad unum* (e.g., the idea «cat» reduces *ad unum* all possible instances of cats; the idea «five» reduces *ad unum* all possible sets of five objects, etc.). Plato, however, argued that these «universal forms» are not just in the human mind, but are also the objective foundation, *which exists in itself*, of all multiple instances of one same species that exist in the world. In this way, he considered as solved a *much more fundamental problem*. Plato's idea-paradigm expresses the unit-form of one genus (e.g., of cats or of numbers «five» defined according to different units of scale...): but what founds the *unit-form* of that genus, or «idea»? This problem can be viewed from two complementary perspectives:

- ◆ From the consideration that any «idea» is in turn *composed of parts common* to the whole genus (e.g., one head, one body, four paws and one set of whiskers for the «idea-cat»; a multiple set of quantitative units for the «idea-five»);
- ◆ Or, most importantly, from the consideration that this *specific*, hence *relative* unity, is *different* from the *absolute* unity of the One in which it participates. The formal unity that holds together the parts of a «dog» is not the same as the one that holds together the parts of a «cat», and none of the two is proper to the absolute unity of the One. How can these differences derive from participation in the One, which only represents absolute self-identity?

Similarly, in the passage from the *specific unity* of a given species of objects (e.g., the genus of cats), which is guaranteed by the form-paradigm or *specific form*, to the *substantial unity* of the single individual (the *substantial form* of the single cat that participates in the unity of the genus-idea that is *common* to all cats, while being distinct from it), what grounds the substantial unity-form of the single living entity (e.g., of a cat)? Given that:

- ◆ Such substantial unity is made of individual parts (e.g., that head, that body, those four paws and that set of whiskers for each single instance of cat);
- ◆ This substantial unity needs a unity-form different from the unit-form of the species (e.g., the substantial form of one cat, different from the genus-idea «cat») in which it participates.

Platonic formal participation cannot give a consistent answer to any of these questions raised by Aristotle.

Difference cannot derive from matter

In sum, «difference» cannot be based on a formal participation that simply multiplies the «same» form in a matter; rather, it can only be based on an additional *form* that is distinct from the original one.

Antinomy of Same and Different

All Western epistemological thought (metaphysical and scientific), from Plato to this day, is stuck on this antinomy between Same and Different, which represents its *original contradiction*. Stating that sameness (One) participates in indeterminacy (Dyad) is insufficient to ground «difference». Conversely, it is then impossible to reduce all differences among things to one original sameness, as any formalist rationalism in metaphysics argues, from Plato, to Hegel and Hilbert.

The One of Plato's Absolute Identity is not adequate to ground specific difference and individual peculiarity, contrary to Aquinas' Pure Act

Plato's One, the absolute Identity, is not the «good» One to constitute the original *arché* of which all beings participate, as they are living, but with their irreducible *specific difference* and *individual peculiarity*. The One of which all things participate cannot be Plato's Absolute Form, which can participate its abstract identity only with itself. It can only be an Absolute Being that contains all differences, all forms, as their single *causal principle*. Of course, such causality is not the same as in the Humian and Kantian modern interpretations; it is rather based on the relationship between potency and act. The One, understood as Pure Act, is the foundation of specific difference and individual peculiarity through a *double causal relation of potency-act*. It causes the various kinds of entities through its relationship with the potentiality of specific essences. Within the latter, it causes the individuality of entities through the relation of form with the potentiality of matter.

This is the core of the Thomasian idea of being-as-act (act of all forms) and of the Self-Sustaining Being as the One that contains the wholeness of being as Pure Act, as Active Principle, Primary Cause of all entities characterized by absolute difference and individuality.

To conclude, Plato's formal participation is insufficient to derive difference from identity. Original difference, or «two-ness», cannot derive from original sameness, or «absolute unity». Neither can difference in all its forms be reduced to one original sameness.

We now turn to the second part of Aristotle's criticism of Plato's formal participation. What is relevant here is whether this theory can justify the derivation of matter from form, through the passage from the One to the Dyad.

Inadequacy of Platonic formal participation to derive «matter» from the pure «form» of the One

Inadequacy of
formal participation
to derive matter
from form

As mentioned, Plato identified the dyad with a *material principle*. Once again, this derived from the legacy of Pythagorean doctrine, for which the dyad identified itself with the *original line segment*. If the dyad (= line segment) could be derived from the unit (= point), then not only could difference be derived from sameness, but spatial continuity (= matter) could be derived from discrete unity (= form). But, just as difference cannot be derived only from sameness, spatial continuity cannot be derived from discreteness. The dyad can be identified with the original line segment only if it is not only composed of a pair of points, but of the *indeterminate length* that the pair of points limits, or «cuts». It is clear, then, that indeterminate length, rather than the dyad, constitutes matter. Conversely, the dyad appears here as *form*, as «act» in its fundamental role of «cutting», «circumscribing», «determining» the *uninterrupted*, the *unlimited*, the *indeterminate*, in one word, *matter*.

The continuous
cannot derive from
the discrete, rather
the reverse

So, if matter is the «indeterminate length» rather than the dyad, then matter will never be derived from unity, given that no set of *n*-dimensional points, even if «thick», will lead to dimensional continuity. One can never pass from the *discrete to the continuous*, but only *from the continuous to the discrete* by «cutting» the first at one point («the form, as act, separates», said Aristotle).

Matter and form are
two original and
irreducible
principles that make
up any physical
entity

Hence, matter (continuous) cannot be deduced from form (= limit, principle that creates discreteness): this means that *matter* and *form* are two *primordial and reciprocally irreducible* causes, or «principles». The essence of physical, as well as of mathematical entities, cannot be made of sole form, but must be made of *form and matter* (Aristotle, *Metaph.*, VIII, 3, 1043a, 33.). According to Aristotle, the essence of any physical and/or mathematical object is *composite* (*σύνολον*, «composite totality») of matter and form (Cf. Figure 5-1).

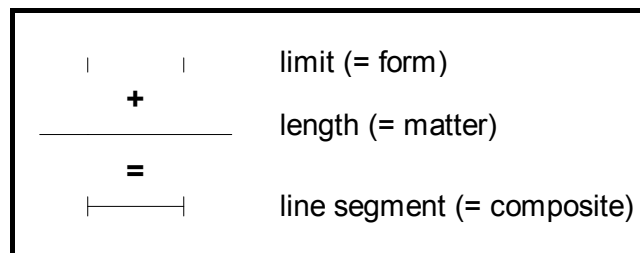


Figure 5-1. The hylomorphic (matter + form) constitution of a mathematical (and/or physical) entity according to Aristotle.

Matter and form are two principles (*id quo*) through which one entity (*id quod*) exists; together, they constitute the essence, substance of physical and/or mathematical entities

The essences of physical and mathematical entities, made of matter and form, are distinct from the related immaterial universals

Matter, therefore, cannot derive from form. This means that «matter» and «form» are two «original», irreducible principles (= «causes» in the sense of material and formal cause that will be explained below, Cf. § 6.3.2.1), which make up the essence of all physical and/or mathematical entities. Each of them is a principle «through which» (*id quo*) a «given entity» (*id quod*), whether a substance or accident, *exists in actuality*. For Aristotle, the nature or essence of any physical entity, any body or «material substance», is «composite» of (*primary*) matter and (*substantial*) form, just as the nature or essence of any mathematical entity is «composite» of matter (intelligible: extended quantity for geometric entities, discrete quantity for numbers) and form (intelligible). For this reason Aristotle's philosophy of nature is a kind of *hylomorphism*, from *u\$h* (*yle*, matter) and *mor fhl* (*morphé*, form).

Given that the ultimate essence of any physical or mathematical entity cannot be a pure immaterial form, simply intelligible, Aristotle introduced a fundamental distinction between *immaterial* logical universals, or purely intelligible forms—which are immanent to the mind—and the *essences* of physical and mathematical entities—which are immanent to these entities. Both of these are made of two ontological principles that cannot be reduced to each other: *matter* and *form*. The essences of things *are not* the intelligible universals (distinction between *natural*, or physical entities and *entities of reason*, or logical entities); most importantly, neither the former nor the latter exist separately from the individual material substances (= bodies) of the physical realm, contrary to Plato's spiritual substances of an ideal realm (as intelligibles/universals/essences).¹¹⁴

¹¹⁴ Here is an excerpt by Aquinas that cleverly explains this Aristotelian doctrine: «*Universals do not exist independently from individuals (universale non est aliquid praeter singularem)*», as demonstrated in the VII Book of the *Metaphysics* (...). If a universal is predicated of a multitude according to a single definition (*unam rationem*) and unequivocally, this universal, in relation to reason—that is, in relation to science and demonstration—is not less, but more of an entity than the particular: that which is incorruptible is more of an entity than that which is corruptible. A universal definition is indeed incorruptible; particulars instead are corruptible, given that they happen to be corrupted according to individual principles, but not according to the definition of species that is common to all, and that is preserved through generations. So, as for what is proper of reason, a universal is more of an entity than individuals. However, as for natural existence, individuals are more of an entity, so that they are called the primary and principal substances. (...) For this (Aristotle) adds that even if in universal propositions and demonstrations something is meant that is *one in itself* (that is, the quality, e.g., the universal of «triangularity», *N.o.A*), for example, the being-triangle, *there is no need to believe that the triangle is a unit existing independently from the many*. Similarly, for those things that do not signify a substance, but some kind of accident when we mean them in absolute terms, for example, when we say *whiteness* or *paternity*, we cannot believe that they exist separately from some existing individual substance. The intellect can apprehend something of the things that are united without understanding the other things (to which they are united), without this concept being false. If a white were also a musician, I can understand the white and attribute something to it

To conclude, formal participation is insufficient to ground the derivation of difference from sameness, as well as the derivation of matter from form. If form and matter — and not just form — are the ultimate and irreducible components of all physical entities, their essences, as «composite» (composed) of matter and form, become distinct from immaterial logical universals, where the former are immanent to objects themselves, and the latter are immanent to the mind that thinks them.

5.4.2 The notions of substance and categories

Distinction between
essence and
substance

The fundamental discovery of Aristotelian metaphysics, the notion of *substance* as distinct from that of *essence*, is implicit in the distinction between intelligible and natural entities. Thanks to this discovery, Aristotle further developed Western metaphysical thought, after the contributions of Parmenides, Democritus and Plato. The main text for this distinction is Chapter Five of the *Book of Categories* (5, 3b, 10-25), the work that introduces Aristotle's *Organon*, the first text of logic in the history of thought.

A universal is
always predicated
of a subject

Plato's fundamental discovery *vis-à-vis* his predecessors was that of the character of reflexive *self-reference* (self-identity) of a quality as such, which makes it *non-reducible* to anything other than itself. Ultimately, a quality refers only to itself to carry out its function, that of differentiating. This is the *being-for-itself* of any essence, any *ousia*, or entity. What Aristotle pointed out is that, as in language (logically) the intelligible quality, the logical universal, is always predicated of a subject (e.g., the «being red» of «blood»), in reality (ontologically) the essence always refers to «something that exists *in itself*» (*tode ti, hoc aliquid*).¹¹⁵ Specifically, it refers to an *individual substratum*, or subject in an ontological sense, which is itself irreducible and that Aristotle called *primary substance*.

and demonstrate something about it, for example that it damages sight, without keeping into account the fact that it is a musician. However, if someone inferred from this that the white is not a musician, this would be false. So, when we say that *whiteness is a colour* (= definition of quality as if it were a separate entity, that is, we define an abstract quality as a «one-of-one» universal, *N.o.A.*) without citing the existing subject (e.g., a white man, a white sheet, etc.) we state the truth. It would be false if we said that *whiteness, which is a colour, does not exist in a subject*. Similarly, when we say that *Man is an animal*, we state the truth, even if we do not mention any particular man. But it would be false if we said that *Man is an animal that exists separately from particular men* (Aquinas, *In Post. An.*, I, xxxvii, 328.330-331).

¹¹⁵ The translation of the Aristotelian *tode ti* is a true *mex* for translators. Etymologically, from Greek, it means «this something», that is, it indicates the ostensively indicated referent of a definition. As if, after defining an object, we were asked: «what are you talking about?» and, while indicating the object we were referring to, we replied: «about this thing». «This thing» is therefore the ontological subject of our definition, the «bearer», the concrete «realization» of a given essence.

The being-in-itself of a subject is irreducible

The *being-in-itself* of the primary substance is also relative to some kind of *self-reference* (*reditio ad semetipsum*, return upon oneself, as Aquinas will call it), which is distinct from and complementary to the self-reference proper of the *being-for-itself* of quality, characteristic of any essence.

In logical terms, the word that denotes a quality (grammatically, an adjective and/or verb) can never be a grammatical subject, but is always a predicate in declaratory propositions. «Man» in the sense of «human» is always the «being man» of an individual subject, e.g., «Plato», «Socrates», «John», etc. Only by considering it in the abstract, as an intelligible idea («Men», predicate made substantive by the plural form, in the sense of «humankind»), an essence can be used as the subject of declaratory and/or definitory propositions (as in «men are living» or «men are rational animals»).

As one cannot go back *ad infinitum* in defining a quality, one cannot go back *ad infinitum* in attributing quality. Any individual is irreducible.

Something similar can be said of the individual substratum. «Substratum» indicates what can never be attributed to something else as a quality, given that it is that to which, ultimately, all qualities are attributed as its «properties». If it is impossible to go back *ad infinitum* in the attempt to *define* a quality because, ultimately, it only refers to itself («red is red»), similarly it is impossible to go back *ad infinitum* in *attributing* a quality (e.g., attributing «being man» to smaller and more specific human groups: «Europeans», «Italians», «Romans» etc.), because, ultimately, it is always one *self-sustaining individual*, «existing in *itself*», that possesses that quality as its attribute.¹¹⁶ In logic, therefore, the individual substratum is what the subject of a declaratory or definitory categorical proposition denotes and that, as such, can never be denoted with a term functioning as predicate.

In both Ancient and Modern philosophy, the two kinds of self-reference, the *being-for-itself* of quality and the *being-in-itself* of individuals, are often confused

Plato's mistake (made also by many modern logicians, starting from Frege) consisted in confusing the irreducible character (ultimate self-reference) of subjects with that of predicates. In other words, he confused the self-reference of individual substrata (self-reference of *being-in-itself*)—which will be called *reditio completa ad semetipsum* (= complete return upon itself) of any substance by Scholastic meta-logic and metaphysics¹¹⁷—with the self-reference of qualities (self-reference of *being-for-itself*)—that will be called *esse secundum se* (= being according to itself) of any quality by Scholastic meta-logic and metaphysics. According to Aristotle, using terms that became classical in the history of philosophy, Plato's was mistaken in confusing «substances» with «essences» or, to

¹¹⁶ As shown above (Cf. § 1.5.2.1), even modern set theory produces antinomies if individuals that are not in turn sets are not taken as primitive elements (which Ernst Zermelo called *Ur-Element*).

¹¹⁷ Cf., for example, Aquinas, *In de Causis*, XV, xv ; or *S. Th.*, I,14,2. For a discussion on the question of the *reditio completa* as the property of self-reference typical of the *being-in-itself* of any substance, Cf. Basti (1991, 144-150).

say it with the words that Aristotle used in the *Categories* (Ch. 5), he confused «primary» with «secondary» substances. Here is Aristotle's explanation of the difference between hypostasis or substratum («primary» substance) and essence («secondary» substance).

Primary substance is the individual subject existing in itself

- ◆ *Primary substance, or individual substance*, is the main, «primary» meaning for the word «substance», which indicates an object able to be *self-sustaining*, able to exist *in itself* (as an individual) and *for itself* (as endowed with an individual essence). Every «primary» substance is therefore an individual ontological (*to ti ti, hoc aliquid*, «one “this” something»: indicative definition) substratum (subject) of property. For example, all individual physical bodies, the material compounds endowed with dynamic stability through time, are «primary» substances: one single dog, one single cat, one single man. In the microscopic realm, individual particles, or «elements», at any level of the organization of matter (in modern words: one single molecule, one single atom, one single proton) are also «primary» substances when *taken in themselves*, as self-sustaining individuals («free» particles), not as components (elements) of a more complex physical substance (respectively, for example, the molecule of a cell, the atom of a molecule, the proton of an atom...). Beyond the physical world, according to Aristotle and the Scholastics, the «separate intelligences», immaterial substances such as souls, angels, or God himself, are also primary substances. These metaphysical subjects cannot be defined as individual substances, given that they are not made of matter in the proper sense; rather, they can be defined as «singular» substances. This, however, is beyond the scope of a philosophy of nature.

Secondary substance is an essence common to many individuals, which does not exist in itself, but in individuals

- ◆ *Secondary substances, or specific essences*, are essences common to more than one individual. For example, the nature or essence of «dog» is common to each single dog; the nature of «man» is common to each single man; the nature of «iron» is common to all atoms and molecules of iron, etc. These specific essences are «secondary» substances —substances in a «secondary sense»— because, while existing *for themselves*, they *are not self-sustaining*, they do not exist *in themselves*; rather, they exist in the *multiple individuals* of which they are the essence. The «primary» substance (e.g., John) is the substratum of which the «secondary» substance (e.g., being man) is an essence. «Secondary» substances, therefore, do not exist in themselves, but «in the many» of which they are the specific essence. These «many», from a strictly logical point of view, says Aristotle, are «immanent» to the «secondary» substance itself. Logically speaking, with «secondary substance» Aristotle indicated, in the intensional sense, a qual-

ity, or property (= *totalitas ante partes*, «totality without parts» or principle that makes up a totality); in the extensive sense (= *totalitas ex partibus*, «totality made of parts», compound) he indicated what one today denotes with the notion of «class», which is determined by a specific quality that all elements that «belong» to that class possess as a common «property». In other words, the «secondary substance» is a notion of both intensional and extensional logic. As opposed to contemporary logic, Aristotle was unable to make a distinction between these two meanings.

If one attributes being-in-itself to essences, as Plato did, one falls into error

As Aristotle pointed out, if one considers secondary substances or essences as primary substances, without distinguishing between them, as Plato did, one falls in the absurdity of the «third man argument». Here is how Aquinas summarized this fundamental discovery by Aristotle:

Aristotle says that if universals were (primary, *N.o.A.*) substances, then in Socrates there would be a (primary, *N.o.A.*) substance in the (primary, *N.o.A.*) substance. Indeed, if all universals were substances, the way that being-man is Socrates' substance, then also being-animal would be man's substance. Hence Socrates would have two substances. (...) It is therefore clear that no universal is a substance, and that none of the things that we predicate as common to more than one entity indicate something that exists in itself as a substance (*hoc aliquid*). Rather, they only indicate some quality (*what*)» (Aquinas, *In Met.*, VII, xiii, 1584.).



An example in common language of Aristotle's distinction between «primary» and «secondary» substances

We could make an example that illustrates the current importance, even in our common language, of Aristotle's notion of «substance». If we are interested in the nature of a certain chemical compound contained in a test tube, we ask: «what substance is this?» Clearly, by the term «substance» we intend to refer to the «secondary» meaning of substance, that is, we are enquiring about the «nature» or «essence» (for the distinction between these two meanings of the word «essence», Cf. § 6.2.1) of that object (e.g., sulphuric acid). Conversely, if we ask «what is the *substance* contained in that test tube?» it is clear that, by the term «substance» we are referring to the «primary» meaning of substance. In this case we are indicating the metaphysical subject, the concrete, existing, individual entity, the **toû ti** (*hoc aliquid*) that possesses a specific nature, in this case that of sulphuric acid.

Aristotle's distinction has often been missed in both classical and modern thought

This fundamental distinction between «primary» and «secondary» substance —between «substance» and «essence», or «nature»— was often misunderstood by ancient thought subsequent to Aristotle. It was rediscovered only in the IV century A.D., in a context deeply different from that of Aristotelian logic and metaphysics. This was the theological context of the disputes on the Divine Persons in the Christian doctrine of

the Holy Trinity, which led to the introduction, in Western theological and anthropological thought, of the notion of «person» as a paradigm of individual «primary» substance, as the most unique of all primary substances, given that it is capable of «freedom». We will deal with this in Part Five of this work.

The distinction between «primary» and «secondary» substance has been difficult even for modern thought. The metaphysical monism that was implicit in the reductionist approach of original modern physics led modern philosophy of nature to accept the position of B. Spinoza. This meant arguing for the existence of one substance, «nature» (a «secondary» substance) which is also «God» («primary» substance: *Deus sive natura*), of which individual beings are only accidents, particular and contingent manifestations (Cf. § 3.2.2.2).

Substance is the first of the ten categories, the ten fundamental predicaments that one can attribute to individuals

In the remaining parts of the *Book of Categories*, starting from the notion of «substance» with the two meanings explained above, Aristotle identified *ten categories, ten fundamental predicates*, or «predicaments», which can be applied to any entity. For Aristotle anything that exists, whatever it is, can be described with one of these ten categories, which represent the most general thinkable concepts, through which one can classify any existing thing.¹¹⁸

The first distinction introduced with Aristotelian categories follows the philosopher's fundamental discovery that we explained above. The first distinction according to which one can classify all entities consists in asking whether this entity exists *in itself and for itself* (= is self-sustaining) or whether it exists in *something other than itself*. This means asking whether the relevant being is a «primary» (*in itself*) substance, with its own essence (*for itself*) or not. It then means asking whether, in order to exist, this entity must exist in *something other than itself*, in a *primary substance*, as something that *happens* to the latter as its *accident*, as an «event», more or less «characteristic», which is typical of the essence of that substance (e.g., growing to that specific weight or height, acquiring that specific colour, reacting to stimuli in that specific way, etc.).

¹¹⁸ One should recall that Aristotelian categories are *ontological* categories, in the sense that they relate to the definition of entities' nature. They should not be confused with *semantic* categories, which are the focus of one of the most studied branches of logic, particularly of contemporary *formal semantics*.

Distinction between substance and accident: what exists in itself and what exists in something other than itself

For example, if the entity we are talking about is Ralph the cat, we immediately realize that it is an individual, so that to describe it as such we refer to it with its first name «Ralph». This is the only way to refer to its individual essence, which is not knowable in itself. The first name represents therefore a «universal one-of-one». Describing by means of a first name is our only way to signify universally, «always and anywhere», the «uniqueness *in itself*» of an individual in relation to *all* other entities.

Essence is the set of fundamental properties that connote what an individual is

Once established that «Ralph» denotes the individual character of this cat, we can use the word «Ralph» as the subject of declaratory propositions that connote it, that «describe» its essence, the «secondary» substance that is predicated of the primary substance. One could start by describing it as «a cat», then as «a feline», «a mammal», «an animal», «an organism», «a body» that is a «primary» substance, an individual capable of being self-sustaining in itself and for itself. All these predicates are «universals one-of-many» that can be applied to many individuals, and that express, with varying degrees of generality, fundamental characteristics of Ralph's essence, of what Ralph «is». These properties *necessarily* belong to Ralph, they express Ralph's *being-for-itself*.

Accidents do not represent what an individual is, but certain properties that an individual has, something that the individual happens to have

Ralph would not be Ralph if it were not cat, feline, animal, etc. But Ralph also has other properties: for example, its fur has a given colour, or it weighs a certain amount of pounds. Obviously Ralph, which *is* a cat, a feline, an animal, etc., «is» neither its fur, nor that colour of its fur, nor that weight x (e.g., when old, Ralph might lose almost all its fur or have gray fur; it could grow fatter or thinner without ceasing to exist and to exist as Ralph). While being-cat, being-feline, being-animal, etc. *are* Ralph's being, they make up its essence, they *necessarily* belong to it, these other features (that fur, colour, weight, etc.) do not *necessarily* belong to it, but they are properties that the individual *has*. These characteristics are not Ralph, but they exist in Ralph as something that *it happens to have*, as *accidents* of the substance- Ralph. «Accidents» —or, to use a modern word, «events»— that happen to a given existing individual subject, or primary substance, are entities, they *exist* just like substances, but *in a way different* from that of substances. Accidents are entities that neither exist *in themselves*, like primary substances, nor *belong for themselves, necessarily*, to «primary» substances as parts of their related «secondary» substances. They exist *accidentally* (*per accidens*) in substance- Ralph —just as they exist in an infinite number of other primary substances, of other individual subjects that may be different from Ralph, from cats or from animals (e.g., weight is a property that exists in all bodies endowed with mass and subject to the force of gravity, whether they are living or not). The ontological distinction «(secondary) substance-accident», therefore, corresponds to the logical distinction between predication *for itself* (= neces-

The Aristotelian doctrine of categories is a theory of *ontological*, not *semantic* categories

sary) and *per accidens* (= contingent) of certain properties or attributes of a particular «primary» substance or self-sustaining individual.

Clearly, then, Aristotle's categories are *ontological categories* whose arguments are names of entities, be they primary substances (existing *in themselves*) or accidents (existing in *something other than themselves*). For this reason, Aristotle's theory of categories must be kept distinct from the modern *theory of semantic categories*. This is a branch of modern axiomatic logic that was developed, starting from some of E. Husserl's arguments, to avoid in semantics some inconsistencies of Russell's theory of types, and to replace the notion of «logical type» with that of the *semantic level*.¹¹⁹

Recalling this fundamental distinction between entities that exist in themselves and for themselves (substances) and entities that exist in something other than themselves (accidents), the following are the ten categories, according to the list (Table of Categories) given by Aristotle in the *Book of Categories*.

The first category is substance, intended as secondary substance

1. The first is the category of substance. If this is understood as «predicament», as one of the fundamental predicates that one can attribute to specific entities, clearly this is not the primary, but rather the «secondary» substance —the expression of what a «primary» substance, an individual, *is*. As Aristotle stated (*Cat.* 5, 3b, 19-21) the secondary substance does not simply express quality (e.g., the colour), but *the quality that characterizes an individual «primary» substance*. In particular, it expresses the *genus* and the *specific difference* that indicate the nature or essence of a given individual in the relevant categorical proposition (e.g., «John is a rational animal»). If the «primary» substance (e.g., John) is «human» (= «secondary» substance, or «class» to which John belongs), «human» is in turn defined as «rational (= specific difference) animal (= (proximate) genus)». Genus and specific difference are what one predicates of a «primary» substance as such, they are what determines an individual subject (substratum) as such. They describe the intrinsic quality of a «primary» substance, an individual —that is, its «essence» or «nature». They do not describe a quality that that individual «happens» to have as its property, or «accident» (e.g., having a given colour). In other words, «secondary» substance is what a given «primary» substance, or individual (e.g., John) *is* (e.g., human), while the other accidents are what the individual *has* as its property (e.g., skin colour).

¹¹⁹ Just as in Aristotelian ontology accidents always have to be predicated of substances, so in semantics, in order to avoid antinomies, the argument of a predicate that belongs to a semantic level *n* must belong to a lower semantic level, at least *n-1*. In simple terms, this is what the theory of semantic levels states.

The remaining nine categories

The remaining nine categories relate to accidents (properties) that can exist not just in one, but in many, even very different, individual substances (e.g., being white does not only happen to certain human races, but also to flour, snow, a sheet of paper, etc.). These are:

2. *Quantity* (discrete and/or continuous), which is immediately linked to the *matter*²⁰ of which a primary substance (its essence) is made.
3. *Quality*, which is immediately linked to the form of which a primary substance (its essence) is made.
4. *Relation*: this and the two preceding categories constitute the «typical» or *intrinsic* accidents of a primary substance, those more directly linked to a given individual's nature, or essence. The remaining five categories are:
 5. *Action* In any relation with other entities, the primary substance will always happen to be in the position of
 6. *Affection* «agent» or «object».
 7. *Place*: another typical accident of a primary substance is its location *in space*.
 8. *Time*: another typical accident of a primary substance is its location *in time*.
 9. *State*: another typical accident is for a primary substance to acquire specific properties.
 10. *Position*: in addition to the spatial-temporal position, any primary substance happens to find itself in a particular ordering relation with other entities²¹.

¹²⁰ Or, in any case, to the potential principle of which a primary substance is made. This notion is not Aristotelian, but Scholastic. Scholastic philosophy admits personal substances, hence primary and non-material (e.g., angels), that can be counted, even if they are not made of matter. Clearly, in this case the principle of quantitative multiplicity — moving beyond Democritus — cannot be matter (in relation to form) but some other potential principle: essence in relation to being (as act). Every angelic «individual» is a *species* in its own right: there are never angels of the same kind. In the case of physical entities, matter is the principle of multiplication of individuals within the same species (hence the definition of «matter as determined by form as a principle of individuation» within a species). In the case of spiritual substances, there can only be a multiplicity of species-individuals, each of them with a given essence, within the same genus.

¹²¹ It should be noted that other Aristotelian texts referred to eight rather than ten categories. The last two categories, «state» and «position», were included, respectively, in the category of «quality» and «place».

Doctrine of categories as the ontological basis of the logic of predicates and of the syllogistic techniques of inductive and deductive inference

The distinction between primary substance and secondary substance and accidents that are inherent to the first—that is, the doctrine of the «ten categories»—is the ontological foundation of the *Aristotelian logic of predicates* (as a logic of properties, having therefore an intensional basis) and of the *syllogistic techniques* (literally: «calculus techniques») for the construction of propositions. It is therefore the basis for associating subjects and predicates in a valid way, with varying degrees of necessity, through *inductive inferences* (inductive syllogism and analytical methods of searching for the middle term) and *deductive inferences* (deductive syllogism, «demonstrative» or scientific with different apodictic and hypothetical modalities, forms and figures). *Prior Analytics*, the first text of *formal logic* ever written in the history of thought, deals with syllogistic techniques, their various modalities and formally valid forms and figures. The other great text of the Aristotelian *Organon*, *Posterior Analytics*, treats the application of these techniques and forms to the construction of syntactically consistent and semantically true *models**, within different sciences and their demonstrative procedures (*material logic* or *logic of contents*).

Essences are the real foundation of logical universals. But where and how do essences exist?

At this point, one question becomes central: if essences, or «natures» of things do not exist in any ideal world—even if they are the *real* foundation of conceptual *universals* that, as «ideas» exist only in the mind that thinks them, as products of the *faculty of abstraction* of the human mind—then «where» and «how» do these essences exist? In what sense do essences constitute some «reality» distinct from the universals of thought and, even more, constitute the foundation of the truth of universals?

Answer to this question through the doctrine of potency and act

In order to answer this question, it is necessary to understand how the distinction between *form* and *matter*—the original and irreducible components of any being that belongs to the physical world, be that a «primary», a «secondary» substance, or an accident¹²²—constitutes (as, respectively, *act* and *potency*) the key for the solution of the last of the three problems posed by Parmenides to his successors: the problem of the consistent or non-contradictory character of *change*.

¹²² Obviously, the constitutive form of a substance will be called substantial form, while that constitutive of an accident will be called accidental form. This distinction will be useful for the following sections of this work.

5.4.3 The third answer to Parmenides

Justification of all forms of change through *act-potency* applied to *form-matter*

By identifying the two constitutive and *irreducible* principles (*matter* and *form*) of the essence of any physical entity, be that («primary») substance or accident, Aristotle also gave an answer to the third problem posed by Parmenides: the presumed *contradictory nature of change*. (Cf. § 5.1.4). For this, he defined matter as «*potentiality* to exist» (or *being in potency*), and form as «*actuality* of being» —as that which makes matter actual, that makes it exist as a given *entity in actuality*.

Change is not the passage from being to non-being, or the reverse, but the passage from being in potentiality to being in actuality, and the reverse

Indeed, once both matter and form are identified as the elements constitutive of physical entities —*substantial* form constitutive of substances, accidental form constitutive of accidents— the passage from an entity A to another entity B, whether substance or accident

$$A \rightarrow B$$

does not imply contradiction, given that it does not imply the passage from A to the absolute non-being of A (or, conversely, the passage from the absolute non-being of B to B). Change does not imply the *total* negation of A (= annihilation of A), but only the negation of one of its constitutive principles: its formal principle. Any entity in change (= «physical» or «natural» entity, one that «is born and dies») is composed of a *potential* material principle, an *indeterminate* « ∞ » that is common to all physical entities; and of an *actual, determining* formal principle, which is different for each type of entity (= specific form) and, possibly, for each individual entity (= individual form). This determining actual principle is the effect of a cause *acting* on the material substratum. So, the change from entity A, composite of matter-form ($x + a$), to entity B, another composite of matter-form ($x + b$) means:

$$A \rightarrow B = (x + a) \rightarrow (x + b)$$

Change as the *transition* of matter through different forms (*transformation*)

For Aristotle, therefore, change is the *trans*-formation of an indeterminate matter. It is the *passage* of *matter* (x) from being actualized in a given entity with a given *form* (e.g., a is the form «wood») to being actualized in a different entity with a different form (e.g., b is the form «coal»), by means of a particular causal agent.

The causal explanation of change implies the distinction between *active* and *passive* potency

The symbol (x) indicates the indeterminacy, *potentiality* of matter. This, while existing *actually* in the form of, for example, wood, stands in *passive* potentiality to other determinations, which it is presently *lacking*. Due to the influence of a further *causal agent* (= *active* potentiality), e.g., fire, it will cease to be actualized in the form of wood to be actualized, for example, in the form of coal. Any matter can be actualized only in *one form at a*

time: it is in actuality in only one form and, therefore, *missing*, lacking an indefinite number of other forms (= *principle of privation*).

Causal explanation
of change, *principle*
of *privation* and
prime matter

This number will be finite for all matter that Aristotle calls «secondary» (*secondary matter*), which is made of particular combinations of elements at varying degrees of complexity (in present physics, they can be situated at the sub-atomic, atomic, molecular levels, considering simply «microscopic» physics). For example, in the words of modern physics-chemistry (molecular physics), «organic matter», the macromolecular compounds of carbon stand «in potency» to an undefined, but still finite number of forms of cells and therefore of organic tissues that are compatible with their nature. Conversely, inorganic matter, so-called «mineral», does not stand «potentially» to organic compounds, but to many other inorganic compounds that cannot be obtained from organic macromolecules. At a lower level of complexity, that of atomic physics, it is obvious that the most elementary compounds (atoms) will stand in a more «distant» «potentiality» to a greater number of physical entities, both organic and inorganic, than the molecular compounds of the higher level, and so on.

One question can be raised at this point: is there a level of organization of matter that is so small (that is, an absolute absence or *lack* of organization or of form) that it stands «potentially» to *all* forms of material entities presently existing in the physical realm (= *prime matter*)? Aristotelian as well as contemporary quantum-relativistic physics give an affirmative answer to this question.

The number of
levels of passive
potentiality to forms
is inversely
proportional to the
degree of
organization of a
material substratum
→ *prime matter*

For both theories the evidence of possible reciprocal transformations between elementary (not compound) particles —the ultimate components of all existing physical bodies (quarks and leptons in contemporary thought; particles of air, water, earth and fire for Aristotle) implies that these material particles are not the ultimate material components of all bodies, contrary to what metaphysical atomism believed (Cf. § 5.2.2). The ultimate component of material entities must be a common material substratum having no form (mass-energy for contemporary thinkers, «prime matter» for Aristotle) that does not exist «actually» as such, given that a physical entity that is not composed of matter and form cannot exist. This common substratum exists always and only as standing *potentially* to the reciprocal possible transformations of elements (e.g., the absolute instability of the so-called «quantum void») (Wallace 1996). In Part Three of this work, we will come back to this interesting point of similarity between quantum-relativistic cosmology and Aristotle's philosophy of nature.

Problem: the modalities of existence of forms in matter are not implicit in prime matter

Another important question is then the following: *in what way do forms exist in matter?* One incorrect way of understanding the «being in potentiality» of forms in matter—which was common in modernity—was to conceive of forms as «implicit» in matter, the way that theorems are implicit in the postulates of a demonstrative procedure. This interpretation of Aristotelian theory was certainly the product of the desire to make Aristotle's philosophy of nature compatible with early modern physics, which considered integrable dynamic systems—those whose final state is implicit in the initial conditions, as theorems are implicit in geometric assumptions—the paradigm of physical systems existing in nature.

For Aristotle, however, the potential existence of a form in a given «matter» depends on the action of a proportionate causal agent that «makes that form exist» in that matter. So, for example, one can turn a piece of wood into a statue by the action of a chisel, or turn it into a piece of coal by the action of fire. Wood stands in passive potentiality to the statue, thanks to the active potentiality of the chisel. Similarly, it stands in passive potentiality to coal thanks to the active potentiality of fire. However, the (accidental) form of the statue is not «implicit» in the wood as such, neither is the (substantial) form of coal.¹²³

Eduction of form depends on the concurrence of causal agents

The fundamental distinction between the «being potentially» of, for example, a theorem in a set of postulates in a «closed» formal system and the «being potentially» of a form in matter in the physical realm will be dealt with in detail below. In the first case, one can state that the theorem was implicit in the postulates: no new information is added that was not already contained in the original postulates. In the second case, form is not «implicit» or «hidden» in matter. Its actualization in matter depends on the concurrence of physical causes acting on matter, which produces «added» information in matter itself that was not there at the first stages of the process.

In the words of medieval Aristotelian philosophy, «form» is *educed* from matter, not *deduced* from it like a theorem from its premises. This causal scheme of «act-potency» is a substitute for the modern Humian-Kantian causal scheme, which was weakened by the discovery of chaotic systems and of their stability outside equilibrium, which could not be predicted

¹²³ To turn a piece of wood into a statue means to provoke an accidental transformation that does not change the object's (physical-chemical) nature. To the contrary, turning the same piece of wood into a piece of coal is a substantial transformation, which causes the «secondary» substance (the nature) of the object in question to change. In Part Three of this work we will come back to these fundamental distinctions in Aristotelian philosophy of nature.

from (= was not implicit in) the initial conditions. This point, already mentioned above (Cf. § 2.7.2), will be dealt with in the final part of this chapter.

Change as the passage from one modality of being to another

For the moment, one point must be made clear: change is not contradictory, because it does not imply the passage from being to non-being, or the reverse. Being is and cannot not-be, as Parmenides pointed out. However, «being» exists in many modes, contrary to what modern Parmenideans still argue. «Change» means the passage from one modality of being to another, from potential-being to actual-being and the reverse, by forms that organize a common material substratum, thus actualizing (i.e. causing to exist) different physical entities, characterized by different «natures» or «specific essences».¹²⁴

Problem: modalities of existence of essences in the physical world

We now turn to the metaphysical question of the modalities of existence of essences in the physical world —given that, for Aristotle, a «world of ideas» does not exist— and in particular of specific essences, or «secondary substances», as treated in Aristotle's philosophy of nature. This is another way of dealing with the epistemological question of the extra-mental foundation of universals that, according to the philosopher, only exist in the mind that thinks them.

5.4.4 The causal foundation of essences

Matter cannot be self-sustaining, without form: matter is not a primary substance

According to the Aristotelian metaphysical scheme, *matter cannot exist in itself*, without form, *neither can form exist without matter*, at least not in the physical world. Matter is not a «primary» substance, that is, it is not an entity capable of self-sustaining as a «metaphysical subject», as a primary substance, as an individual «that-which-exists» (*id quod existit*). Aristotle often calls it *substratum*, where the term does not refer to the «substratum» as a metaphysical subject, a primary substance, an *individual* existing in actuality.

¹²⁴ Strangely enough, even over two thousand years after Aristotle, some still argue for the absurdity of change, stating that it would presume the absurd possibility of the existence of non-being or of the non-existence of being. This is the case, for example, with Emanuele Severino who, for over thirty years, has refused a fundamental point established in Western metaphysics after Plato, Aristotle and Aquinas. This point relates to the fact that being can be stated in many ways, first and foremost as «being in potency» and «being in act». On this issue, Cf. Basti and Perrone (1996).

Matter = substratum of forms common to more than one individual *versus* substance = individual substratum of accidents

Matter is instead potential substratum, *common* to all material entities, whether substances or accidents. As the *substantial or accidental form* of each entity (respectively, of a substance or of an accident), matter is only a principle «through which» (*id quo*) natural entities (substances or accidents) exist as such. In sum, matter is not a «that-which-exists» (*id quod existit*), an *individual* substratum or primary substance; rather, it is a *common* substratum, a «that-through-which something exists» (*id quo aliquid existit*). More precisely, matter is «that through which» all physical entities (substances or accidents) exist.

Common matter is then an *id quo* common to all physical beings, and not an individual *id quod*. In this sense, Aristotle's philosophy is not a form of «materialism». In Aristotle's words, all forms of materialism are philosophies of nature incapable of distinguishing between the *common substratum* (matter) and *individual substratum* (primary substantial nature, existence in itself) of physical entities.

Forms are not self-sustaining, neither is matter

According to Aristotle any *form*, even the *individual, substantial* form of a «primary» substance —not just the *accidental* form of an entity's event that can «occur to» (can *exist in*) many individual substances (e.g., a given colour)— is *educted*, «taken out» of the potentiality of matter, through the action of another physical entity having the function of *causal agent*. Stated otherwise, all forms (with the exception of the human soul) are not added «from outside» matter, since the forms of physical beings cannot exist in themselves «outside» matter. As matter, they are not «primary» substances, entities capable of being self-sustaining as «metaphysical subjects», as a «that-which-exists» (*id quod existit*). They are principles «through which» metaphysical subjects, individual entities, exist as such. In sum, they are a «that-through-which» something exists (*id quo aliquid existit*). Matter and form, therefore, are *id quo*, and their substance, or «composite», is *id quod*. For this reason, Aristotelian philosophy is a form of *hylomorphism* rather than of «materialism».

In order to understand the mechanism of «eduction» of form from matter, a step back in Aristotle's criticism to Plato is necessary.

Eduction of forms
from matter =
dynamic justification
of the existence of
forms

Given that difference cannot be derived from identity, neither can matter be derived from form (Cf. above 5.4.1), Aristotle justified *dynamically* the diversity and multiplicity of entities. «Primary» matter (= potential substratum, which does not exist actually as such and is common to all physical beings, starting from the elementary particles that make up all bodies) does not denote «something» (*id quod*), but the infinite number of an entity's potential material forms, given that physical matter is *in continuous change*.

Physical matter is
not static geometric
extension (vs.
Descartes)

The *potentiality* (= indeterminacy) of the prime matter of physical beings to *infinite* forms should not be understood in a static way, as the geometric example of the length of a segment might lead one to think (Cf. Fig. 5-1, p. 332). In other words, it should not be understood as if prime matter were the same as the modern notion of geometric extension: as a «compact» infinity of points, so that any line, figure or solid that can be described in it are just a particular way of uniting (combining) points in this extension according to a specific order or «law» (function), which can be deduced from the axioms that define the geometric space itself.

The being-in-
potency of a
geometric entity in
continuous space is
being *implicit, static*,
as theorems are in
postulates

In this sense, the «being-in-potency» of a given form in geometric extension reveals a fundamental *static nature*, which implies no change or transformation. In the geometric continuous space of modern mathematics, points already exist *in actuality*. Making a form actual in this geometric continuum just means making explicit what is logically *implied* yet *implicit* («hidden» or «not-yet-evident») in the original postulates. The «being-in-potency» of a mathematical entity is something that can always be removed. It is not absence of determination, but the not being explicit, not having made evident that which is already implicitly determined.

The being-in-potency of physical matter means *indeterminacy* in a *dynamic* sense: «being delimited yet always different».

Aristotle's notion of potential infinity of matter reveals the fundamental, irreducible *indeterminacy* of physical entities' *finite* material substratum. Aristotle defined this indeterminacy of matter as a «delimited, yet always different being» (*Phys.* III, 206a, 34), as the «indefinite variation of the finite» (G. Cantor). The potential infinity of matter is that for which matter, without form, would be an «always different being», it would be the *pure casualness of becoming*. To use a mathematical analogy, a random sequence of numbers has no repetition, no periodicity and 'whereabouts'. In this sequence, therefore, there is no function or «law» through which one can predict a given value on the basis of the preceding values.

As Aristotle stated, physical matter «stands (potentially)» to a form but not in the way a figure «is implied» in geometric extension or, more precisely, in the axioms that define the object in question. «In mathematical magnitudes, said Aristotle, there is infinity because what we have assumed when stating the infinite remains» (*Ini*, 35). The form, the information is all in the premises, in what has been stated in the initial assumptions. In classical mathematics, therefore, as in all deductive thought, all things are predictable and just need to be made explicit. According to Aristotle, instead, physical matter «stands potentially» in the sense that

One day *is* or a contest *is* because they always become something *different*, and indeed, in these examples, being is both in potency and in act (*Phys.*, III, 6, 206a, 23-24).

Matter is a changing finite for which there is no law allowing predicting its subsequent stages. It is finite in act, undefined in potency, hence *dynamically undefined*

Is there a moment of the day or of the contest that is the same as the preceding or the following ones? Is there a «law» through which one can tell with absolute certainty what will happen at the next moment of the contest or of the day? By definition, each moment of a day or of a contest is different from the preceding ones. At any moment of a contest, a well-defined reciprocal position of players, relative to the given location in the stadium, is *in act*. This position will be different and unpredictable at the subsequent moment of the contest. Each matter, thanks to its form, is *in act* something finite and definite, but matter in itself, without form, is an un-defined that can be defined. All matter is always a «finite» and «defined» *in act* that is an «infinite» and «undefined» *in potency*.

Matter relates to the intrinsic instability of a dynamic system

Matter, as the constitutive principle of any physical entity, essentially relates to the intrinsic instability of motions of an element's substratum, as we will see in § 6.3.2.1. According to Aristotle, matter relates both to being in potency (taken in itself) as well as to being in act (taken in relation to form). The potential infinity of physical matter relates therefore to an absolutely unpredictable becoming, with no stability or periodicity (=

form in a Pythagorean sense) that is, *with no reciprocal order of its parts*, spatial or temporal, that is *predictable* through any law. For this reason, according to Aristotle, and contrary to most of Greek and modern thought, physics cannot be reduced to geometry (Cf. § 1.1). Like many other authors that dealt with the issue, we were right in referring to Aristotle's philosophy as that most capable of explaining the state of the art of contemporary physics after the discovery of the primary role of dynamic instabilities in the study of *real* physical systems (that is, «complex» Cf. § 2.7.2).

The dynamic, physical potentiality of matter stands in opposition to static logical possibility

Matter's *physical potentiality* of is not a static *logical possibility*, as with geometric extension, contrary to Descartes' and modern physics' mechanistic interpretation. Forms do not exist potentially in physical matter the same way that theorems exist potentially (= *implicitly*) in the axioms of a formal deductive system (e.g., Euclidean geometry). Forms are not «hidden» in matter; rather, they exist in matter in *passive potentiality*.

Eduction of a form from matter= partial stabilization of a dynamic instability by means of external agents

In other words, forms do not exist in actuality until a sufficient *causal agent* (= *active potentiality*) is able to *determine them* and therefore to «make them exist» in matter—in the dynamic substratum of elements' unstable motions. Given the similarity with dynamic instabilities, we can use the metaphor of the stabilization of random sequences of symbols to give an intuitive example of Aristotle's causal mechanism of «eduction» of forms from the undefined/infinite potentiality (instability) of matter.

The sequence:

XCGDXWQCOXUIZWNPFGYTRQWEXCVU....

is clearly random. In Aristotle's words, it is an always «different» sequence. However, if the sub-sequence «XCGDXWQCO» were stabilized in this sequence, so to make it periodic:

XCGDXWQCOXCGDXWQCOXCGDXWQCO

a new form would have been educed from the potentiality of a matter that was «amenable» to it.

In this sense, form is «limit» «threshold» «end» of matter, which encloses and organizes the motion of an element's substratum

The 'eduction' of form from matter by means of a causal agent (i.e., by means of another entity) means therefore to set a *limit* to matter's becoming, to *de-termin*e it (= setting a term to it), after another causal agent, at the beginning of the process, has induced instability in it by «breaking» the preceding stable state (= periodicity). The eduction of a material form (substance or accident) from matter means *to stabilize*, at least temporarily (= for the time of physical existence or «duration» of a given physical entity) the dynamic substratum of an element's chaotic mo-

tions, according to one or more of the different ordered (periodic or pseudo-periodic) possible configurations.

Aristotle called these partial arrangements of the substratum, which can be acquired or lost, «dispositions»

Aristotle called these potential arrangements of the material substratum *dispositions*. He intended them as «qualities» that can change, can be «lost» or «acquired» through the action of physical agents and that, as more or less provisionally «stabilized», «acquired» in a material substratum, become «states», *stabilized* dispositions. Clearly, «quality» and «state» must be understood according to the Aristotelian categories (respectively the third and ninth categories) as explained at pp. 340-ff.¹²⁵ The following is one of Aristotle's texts on this point that we will deal with in more detail below (Cf. § 6.3.2.1):

Form is a *limit* (τελος) and *something-by-means-of-which* (τοῦ οὐκ ἐνεκα). Given that the motion (of elements) is *endless*, one needs a limit and a something-by-means-of-which such a limit is reached (*Phys.*, II, 2, 194a, 27-29).

In another text, Aristotle was even more explicit:

When the agent is present, the object undergoes a change. But when the states (literally, the «habits») are present, the object is not becoming any longer, rather it is. Now, the acts (= forms) and final states of motion are a kind of habit (*De Gen. et Corr.*, I, 7, 324b, 15-18).

Common point with the theory of complex systems vs. geometric dogmatism in dynamics

In contemporary physics, the theory of dynamic instability and of complex physical systems starts from the same criticism of the «geometric» dogmatism of modern physics. For this reason, the best examples illustrating this point in Aristotle's philosophy are taken from the modern theory of complex systems (Cf. §§ 2.6.1 – 2.7).

«Cause» is not a category, a «primary» predicate, but rather a composite notion

In any case, it is clear that, for Aristotle, the notion of «cause» was not a category, a «primary» predicate or a «predicament» as Kant considered it later. Rather, it was a notion derived from three categories combined: relation, act and potency. In this sense —similarly to the theory of unstable systems that broke the Kantian causal scheme in contemporary physics— the new model able to explain the logical unpredictability of «complex» systems in physics, as well as their causal determination, is rooted in Aristotle's philosophy of nature.

¹²⁵ In Aristotelian cosmology, only earthly entities are corruptible, given that their matter is made of elements (particles of fire, air, water and earth). Conversely, celestial bodies — the transparent spheres with planets and stars fixed on them, which, according to Aristotelian-Ptolemaic cosmology, make up the sky with their concentric motions— are incorruptible, unchangeable, and their circular motions are perfectly geometrically predictable, given that they are made of matter that is not composed of elements (*ether*).

Form is an act because:

1. It is relative to potency

Returning back to the guiding principles of Aristotle's philosophy of nature, the material form of a physical entity is an «act»:

◆ Because it is always *relative to potency*. Act/potency and form/matter are correlated terms. In Aristotle's logic, to take them separately is meaningless.

2. It derives from an agent

◆ Because it derives from an extrinsic causal «agent», which is external to the material substratum from which the form is educed.

3. Because it is the «intrinsic limit» of a chaotic substratum of motions, *entelechia*

◆ Because it is for itself, *intrinsically*, the limit, *pe/ra/j*, which *de-termines* matter in constant 'becoming'. It thus gives it an *order* it did not possess¹²⁶, by stabilizing it and therefore diversifying it as an ordered *whole*, or *entity*. In Latin, the word *actus*, which stresses the dependency of form on a causal agent, translates Aristotle's Greek term for *entelecheia* (= that-which-has-the-end-in-itself) that instead stresses the idea of form as *intrinsic limit* that «contains in itself» (and therefore organizes) matter in chaotic becoming.

More generally, to state that form is a limit that arranges matter's motions meant, for Aristotle, to set the changes of matter within a min-max *limit* of possible ordered mutations. These constituted the set of events, or «accidental transformations», that are possible *of* a given entity (e.g., the cat's growth to a given adult size), without it undergoing a substantial transformation that would give its matter one (or more) completely different nature (e.g., the cat's death and the subsequent «corruption» of its body). Form is «limit» (*pe/ra/j*) in the sense that it «delimits» the possible mutations of an entity (= accidental mutations) without provoking the latter's loss of identity or of substantial form (e.g., of cat). When this limit is overcome, for example by means of a violent action, we have the «corruption» of that (substantial) form and the «creation» of a new entity (e.g., in the case of a cat being killed, the corruption of its body means the generation of many other organisms, vegetal and animal alike, as for the corruption of any other animal organism).

The following is a text from Aquinas commenting the text of Aristotle's *Metaphysics* that argued, contrary to the «Platonists», the character of «composite», *suholon*, (matter-form) of any essence, be that a «secondary substance» (kind, species) or a «primary substance» (a concretely existing individual entity, defined by that essence: Cf. § 5.4.2). In Aristotle's text, the distinction matter-form was illustrated through the example of a house. The form corresponds to the roof and to the order of stones and concrete that, in turn, constitute the material principle, con-

¹²⁶ It should be noted that to organize a process it is sufficient to determine at least one of its limits (upper or lower).

tained, ordered «under», «inside» the formal principle. This is the concept of matter as *subliminal* (= located under the limit, within the threshold) to form in the constitution of the essence of any physical entity.



First of all, Aristotle asks (...) whether the name «genus» refers to a compound (secondary, *N.o.A.*) substance, or to a form, or to something that replaces the act. For example, the question is whether the name «house» commonly refers to a given matter with a given form, which could be a shelter (*tegumentum*) made of concrete, and stones and organized as it should be (the being-shelter is like the form, the concrete and the stones are like the matter); or whether it only refers to the act and the genus, that is, the being-shelter (Aquinas, *In Metaph.* VIII, iii, 1705).

The essence of a material entity is not pure form

Aquinas stated, against the Platonists, that the essence of any physical entity is made of matter and form rather than of only form. In agreement with them, however, he criticized the opinion that form is something that belongs to an entity's matter. Said Aquinas:

They speak the truth, because if form were a part of matter, it would depend on it. But this is clearly false. Indeed, composition and mixture, which are formal principles, are not made of the things that are composed or mixed, the same way that, in general, nothing that is formal is made of its matter; rather, the reverse is true. That which is under a threshold (*subliminare*) is made of the compound that is its form, not the reverse (*Ivi*, 1713).

Again on the form as the «limit» of matter

Comparisons with artificial entities such as houses taken aside, what Aristotle meant with his concept of form as the intrinsic limit of matter's change in a natural entity is clear in this other text by Aquinas commenting Aristotle:

The «limit», «threshold»¹²⁷ (*limen*, the form is the limit of a substratum's motions, *N.o.A.*) is the following: it is the «being-placed-in-this-way» (of one thing, *N.o.A.*). The same being-placed-in-this-way (of one thing, *N.o.A.*) is its «being», its «reason of being» (*idest propria eius ratio*). Similarly, the being of a crystal consists in growing as far as a given limit (*taliter ispissari*). (*Ivi*, VIII, ii, 1694).

¹²⁷In the excerpt of the *Metaphysics* commented upon by Aquinas, Aristotle explicitly speaks about «threshold» (*oudoj*), the «limit» defined by the house's door (*Met.*, VIII, 2, 1042b, 25-27). In other words, form is what encloses within (min-max) limits the fluctuations of quantity induced by external agents on the individual substratum, thus «de-termining» (making them the «term-of») the quantities characteristic of a subject and making them the characteristics «of that subject». This is the meaning of the Aristotelian-Ptolemaic principle discussed at length in Aquinas' philosophical treatise, *De Natura Materiae et Dimensionibus Interminatis*.

One could take the well-known example of the experiment to crystallize a liquid. In each crystallizing process there is an intrinsic ordering law (crystals are dynamically stable systems) for which the different parts of the crystal overlap in perfect geometries until this growth reaches a limit—usually linked to thermodynamic variables such as density, temperature and pressure of the given liquid. This is intuitively what Aristotle meant by form as the limit of a set of modifications and intrinsic ordering principle of this set in its temporal development, even if the example of the growth of a crystal, typical of a perfectly predictable, dynamically stable system, could be misleading for today's physics.

Being-in-potency of form in matter as being-dependent (passive potentiality) on a physical agent (active potentiality)

For Aristotle, the being-in-potency of form in matter relates to its dependency (= passive potentiality) on the causal action (= active potentiality) of a physical agent, which is able to stabilize and hence arrange *partially and temporarily* the unstable motions of a material substratum. It then becomes clear why and how, for the philosopher, the essence of any physical entity cannot be immaterial, but is «composite», inseparable composition of matter and form.

After dealing with the conceptual scheme of the «eduction of form from matter» through the action of a given set of physical agents, one last step is necessary to understand Aristotle's solution to the problem of the causal foundation of essences («secondary substances»).

Permanent character of species through the succession of individuals

Essences, or «secondary substances», are characterized by specificity, that is, by the fact of being common to many individuals or primary substances. First and foremost in the biological realm, any species is characterized by the capacity to maintain itself through the succession—«generation» and «corruption»— of individuals that belong to it. In order to explain this perpetuation of species, despite the corruption of the individual material subjects that belong to them, Plato had argued that these subjects' essences belong to an invisible world of incorruptible spiritual substances, just like the intelligibles of the mind—indeed as part of the latter.

Causal explanation of the stability of species through the distinction between two levels of physical causality: celestial and earthly, where the first exerts active control (without reaction or counter reaction) on the second

Aristotle found a *causal explanation to the stability of species*. This explanation is synthesized in a line at the end of the Second Book of Physics, which was famous in its Latin translation: *homo generat hominem et sol* («a man together with the sun engenders another man»). To justify how a *corruptible* physical agent (a human parent, in this example) can engender a fellow entity and how this has happened throughout thousands of subsequent generations, it is necessary that another physical agent, which is *incorruptible*—given that it is made of a different matter, simple, not composed of elements («ether» or «quintessence» in ancient cosmology)—and that is not subject to the instability typical of «earthly» bodies (which are made of elements) controls all the unstable, otherwise unpredictable, processes of generation/corruption of bodies composed of elements, through the perfect stability/predictability/symmetry of its motions.

In order for this control to succeed, it is necessary that the motions of corruptible bodies do not influence, in turn, the motions of the incorruptible ones, thus destabilizing them. These are the bodies that, thanks to the perfect stability of their motion, control the processes of eduction (stabilization) of forms in matter, therefore of generation of corruptible bodies, without being influenced by them. They do so by acting causally on the material substratum of elements, which is in unstable motion, through light and hence warmth.

For Aristotle, as for later Ptolemaic cosmology, these bodies are the heavenly bodies with their circular movements, perfectly symmetrical and therefore predictable and measurable, as they had been known since the times of Babylonian Assyrian astronomy. These celestial bodies are the concentric spheres, with the earth in the middle, of the ancient planetarium. They are solid spheres, made of ether, perfectly transparent, in which stars and planets, as concentrations of ether, are «fixed like nails in a wheel», according to Aristotle's telling image.

Possible parallelism with the quantum foundation of atomic stability based on the nucleus' active control on the structure of atomic «orbits»

In the systematic parts of this work in the Second Volume, and particularly in the Third Part, dealing with the philosophical-natural enquiry into inorganic physical entities, we will see the continuity of these principles with that which was suggested by quantum mechanics concerning the structure of atomic and molecular compounds, intended as the foundation of the physical-chemical properties of physical bodies. This parallel would be even stronger should dynamic instabilities be found even in quantum systems, whose stability would then derive from a more radical instability (Cf. the notion of «quantum chaos» at the end of Chapter Two). Modern quantum physics, just as Aristotle, is concerned with finding a physical foundation to the physical and chemical properties of chemical elements and of their compounds (the «bodies» in our common experience), with no regard to the history of their interven-

tions and therefore independently from their initial conditions. The «history» of an oxygen atom on the earth and that of an oxygen atom in the tail of a comet that goes past hundreds of thousands of miles from the earth, after entering the solar system from an unknown part of the galaxy, are indeed very different. Yet, the physical-chemical properties of the two atoms are the same, starting from their *discrete spectrums of electromagnetic emission* —which allow us to distinguish among different species of atoms and therefore of chemical elements (Cf. above § 2.4.2). How is this possible?

Some parallels with modern materials physics: «the sky» of ancient thinkers is «in the heart» of matter

As summarized by the famous quantum physicist V. F. Weisskopf (Weisskopf 1990), quantum physics has exchanged the «sky» of ancient thinkers for the «heart» of matter. For Aristotle, the perfect stability of celestial bodies set limits, «cut» the indeterminate continuity (potential infinity) of motions in the material substratum, thus organizing it with the forms of the different species of «secondary substances», both living and inanimate. In the modern physics of matter, this is equivalent to the perfect stability of a nucleus —whose elements are linked by the «strong» nuclear force —compared to the physics of the electronic exchanges in atoms and molecules— which are governed by the weaker electromagnetic force. The perfect stability of the nucleus forces electrons' motions (within a «range of potentiality»), binding them around itself and organizing them according to the quantization rules illustrated above (§ 2.4).

Causal foundation of stability in organism species, in Aristotelian and quantum physics

Whatever the origin of those electrons and stable nuclei of protons and neutrons, which could be, for example, the explosion of two stars located millions of light-years apart, their union engenders the same stable compound, for example an oxygen atom. The secret of this phenomenon, which is the basis of the *specificity* of the physical-chemical properties (= nature) of the various kinds of existing materials (both organic and inorganic), is equivalent to the principle outlined by Aristotle. This is true even if the quantum and the Aristotelian theories are developed within two completely different, indeed opposed physical phenomenologies. For example, Aristotle's distinction between celestial and earthly physics has been derided by modern physics since the discovery of the law of gravity.

Identity, stability and specificity of atoms of different elements and Schrödinger's equation

The logical-structural resemblance of the argument, despite the different phenomenologies, is in the fact that the physical-chemical properties of chemical elements in Mendelejev's table and of their molecular compounds depend on the undulatory configuration of electrons' motions around the atom nucleus. According to Schrödinger's equation (Cf. above § 2.4.6), these configurations are always the same for each atom of the same chemical element, given that they are determined by the

ways in which electronic waves are limited by the action of the nucleus. All atoms of one same element (e.g., oxygen) possess a fundamental *identity of species*. Whatever their past history, the electronic wave in all oxygen atoms is subject to the same limitations. These, in turn, are given by the attraction to the nucleus and by the electric interactions with the defined number of electrons that make up that atom, a number that, once again, depends on the nucleus structure.

From this derives the *specific stability* of atoms in a given element, that is, their capacity to regain the original configuration after a disturbance — which would be impossible in a classical Newtonian mechanics system. This is possible because such configurations only depend on the conditions in which electrons move, which are in turn determined by the atomic structure and are *absolutely independent from the intervening history*. This dependency is on *actual*, rather than *initial conditions*. The same occurs in Aristotle's theory that justifies the *stability of species* through limitations to motions in the elementary substratum, which are due to the hierarchy of causal actions on such a substratum.

Development and enlargement of Aristotle's theory of actuality and potentiality with Aquinas

Despite their simplicity, the preceding paragraphs can properly introduce the guiding principles of Aristotle's hylomorphism, which will be used in the systematic parts of this work, particularly in the second volume. The following sections will briefly illustrate the main developments that led to the Thomist theory of being-as-act, starting from Plato and Aristotle. Of course, between the two latter and Aquinas there are almost a thousand years of philosophical thought, which should be taken into account to understand how this development was possible. However, given the limits of this work, it will be necessary to sacrifice part of the account of these years.

Thomist metaphysics as a synthesis of Platonic and Aristotelian thought

On the other hand, and in order to give historical credibility to this *exkurs*, one should note that Aquinas lived at a time in which the recent translation of Aristotle's works into Latin — works that, with the exception of those in logic and poetics, had been almost entirely forgotten— during the first millennium of the Christian age forced him — and all philosophers and theologians of his time— to integrate Aristotelian metaphysics and ontology with an essentially Platonic Christian culture. To state that Aquinas tried — or was obliged by historical circumstances— to synthesize Plato and Aristotle is not just a simple expository technique, but also an accurate picture of the period in which Aquinas lived.

5.5 Aquinas' being-as-act

5.5.1 The limits of hylomorphism

Risks of a
materialist and
monist
interpretation of
Aristotle's
metaphysics

From the point of view of philosophy of nature —that is, of the metaphysics of physical entities— the main contribution of Aquinas' theory of being-as-act was to solve a fundamental *inconsistency* in Aristotle's hylomorphism, which one could consider its *materialist flaw*. This can be summarized as follows: if the *foundation* of any entity's *being* were simply the being of its form as educed from the potentiality of matter through the action of «earthly» and «celestial» physical causes, then all entities would be some kind of *accident*, some sort of events of matter.

Matter, in turn, would be a kind of *unique*, hence *universal*, *primary substance*, which «contains in itself» all *individual entities* that would be reduced, at this point, to simple «modalities» of matter's becoming. In this sense, Aristotle's system would be an *ante litteram* version of Spinoza's idea that there exists a single substance, containing in itself the attribute of *thought* (due to its *uniqueness*) and the attribute of *matter* (due to its *potentiality*, considered as a sole logical possibility), and where matter is reduced to pure geometric extension in the Cartesian sense. Aristotle, however, was not Spinoza, and such a materialist reading of his theory was not characteristic of his own thought. Rather, it was characteristic of later representatives of materialist Hellenic thought, such as the Stoics, the Epicureans and, above all, Alexander of Aphrodisia, the great ancient commentator of Aristotle's work (2nd – 3rd century, A.D.).

Fundamental
Thomist
development:
causal explanation
not just of
«secondary»
substance
(essence), but also
of «primary»
substances (being)

However, as Aquinas noted, some reason must be present, probably rooted in some inconsistency, if many modern and ancient thinkers alike have given an essentially materialist interpretation of Aristotle's hylomorphism. Stated otherwise, a fundamental point in Aristotle's metaphysical doctrine was the distinction between the «primary» substantiality of self-sustainment and the «secondary» one of essence, where the latter derived from a *causal foundation*. Aquinas further developed this notion by finding a *necessary causal foundation even for the «primary» substantiality (self-sustaining character)* of individual physical entities. Aquinas, in this way, removed the otherwise inevitable assumption that the only self-sustaining entity, to which the primary substantial nature of *all* physical entities can be reduced, is *common matter*.

Act-potency, if applied to form-matter, can causally explain essences, but not the existence of primary substances

On the other hand, as Aquinas noted, the principle of act and potency, when exclusively applied to form and matter, can ground the being of «essences», particularly of the specific essences common to various entities. However, this principle cannot be the basis of the existence of (primary) «substances» and of accidents that relate to them. In other words, if one applies the principle to form and matter only, one can justify why entities are *different*, but not why they are *entities*, why they exist and, above all, why some of them *are self-sustaining* individuals, primary substances.

Aristotle developed metaphysical thought by identifying the *to ti ti*, the existential substratum of essences as distinct from essences themselves. With the discovery of essences and their causal explanation through act and potency, as applied to form-matter, he could causally explain *diversity*. However, diversity is always of something that exists in itself, of a substratum of differences that is self-sustaining.

Materialism commonly argues that matter is the metaphysical subject of diversity. This, however, would imply that forms, the principle of diversity, are not educed from matter. In Aristotle's hylomorphism, this would be in contradiction with the whole Aristotelian system, particularly with the fact that matter is not *self-sustaining*, rather it is what makes possible the transformation of what is self-sustaining as a physical body.

If matter were «that which exists in itself», from which all differences and different properties derive, Aristotle's system would fall into an unsolvable antinomy. Differences themselves could not be causally explained through act and potency, since if matter existed as a «primary» substratum, it would be an entity in actuality and not in potentiality. Then matter would exist as the single «primary» substance, as the subject of all properties, while all physical entities (dogs, cats, men, etc.) would be mere events, accidents of the single matter intended as a universal substance, unique substratum, sole metaphysical subject of all properties, of all accidents.

If we reduce entities' primary substantiality to sole primary matter, the latter cannot be pure potentiality and Aristotle's explanation of forms becomes invalid

If this were the case, the fundamental distinction made by Aristotle between «primary» and «secondary» substances would lose meaning. More generally, the same would happen to the metaphysical foundation of the individuality of existing entities, as well as to any real foundation, ontological and epistemological, of qualitative differences among entities. These foundations cannot have the metaphysical grounding argued by Plato. However, neither can forms be based on the metaphysical foundation that Aristotle argued for, if one considers matter as a «primary» substance (*id quod* rather than *id quo*). Hence, one should turn back to Democritus, and to the reduction of «quality» to sole «quantity». This is

what modern physics did, by distancing itself from Aristotle's philosophy of nature and embracing a form of atomist and mechanistic determinism. Modern physics was credible until the idea of the integrability of dynamic systems, hence of the possibility of replacing Aristotle's causal determinism with mechanistic geometric determinism (antecedent → subsequent, temporal *a posteriori* + *a priori* logical law that guarantees this necessary relation) remained in place. This, however, is not the case today (Cf. § 2.7.2), when it has become necessary for philosophy of nature, as well as for post-modern science, to go back to interpretations that had been considered outdated.

5.5.2 Participation of being

From Plato's participation of forms to Aquinas' participation of being as act

The materialist flaw explained above is the core of Aquinas' criticism of Aristotle (Cf. Aquinas, *S.c.Gent.*, II, xvi, 932-935). An entity's being cannot be reduced to the being-in-act of a form in a given matter by means of a material agent whose existence and causal action must be in turn justified. Plato's theory of derivation of matter from form through «participation» had flaws that Aristotle had pointed out. Aristotelian doctrine, however, showed some inconsistencies as well. In order to overcome these two thinkers' inconsistencies, Aquinas developed the doctrine of being-as-act and of the participation of being to single entities in reason of their essence. This doctrine thus aimed at synthesizing Platonic and Aristotelian metaphysics.

The causal explanation of essences makes a body's properties *passiones*, effects of causal agents

The starting point is the Aristotelian metaphysics of act and potency as applied exclusively to matter and form, through which Aristotle causally explained the essence (nature) of physical entities. Clearly the causal (active) influence of causal agents — already existing physical entities that are able to exert causal action — on the unstable material (passive) substratum, which is at the basis of all physical processes, could justify the being of essences in certain physical entities. It could justify the set of properties that characterize the nature of a given physical entity, in the sense that without that specific causal influence, that given kind of entity *could not* exist. To use an example in bio-physics, the existence of given species of animals and plants would not be possible without a given «ecological niche» made of environmental causes. In case these causes became impossible or started to lack, they would determine the *impossibility* of existence for a given biological species. Another example, this time related to quantum physics, is that of «Feynman's diagrams»: each quantum particle exists as the intersection of a particular set of interactions with other particles.

The intrinsic «nature» of an entity is the passive principle of a particular causal relation

For this reason, Aquinas usually called *passiones* — the passive term of a causal relation — the «properties» of a given subject. In a text cited above, dealing with the Newtonian criticism to a kind of Neo-Platonic Aristotelianism that was common during the late Renaissance, Aquinas considered some pseudo-Aristotelians as «ridiculous»: he was referring to those who tried to interpret the notion of «nature» or «essence» of a body as a «source of obscure forces» that would explain the chemical-physical interactions among chemical elements (Cf. footnote 40). Aquinas defined the specific nature of a body as a passive principle, that is, as the disposition of a given material substratum to a given type of causal action by other physical entities, which determine its existence and its specific properties. As a biological species in its ecological niche, or a quantum particle in its Feynman's diagram, the specific nature of an entity in the physical realm is the result of a characteristic set of stable causal interactions with the surrounding environment.

The essences or natures of bodies are potentialities, compared to the actuality of the related bodies' existence

Aquinas' fundamental intuition was that Aristotle's causal explanation of essences justifies the being of species, of «secondary» substances within the material universe, with no need for a Platonic «hyperuranium». However, these essences, or «modalities of being» are «possibilities of existence», they are a *potentiality* in relation to the concrete existence of an individual metaphysical subject, of a *toide ti*, a concrete «primary» substance possessing that essence in *actuality*. To use again the example of the «ecological niche», as employed in the famous recent movie by Stephen Spielberg (*Jurassic Park*), one thing is the reconstitution of the ecological niche of a given biological species with its different causal factors (dinosaurs, in this case), including the availability of the appropriate genetic set in the DNA strings of that specific species, which has been extinct for years. Another thing is to 'create' dinosaurs, for which the *in vitro* insemination of dinosaur gametes and the artificial gestation of the resulting embryo will be necessary. In other words, just as a causal explanation of the *being of essence* of a given «secondary» substance — species of entities — is necessary, so is the causal explanation of the *being of existence* of «primary» substances — the concrete individuals that possess that essence, one or more individuals that realize concretely, *actually*, that «possibility to be like this» with given specific characteristics, which we define as the *specific essence* of a given set of entities.

Fundamental distinction, in being, between existence (common being) and modality of existing (essence)

Compared to Aristotle, Aquinas introduced a new way of interpreting a fundamental metaphysical distinction:

- ◆ One thing is the *existence* common to all entities (*common being* or *esse ipsum*, «being itself»).

- ◆ Another thing is the *modality of existing or being of an essence* (*entitas*, «entity»), which is specific to different kinds of entities and possibly to individuals.

Being and essence
are related as act-
potency

One entity's being, therefore, can be distinguished in *being of existence* (common being or being itself) and *being of an essence* (entity). The *being-as-act*, which «includes» both, will be participated in any thing in reason of its essence, determining that it *exists* and *exists as such*, as a particular *entity*, with given essential and accidental properties. «Being» and «essence» are related as *act and potency*, the same way that, for Aristotle, *form and matter* were related in the constitution of the sole «essence» of material entities.



In substances composed of matter and form, there is a *double composition* of act and potency. The first is the composition of the (= secondary, *N.o.A.*) substance itself, as made of matter and form; the second is made of the *already constituted substance* (= essence as *potency*, *N.o.A.*) and *being* (= being-as-act, *N.o.A.*) (Aquinas, *S. c. Gent.*, II, 54, 1295).

Being and essence
are distinct and
complementary;
therefore they need
two distinct and
complementary
levels of causal
explanation

Together, *being* and *essence* constitute the actual and potential principles of any entity's *absolute being* (entity and existence). They are *truly distinct*, because they refer to distinct levels — *metaphysical* and *physical*, respectively— of the unique but complex causal relation that produces existence in any physical entity. The being-act of form, characteristic of the nature of a given kind of entities, refers to a causal action that is common to all individuals of the same species. Similarly, the being-act of existence *common to all entities* (*common being*) refers to a *metaphysical, unique, universal* causal action for all entities that make up the universe and that exert causal physical action on one another. No entity can cause unless it *previously* exists



That which belongs to something as an act depends on the action of an agent. It is the agent that realizes something *actually*. (...) For this reason, substances themselves, as they are caused, are things that obtain being from other things. Being itself belongs to substances as one of their acts. Now, potentiality is that to which an act belongs. Hence, both potency (= essence) and actuality (= being) are in each natural substance (Aquinas, *S. c. Gent.*, II, 54, 1284).

The reciprocal association between subject and predicate in analytical propositions in meta-logic is grounded on the reciprocal association between being and essence in metaphysics

As one can see, we find here the Thomistic solution to the fundamental ontological and epistemological problem of modern thought: the ontological foundation of logical necessity, particularly the foundation of the reciprocal association and implication of subject-predicate that constitutes the result of the inductive, analytic procedure of syllogistic *inventio medii* (Cf. § 4.5.2). The fact that given predicates necessarily relate to a logical subject is grounded on the fact that given properties necessarily relate to a metaphysical subject (substance). This, in turn, is grounded on the fact that both the properties and the subject are the result of the unique, but complex, causal «action» at two levels —respectively *physical* (for the properties) and *metaphysical* (for the substance and the properties) — that make that entity *exist*, and make it exist *as such*, as a particular *entity*.

Contingency makes the double and complementary causal level of explanation of being and essence necessary

But how should we understand the double structure of this causal action, intended as a joint physical and metaphysical level in the single causal relation through which any entity comes to exist with certain properties? At first blush, it would seem unnecessary to distinguish between these two levels. For an entity to come to existence, and for it to do it with given specific characteristics (a given essence), physical causality could seem sufficient. However, the need to assume a *metaphysical* level that adds to the *physical* level of causality, including it and making it ontologically possible, becomes apparent if one thinks of the *contingency* proper of a physical entity's existence, as well as of the *necessity* that should characterize the causal concurrence that makes this existence possible.

The existence and essence of all entities depend on the concurrence of causes that are necessary, even if contingent, and on their order

All entities in the universe, including men, are contingent. They do not come to exist by themselves, but need a concurrence of necessary and sufficient causes that make them exist and make them exist with those specific characteristics. In order to exist, all physical entities — «substances» or «accidents», natural or artificial entities— need:

- ◆ A set of causes, that is, of entities that already exist and that make them exist.
- ◆ A spatial-temporal order for these causes. In other words, in order to exist with those particular modalities of being (essence) entities need these causes to interact according to a given order so that they are not prevented from causing a given entity by the causal influence of other entities.

A contingent entity does not possess being from itself; therefore, even an eternal or sempiternal, spiritual or material entity can be contingent

All entities that, in order to exist, must *necessarily* be caused by other entities, which can be prevented from acting either by themselves (if these causes are endowed with freedom, as with human beings) or by something other than themselves (as with all physical entities that enter the causal concurrence determining the existence of other entities) are *contingent*. They are entities that *do not necessarily possess being (essence and existence)*, because they *do not possess it from themselves*. Possibly, even entities that are eternal in *one or two directions* of the *arrow of time* (sempiternal or immortal entities) can be contingent.¹²⁸ Given that the same reasoning explaining the existence of any entity that belongs to the physical realm can apply to their causes, which are in turn necessarily caused — because they do not possess being from themselves — then the being of all entities that made, make and will make up the universe cannot be grounded on an *ultimate*, definitive justification, unless one assumes a *primary causality* that gives meaning to these relations «from the outside».

This is even more valid for entities with limited temporal existence

The causal concurrence of a set of physical causes is *necessary* to justify the existence in a given *spatial-temporal location in the universe* of a given entity with its essence (e.g., a lizard's existence «here and now»). It is also necessary to justify the non-existence in a given *spatial-temporal* location of another entity with another essence that is incompatible with that causal concurrence (e.g., a dinosaur's non-existence «here and now»). However, it is not *sufficient* to justify in *absolute* terms the being of any entity that needs to be caused in order to exist.

Concurrence of physical causes can only justify the spatial-temporal location of existence

Conversely, the causal concurrence of physical causes is sufficient to justify the existence of physical entities of a given kind in a given spatial-temporal location, with a given essence and not another. However, these, by themselves, cannot completely justify the existence either of entities that are the effect of a given causal chain, or of entities that make up the causal chain itself, or of the order that characterizes such a chain. Even if one closed the chain in a «circle», according to the Greek-pagan notion of eternity of the cosmos or of the indefinite succession of «worlds» —which, as shown in the next chapter, is compatible with the Thomist idea of participation of being, but is incompatible with the Christian theological notion of creation (Cf. § 6.3.3.2.) — once the principle of contingency is accepted, the existence of a «non-caused-cause»

¹²⁸ In Thomist cosmology, both physical entities (e.g., celestial bodies) and spiritual entities such as angels and human souls were sempiternal. However, in his metaphysics Aquinas acknowledged the possibility that the universe itself be eternal, while remaining *contingent*, therefore in need of a causal justification to its existence.

that gives substance to the chain and to each of its rings is still needed, whether the chain is «open» or «closed», «finite» or «infinite».¹²⁹

Aristotle's
explanation cannot
justify entities'
primary
substantiality

This was the core of Aquinas' criticism of Aristotle. The philosopher's causal explanation of essences (secondary substances) through act and potency reduced to form and matter could not justify the primary substantiality, the *to ti ti* of metaphysical subjectivity, of the individual substratum of entities that make up the ordered set of «causing» causes, which are in turn «caused» and make up the physical realm.

Same limitation as
with neo-Positivist
ontology: one
entity's being is
reduced to «being-
something», the
being of essence,
but not of existence

At the metaphysical level, one finds here the same limitation that was present at the logical level in the ontology of logical neo-Positivism, related to what Quine defined the «inscrutable nature of reference». The problem with that approach, as with any other ancient or modern «formalist» approach to being, relates to the justification of the existence, or absolute being, of entities. The statement «*x* exists» can be interpreted, within that ontology, only in terms of a definition of essence (belonging to a class), so that that statement is reduced to the other statement «*x* is *y*» (Cf. § 4.3). In this way, however, the justification of the existence of any entity is deferred *ad infinitum* and, as noted by Aquinas, one cannot go back indefinitely with «essential» causes. The universe as a whole can both be finite and infinite, eternal or sempiternal (with a beginning). But, since it is made of contingent entities, which do not give being to themselves, given that both their essence and their existence depend on a concurrence of causes, one must «close» the system of causes. If it were otherwise, such a system would lack metaphysical substantiality both as a whole and in each of the intersections-entities of causal relations (Cf. Fig. 5-2).

¹²⁹ Taking the simple example of a chain of people that pass water buckets to one another to extinguish a fire, if I asked why the water is in bucket *n* at time *t*, it would be correct to state that «it is in *n* at time *t* because it was in bucket *n-1* at time *t-1*». If the chain closed in a circle, this explanation would be indefinitely valid. However, in both cases water must have been taken from a fountain in order to be able to exist in the buckets. The water, in other words, must have been taken from something that has *water from itself and not from something other than itself* even if, when the chain is closed, it makes no difference that this has occurred in any bucket *n* at any time *t*.

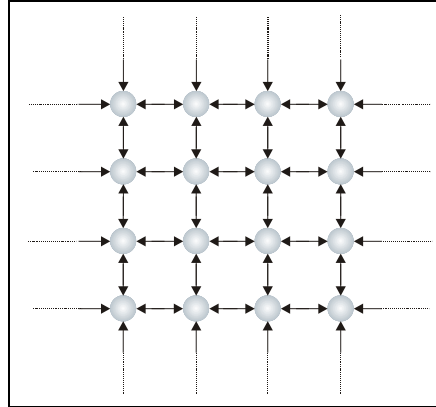


Figure 5-2 Diagram of the causal network of contingent entities according to Aristotle's approach, which lacks metaphysical substantiality, both as a whole (causal lines are shaded at the extremes) and in each of the intersections-entities (non-definition of the content in each circle).

The causal network of contingent causing-caused entities needs an uncaused cause that gives substantiality to the network as a whole, as well as to each of its intersections

Stated otherwise, a given set of physical causes that are *necessary* to the existence of a given entity cannot give the ultimate justification for the existence of that entity, since they are *contingent*. In order to be metaphysically substantial, the ordered set of caused-causing contingent entities, the so-called «cosmos», needs a common dependency on a *Self-Sustaining Being*, a Universal Primary Cause that is not caused,¹³⁰ in each of the intersections (entities) of the network of causal relations. This Primary Cause *transcends* the universe of contingent entities, which are causally ordained at the *immanent* physical level; *independently from time, it participates and preserves* the being of these entities, both taken individually, and as a whole (Cf. Figure 5-3).

¹³⁰ All contingent entities that are necessary to cause the existence of a given entity are therefore defined as «secondary causes» in relation to the Primary Cause. To avoid confusion between the notions of «Primary Cause» and «secondary causes» on the one hand, and «proximate causes» and «ultimate causes» on the other, one should keep in mind the inverse relationship that exists between ontological order (of being) and epistemological order (of knowledge), which will be dealt with below (Cf. 6.2.1). The causes that, at the level of the various sciences, can be defined as «proximate causes» of a given entity's being and action will be, at the level of the metaphysical constitution of this entity, its «secondary causes». Conversely, the «Primary Cause» of an entity in its ontological constitution is, in relation to the epistemological order of sciences, the «ultimate cause», arrived at through metaphysical investigation.

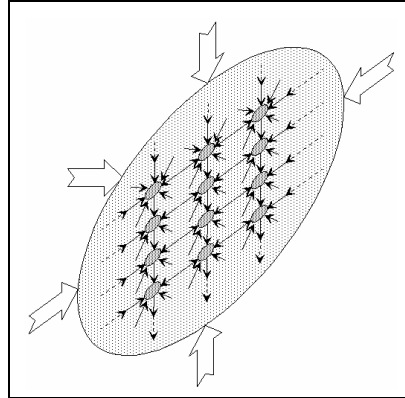


Figure 5-3 Diagram of the inter-relation between Primary Metaphysical Cause (large arrows), outside the spatial-temporal universe (large light-coloured circle), and secondary physical causes (small arrows) within this universe that determine the existence (the content) and the essence (the borders) of the being of individual entities (small darker circles) —on a different level than that of the Primary Cause, as represented in the diagram (secondary causes are on two dimensions, while the Primary Cause on three). The being of any entity, essence and being, entity and existence, is the result of the causal concurrence between the Primary Cause and the secondary causes (the Self-Sustaining Being and other physical entities)

Aquinas' *proprium* compared to Plato and Aristotle, Kant and Husserl

This is the core of the Thomist doctrine of being-as-act, compared both to the «champions» of transcendentalism in modern philosophy —Kant and Husserl— and to those of classical philosophy —Plato and Aristotle.

Contrary to transcendentalism, being is based on content, rather than on form (*versus* Kant)

- ◆ The Thomist doctrine of being is based on content and is intensive (Cf. § 4.5.2.), rather than being based on form and extensive (Cf. § 4.5.1.), contrary to Kant's transcendentalism. Existence is not extrinsic to essence, but rather includes it as the surface of a shape includes its borders, while being delimited by them (double and complementary determination between being and essence). *Ontologically*, existence is not a subject's property, but what gives substantiality to the whole of the subject's properties. *Logically*, the being of existence is not a predicate, but a meta-predicate that has, as argument, any other predicative attribution (proposition) that can be validly constructed between a subject and a predicate.

Being's content does not depend on a transcendental intuition, but on an act-potency causal network on two complementary levels, the immanent and the transcendental (*versus* Husserl)

- ◆ At the same time, being is endowed with content, it is intensive being, as Cornelio Fabro (Fabro 1961) defined it, but not because it is the object of transcendental or eidetic intuition by some trans-individual «phenomenological subject», as in Husserl's transcendentalist phenomenology. It is intensive because it is part of a causal scheme of act-potency, different from the temporal scheme proper of Hume and Kant (time has no role here: transcendental causality is simultaneous in each fraction of time) in which two distinct causality levels — transcendental and categorial, metaphysical and physical— intersect with no confusion, each autonomous in its realm.

The act-potency causal relation in relation to being-essence is defined *participation* of being

Aquinas referred to this particular causal scheme of act-potency with the Platonic term 'participation'. This, however, does not refer to the participation of «form as act» (*forma ut actus*) in relation to matter, but rather to participation of «being-as-act» (*esse ut actus*) in relation to essence, in relation to form and matter. With Aristotle, this network of act-potency relations was considered as a set of causal relations between already «actual» entities that, with time, act on matter's potentiality «educing» forms from it that, for their uniqueness, «qualify» the new entities, making them exist with specific essences. Compared to this interpretation, metaphysical participation is what gives the «ultimate» ontological consistency to the whole network, as well as to each of its intersections and relations, outside time and simultaneously at each moment within the material realm.

Aquinas gave consistency to the Aristotelian causal foundation of essences and to Plato's doctrine of participation by moving the latter from forms to being

The Thomist reinterpretation of the Aristotelian doctrine of actuality and potentiality was able to give consistency to Aristotle's fundamental idea of the causal foundation of essences. It was also able to solve the intrinsic contradiction in Plato's doctrine of participation. «Form» and «matter» cannot be related as «participated» and «participating», as Plato argued. Given that one is complementary to the other, it cannot be that one derives from the other, but both must derive from something in which they both participate. The only thing that both have in common, even if with different degrees and modalities (being in potentiality and being in actuality) is existence, the *being common* to all entities. This was Aquinas' solution to the fundamental dilemma of classical Greek philosophy. That in which any entity participates is being. It is not the form in the Supreme Unity, as with Plato, but it is the being of the Pure Self-Sustaining Act, the One in which essence and being coincide, so to ground their difference and complementarity in all other entities in which they do not coincide. All forms that diversify physical entities

originate as acts-relative-to-potency¹³¹ from the Pure Act, the Sole Principle source of the being of all matter. This is Absolute Act, Absolutely Simple and Actually Infinite in relation to any determination, so that it remains completely ineffable.

Consequences for today's evolutionary cosmology

For a philosophy of nature that is appropriate for the present state of the art of physical and cosmological sciences, there are two useful consequences deriving from a metaphysics of act and potency that is moved from form-matter to being-essence:

1. Possibility of a metaphysical foundation of the cosmos and of its evolution that is independent from temporal questions on the absolute or non-absolute beginning of the universe

- ◆ *From the point of view of evolutionary cosmology*,¹³² this theory offers a metaphysical foundation consistent with the fact that matter itself (mass-energy), with its present characteristics (preservation, distribution, structure, universal physical constants, etc.) needs an adequate causal foundation, even if such a causal foundation cannot follow the modern simplistic Humian-Kantian scheme of predecessor-successor + necessitating logical link. This is because it is impossible, in principle, to deduce the initial conditions of the universe from its present state. The principles of general relativity are not valid on an infinitely small scale, hence they are inadequate to define the initial conditions of the universe close to the big-bang, just as, in unstable systems, it is impossible to go back precisely to the initial conditions (Cf. § 2.7.2). In order to metaphysically justify being and the development of the universe in a consistent manner — even taking the assumption that the universe is eternal, as it will be shown below — the Humian-Kantian causal scheme is totally inadequate. However, at the ultimate level, in which the foundation of the «being and changing of matter» themselves is required, the Aristotelian scheme of act-potency/form-matter, which was so useful for the philosophy of nature of ordinary unstable systems (Cf. § 6.3.2), is also insufficient, and it becomes necessary to look to Aquinas for a metaphysical theory that is richer, more articulate and inclusive (Cf. § 6.3.3).

¹³¹ Because of this *relative* actuality, they are limited, different and somehow definable, even if they are actually infinite at the level of some of their properties — think, for example, of the infinite actuality of numerical forms in relation to infinite increments.

¹³² Not «*evolutionism*». «*Evolution-ism*» and its dialectical antithesis, «*creation-ism*», like all opposed and reciprocally alternative «isms» (as well as the Hegelian dialectic that formalizes them) belong to the age of ideologies or «world visions», that is, to the modern age from which we are slowly distancing ourselves.

2. Explanation of the virtual existence of elements in molecules and atoms, through an explanation of the substantial unity of bodies

- ◆ From *the point of view of the substantial unity of bodies*, the metaphysical distinction act-potency, as applied to being-essence and, more precisely, to the pair existence-entity, has another precious use in contemporary philosophy of nature. This aims at explaining one of the most useful, and at the same time most confusing approaches of the medieval philosophy of nature in the Scholastic doctrine of Aristotelian inspiration. This is the doctrine of the *virtual existence of elements* in stable physical compounds —e.g., of atoms in molecules, of electrons and nucleons in atoms, of quarks in neutrinos (protons and neutrons), etc. — of units in numbers, of points in mathematical continuity¹³³ and, more generally, of «parts» in «wholes» in metaphysics. Without the Thomist doctrine of the real difference between being and essence and of being-as-act, it becomes impossible to justify the different virtual and actual degrees of existence of being in the elements of the compound, as a doctrine that is distinct from the Aristotelian one of the indistinct, potential existence of form in matter. In fact, without the Thomist development, the medieval concept of virtual existence of elements in a compound in physics, of parts in the whole in logic, of points in continuity etc., resulted insubstantial even to well informed medieval thinkers such as Anneliese Meier (Meier 1984). The elements of a compound, the units in a number, or the subsets of sets do not exist as forms in matter in an «indistinct» way.

Virtual character not of forms, but of the being of parts compared to the whole to which they belong

Without the Thomist doctrine, the virtual existence of elements in a compound is intended as the virtual existence of the elements' form compared to the actuality of the form in the compound. In this way, for example in physics, the doctrine of the virtual existence of elements is presented in opposition to experimental evidence. Atoms in a molecule are not «indistinct», neither are quarks in protons and neutrons, unlike the final stages in the initial conditions of an unstable system's dynamics, that is, unlike forms in the potentiality of matter according to Aristotle.

¹³³ This is a doctrine that could be extremely useful as a foundation of mathematics, as Cantor first realized (Hallett 1984, 119-146). In order to avoid antinomies, while limiting constructible sets (classes), one could use one single axiom of the degrees of existence that justifies the notion of virtual existence, e.g., of units in numbers. This would be an alternative to axioms of existence of non-constructible totalities, such as the set-potentiality axiom or the «well-ordering» one. However, as Cantor bitterly realized, without the doctrine of virtual existence as applied to the virtuality of parts' «being» in a totality —rather than to the parts' «form»— while keeping forms in their full actuality, all numeric sets «collapse» at the unitary cardinality. A set foundation of numbers without the well-ordering axiom becomes then impossible, thus making the theory of parts' virtual existence completely useless (Cfr. Hallett 1984, 119-184; e Basti & Perrone 1996, 224-241).

The elements of a physical compound can be identified and distilled from the systems to which they belong. However, if parts are distinct in the whole, it means that their forms are actual, otherwise parts would not be distinguishable and the whole would be *homogeneous*. What is virtual in a «part» compared to the «whole» is not the actuality of the form, but the actuality of the part's being, its degree of «existence». In being there exist, due to the differences in essence, multiple degrees between the minimum participation of being (= «pure» potentiality of prime matter) and the maximum of what is participated (= «pure» actuality of the Self-Sustaining Being). Parts are therefore distinct in the whole and their forms are completely actual, and this is why parts can be «re-extracted» from the whole when the latter is destroyed (e.g., in chemistry, through the electrolysis of water).

Parts, as components of an entity whose essence is by definition more complex, participate in being to a lesser degree hence their existence is virtual compared to the actuality of the whole to which they belong

When parts exist as individuals, separated from the whole, they participate in being in proportion to their essence. Once they become part of a more complex entity — physical, mathematical, logical — the latter participates in being to a higher degree, given its more complex essence, and the parts' entity has changed. The constituent parts, which have a lesser degree of being due to their very essence — and not simply because they belong to the whole — are virtually existing compared to the actuality of existence of the whole to which they belong. The fundamental consequence of this doctrine is that the definition whole-parts becomes predicative with Aquinas: the whole *cannot* exist without the parts, because its modality of participation in being, that is, its essence, presupposes them. However, parts can exist without the whole — naturally not as parts, but as individuals — since their essence, even if «simpler», allows them to exist without the whole to which they *can* belong.

As will be shown in the Third Part of this work, in the Second Volume, the doctrine of the virtual existence of elements in a compound is useful in the philosophy of nature dealing with the structure of material entities (e.g., atomic, molecular and organic compounds).

These intuitions — that are extensively treated in classical texts of neo-Thomist metaphysics such as *The Spirit of Medieval Philosophy* by Étienne Gilson (Gilson 1932) and, above all *Participation and Causality* by Cornelio Fabro (Fabro 1961) — are important to understand the classical Thomist metaphysical formulation of «being as act», *esse ut actus*.

The essence of an entity is related to being-as-act like potency is related to act

Entities participate of being-as-act, which is participated by the Pure Act, according to their different essence, or nature. Each entity's *essence* is therefore related to the *being* participated by the Primary Cause like *potency* to *act*. Essence coincides with being only in the Self-Sustaining Be-

ing, which is therefore *Pure Act* with no potency. Only the Pure Act is «being as essence», it is the *Being-necessarily*, as Aquinas defined it, following Avicenna. In all other entities, which have being through the participation from the Self-Sustaining Being, the essence «draws» the actuality of being with its potentiality, thus determining the differences among the different entities, just as matter «draws» the actuality of a specific form, so that no individual or group of individuals exhaust the richness of the species to which they belong. No man or group of men exhaust the richness of the human species, of the human essence, of «human-kind», contrary to what racists all around the world claim. By the same token, no entity, no kind of entity, nor all species of entities that make up our universe exhaust the richness of being, of its actualizations and manifestations...

The following are some of Aquinas' excerpts that well explain the heart of his metaphysics of the *act of being*.



Indeed, being is the most perfect thing of all: it is related to all things as *act*. Nothing has actuality if not in reason of what it is: being itself is what actualizes all things, as well as forms (*S.Th.*, I,4,1, ad 3, emphasis added).

It is clear that the First Entity (...) is actually infinite, that is, it has in itself the entirety of being, which is *not drawn from any nature of genus or of species* (...). Hence, any entity that is after the First Entity, given that *it is not its being* (being and essence are not identified in it, *N.o.A.*) receives being into something (into the essence, with its generic and specific components, *N.o.A.*) *by means of which being itself is drawn*. So, in any created entity, one thing is the nature of the thing that participates in being; another thing is the participated being itself (...). It is then necessary that the being participated in each be related to the participating nature, the way that act is related to potency (*Q. de Spir.Cr.*, 1, emphasis added).

In each thing, there are always two principles, one being the counterpart of the other, the relation (*proportio*) of one to the other being like the relation of potency to act; indeed, nothing is completed other than by its act (*S.c.Gent.*, II, 53, 1283, emphasis added).

What type of causal relation is the «participation of being»?

But in what way is the participation of being a kind of causality and what makes metaphysical causality, or «participation of being» different from the other forms of physical causality? In order to understand this, it is necessary to go further into the analogy between the ontological and the logical participation of being by the Self-Sustaining Being, that is, into the analytical foundation of «truth» of an entity's being, which constitutes the referent of a linguistic proposition.

5.5.3 Causal structure of participation

In many parts of the *Summa contra Gentes*, the principal Thomistic text of metaphysical theology and investigation on the notion of the Absolute and hence of the «God of philosophers», independently from Revelation, Aquinas delves deeper into the fundamental structure, both metaphysical and logical, of the notion of *participation of being*.

Two questions:

The main questions are the following:

1. In what way is participation a causal relation?

- ◆ In what way and up to what point can the relation of participation of being —from the Absolute Being to the universe of contingent entities— be defined as a *causal relation*?

2. Can participation be understood as a relation between parts and whole?

- ◆ Can the relation of participation of being —from the Absolute Being to the universe of contingent entities— be understood as a relation between parts and whole, as if Absolute Being were the totality of the universe, as argued in some metaphysical systems, particularly oriental ones?

Participation is a causal relation only in an *analogical* sense

Concerning the first question, Aquinas often referred to the Self-Sustaining Being with the term «First Cause». However, the philosopher clearly stated that this use of the word «cause», as applied to the relation of participation of being, could only have an *analogical* value. As Aquinas noted (*S. c. Gent.*, II, 12, 913)¹³⁴ the notion of causality is characterized, in logical terms, by a relation of *double implication* and therefore by the *symmetry* of necessitating relations between cause and effect (Cf. § 4.4). Indeed, given a causal law, not only does the effect *necessarily* refer to the cause, but also the cause *necessarily* produces the effect. But if this were the case also with the relation between entities and the only Self-Sustaining Being, then:

Asymmetry of the relations between Self-Sustaining Being and entities that participate in It

- ◆ The relationship with the Self-Sustaining Being would not have 'foundational' value towards the set of causal relations characterized by the double necessitating nature mentioned above;
- ◆ If it were subject to necessity — the way that Greek gods were subject to Fate— the Self-Sustaining Being would not be as such, it

¹³⁴ Here is the excerpt: «Given that 'relations are those things that refer to something other than themselves according to their being,' as the philosopher stated in the book of *Categories* (7, 6a, 36-37; 6b,6-8), it would derive that God's substance would be this same entity that is related to something other. But that which is defined as something related to something other, in some way depends on the latter, because neither it can be, nor can it be intended without reference to it. This would mean that God's substance would depend on something else that is extrinsic. But then it would not be the «necessary being». Hence, in God, there are no *real relations* of this kind».

would not be Pure Act any longer, but would possess *potency*. Indeed, that which is subordinate to necessity is *contingent*.

To use the Arabian philosopher Avicenna's words,¹³⁵ if the relation of all entities with the Self-Sustaining Being (= participation of being) were an ordinary causal one, the Pure Act or Self-Sustaining Being would cease to be that *Being Necessarily*, that Absolute Necessity that the Absolute instead is, so to be the Foundation of any other type of necessity, including causal laws.¹³⁶

Difference between
modern and
Thomasian
causality schemes

The profound difference between a realist foundation of causal determinism and a logical one, such as that of modern and Greek science alike, appears then clear. The latter were both grounded on the logical and geometric notion of *symmetry* and therefore on the ultimate *reversibility* of processes — as with the ergodic hypothesis discussed above (Cf. § 2.7.2). Clearly, the consequence of the logicians' approach is *im-manentism*: the absolute identifies itself with the necessity of totality. To-day, however, the discovery of logical and mathematical antinomies has shown that this absolute (i.e., actually infinite) totality of perfectly determined relations cannot be demonstrated or defined in any substantial way, given that any attempt of this kind contradicts itself. Hence, either one nihilistically gives up any foundational attempt — the *Grund*, or «foundation», is an *Abgrund*, an «abyss» or «absence of foundation» (Nietzsche) — or one understands this infinite absolute totality in actuality as a kind of «primitive» that cannot be further analysed. Such a primitive stands prior to any foundational argument, and formally expresses this assumption through axioms such as that of the *well-ordering for infinite sets* (Cf., for example, Fraenkel 1968). Knowing that an alternative to the *impasse* common to Greek and Modern culture exists can be challenging for a post-modern thought that is intellectually honest and that is not closed by way of principle to the Arabian-Christian Middle Ages. It is no coincidence that the oriental mind-set most open to science and least influenced by the Enlightenment, in fact more open to a revision of its own medieval culture against the attack of Western Greek-Modern culture — this is the philosophical-scientific thought in

¹³⁵ He was the first to introduce the notion of «participation of being» in western thought, even if in an extrinsic sense, which is very different from the Thomasian interpretation. Indeed, it is not grounded on the double level of causal determination that we have just illustrated.

¹³⁶ At this point we should recall how Hegel, in order to challenge the Arab-Christian notion of transcendence, developed the concept of the «god that needs the world in order to be god». This is the «god» of reciprocal determinations, absolutely immanent, that it would be contradictory to indicate with a capital letter.

Japanese culture— has resulted as the best «breeding ground» for similar foundational notions.

These reflections, however, bring us to the second question on the Thomistic relation of participation of being from the Absolute to the universe of contingent entities: can this be understood in terms of a relation between *Whole and parts*, as if the entities that make up the universe were parts of the Absolute, and the latter represented the universe itself?

The relation
between Being and
entities cannot be
one between Whole
and parts

On the basis of the previous discussion, the answer to this question must be negative: theologically, this would be equal to affirming *pantheism*. The metaphysical reason given by Aquinas for this negative answer is indeed interesting. This reason is still valid today, despite it touching upon questions on which both logical and mathematical thought have greatly evolved, starting from Cantor's work of last century.¹³⁷ Aquinas set out the issue in the following way: can the «Being-Necessarily», the Absolute Principle, be defined without contradiction as an *all-embracing totality of relations*?

Contradiction in the
notion of actually
infinite totality of
relations

Clearly, this totality must be intended as an *actually infinite totality of relations*, given that it must contain all possible relations in itself. However, long before Cantor and Gödel (who used the method of *diagonalization*), Aquinas demonstrated that the concept of infinity in act including all possible relations is intrinsically contradictory, given that it is already contradictory in one subset of these possible relations: the set of whole (positive) numbers and their combinations, generated from the unit and from the relation that sets the successor as $n + 1$ (Aquinas called it the *binarius*). If one considered such a totality,

It follows that there would be *actually infinite relations in the same totality*, given that numbers that are infinite in potency (hence a subset of the totality of possible relations, *N.o.A.*) are in any case *greater* than the initial successor (*binarius*, Plato's «two-ness»), even if the latter contains them *all* (*cum numeri infiniti in potentia sint maiores binario, quibus omnibus ipse est prior*)» (*S.c.Gent.*, II, 12, 915).

In modern terms, the characteristic function of an infinite set of elements (in the case of the set of positive numbers, the relation of succes-

¹³⁷ On the other hand, it is well-known that Cantor studied Aquinas' work on these issues, and looked for assistance from Professors of the Gregorian University in Rome to understand Scholastic philosophers' and theologians' interpretations on the issues of infinity and its actuality (Hallet 1984). Unfortunately, he did not have access to the research on the Thomistic doctrine of being-as-act that was developed during the XX century, so that his understanding of Aquinas' thought was strongly limited. In different circumstances, his late theory on the virtual existence of elements in numerical sets—with which he tried to avoid antinomies— would have been developed differently, and would have led to very different results (Cf. Basti & Perrone 1996, 224ff).

sion) and of all their possible combinations (subsets) cannot be contained in that set. As recalled, the modern way, discovered after Cantor, to avoid antinomy in set and/or class theory is to distinguish between non-constructible sets-potentiality (= classes) that determine (organize) other sets, and constructible sets (determinable) that derive from the first (Cf. § 1.6), through axioms of existence such as Von Neumann's axiom of the set-potentiality. Aquinas' distinction went in the same direction, as shown both by the preceding text as well as by another that will follow.

The principle that gives actuality (being) to an infinite totality must be «outside» the totality itself

If we consider the universe of existing things as the actually infinite totality of *real* (causal) relations among the entities that make it up, the relation(s) with the Absolute that *engenders* such a totality *by determining it* must be «external», «transcendent» to the totality itself. But, if the relation(s) that the Absolute has with the set of contingents is not a real, causal relation—indicating at least the symmetry of necessity between cause and caused—what kind of relation is it?

5.5.4 Inductive foundation of truth

In order to understand the asymmetrical relation of ontological participation, Aquinas used an analogy with the asymmetrical relation of logical reference between a proposition and the entity it refers to, which grounds the *logical truth* of the proposition itself. This grounding of logical truth might then be defined as the grounding of *logical participation* by the logical entity in the proposition to the extra-logical entity of its referent.

Reference as a relation of foundation

As Aquinas explained, relations with creatures can be attributed to God, but only in the way that the *knowable* (object) is related to the *knower* (subject). This relation is always *asymmetrical*. As the knowable determines with its being the truth or falsity (existence or non-existence as a logical entity) of the knower's proposition that refers to it, the cognitive relation *as such* between knower and knowable is not in the latter, but rather in the knower. It is the proposition that *necessarily refers* to the knowable (hence it is the entity that necessary refers to the Self-Sustaining Being) and not the opposite. This is because the knowable (as an entity) determines the «being» (truth or falsity) of the proposition, but the proposition cannot determine anything in the being of the knowable to which it refers. If I say that «the sky is blue» this does not mean that the sky is or is not blue; it is the blue colour of the sky that determines the logical being (*truth*) of the proposition «the sky is blue» or the logical non-being (*falsity*) of the proposition «the sky is not blue» (Cf. *S.c.Gent.*, II, 12-15).

Parallel between the referential relation proposition-object and the relation universe of entities-Self-Sustaining Being

Metaphors aside, *reference* is an asymmetrical R relation, that is, $xRy \neq yRx$, as Russell explains in his *Principia*. Aquinas explained this asymmetry in the following way, when applying it to the justification of the relational structure between entities and the Self-Sustaining Being. Just as an entity gives being to a proposition, which necessarily refers to that entity in order to be «logically» *true*, the Self-Sustaining Being gives being to an entity, which necessarily refers to It in order to “physically” exist.

The relation reference-to-object is one *immanent* to the knowing subject — and/or to the logical system— and does not causally influence the object

In the cognitive order, the relation reference-to-object remains in the knowing subject, rather than in the real known object. It is an *immanent action*, as will be explained in Parts Four and Five of this volume. The act of referring-to-an-object (*adequation*) is one through which the cognitive operation — or the corresponding logical operation— modifies its own logical form, in order to adapt to the natural form of the known extramental object. In this way, as Aquinas concluded:

To see, understand and other similar actions remain in the things acting and do not pass over into those which are acted upon. Hence, what is visible or what is knowable is not acted upon by being known or seen. And on this account, *these are not referred to other things but others to them* (Aquinas, *In Metaph.*, V, xvii, 1027).

Other examples of similar relations in epistemology and economics

The «right-hand» or «left-hand relation» of objects, which are always in relation to the observer and are never in the objects themselves, are other examples of asymmetrical relations. For example, I might say that a column is «at my right» simply because I am «at its left». The same applies to an image in relation to its original: the image is similar to the original, not the opposite. Similarly, for the relation between currency and purchasing power, it is the value of the currency that changes in relation to the variation of prices — or of the market flow, as one would say today — not the opposite. All *foundational relations* are asymmetrical, hence they are «causal» only in an analogical sense.

It is an object's being that grounds the truth of a proposition, not the opposite

From the point of view of logic and epistemology, given that declaratory or cognitive acts do not modify the referential object, it is the object's being that grounds the *truth* of the corresponding proposition and/or the *adequacy* of the corresponding cognitive act. From the epistemological point of view, concluded Aquinas, one can also speak of the relation of a cognitive act to the knowing subject. However, it would be a mistake to talk of a «relation of reference» of a cognitive act to the knowing subject. The reference is only from the proposition to the object, which then is *constitutive* of truth —of the logical being of the proposition itself. The proposition is then intended as a result of the cognitive act. The relation between a cognitive act and a subject — said Aquinas (Cf.

Aquinas, *In Metaph.*, V, xvii, 1029) —is not one of reference, rather one of an accident to a substance and/or of a faculty (= active potentiality) to its subject.

Decoherence vs. reduction of the wave function eliminates the last possibility of refuting this epistemology and meta-logic

In quantum mechanics, the reduction of the wave function seemed the only sphere — which is of no little epistemological importance for philosophy of nature and of science— in which, contrary to what is stated here, a change in the system of representation radically influences the observed physical system. Today, there are explanatory models of quantum indeterminacy, such as the *decoherence of the wave function*, which are more effective than the wave function reduction. These stress that the interaction of the observed quantum system with the representation system as such is not the important factor; rather, it is the interaction of the observed quantum system with other physical systems. This refutes the last «scientific» principle of modern transcendentalism. In other words, the phenomenon of decoherence— hence the passage from determinism to indeterminacy— is not upheld by the relation with the observation system intended as a representational, or cognitive, system, but intended as a physical system among other physical systems.

A proposition does not change anything in the referent, and the latter grounds the truth (logical existence) of the former; the same occurs between an entity and the Self-Sustaining Being

An entity, with its existence, does not modify anything in the Self-Sustaining Being to which it necessarily refers in order to be an *entity*, the same way that a proposition does not modify anything in the being of the entity to which it necessarily refers in order to be *true*. The meaning of this analogy is better understood when thinking of the *asymmetrical, reflexive* and at the same time *open* nature of both a *reference* in the logical order (which is always a reference in itself to something other than itself) and of the *being-in-itself* of an individual substance's existence in the ontological order (Cf. § 5.4.2, the notion of *reditio ad semetipsum*, «return upon oneself», with which Aquinas defined this immanent character of being-in-itself of individual substances as discovered by Aristotle). Both are cases of self-referentiality that, in order to be substantial, must be *partial*. That is, the entity's existence and that of the proposition's adequacy (truth) must be *grounded on something other than themselves* (Cf. § 3.1.4.2, on the «semantically open» character of truth in all formal systems as demonstrated by Tarski; and Cf. § 5.5.1, on the insufficient character of a substance's being-in-itself in the Aristotelian sense, since it is not «ontologically open» to the participation of being, as argued by Aquinas). Just as formal systems must be syntactically and semantically «open», in order to be substantial and true (that is, in order to exist logically), substances must be metaphysically «open» in order to exist as physical entities.

The actual infinity of the Self-Sustaining Being is absolutely simple



Differences with the modern, Hegelian reading of the dogma of creation

The relation between the Self-Sustaining Being and the universe of entities, concluded Aquinas, is similar to the intensional relation between subject and object in a cognitive act. In the knowing subject there exist both the intensional object (e.g., the concept in the metaphor ‘the world’) and the relations between this and other objects (Cf. *S.c.Gent.*, II, 13, 919).¹³⁸ The fact that all entities refer to Being, because it is from this Being that they receive existence, is not in contradiction with the absolute simplicity of the Self-Sustaining Being.

Indeed, nothing prevents our intellect from understanding many things and from referring in many ways to what is simple, so to consider it under multiple relations. The simpler it is, the greater its power and the number of things of which it is the beginning (...). For this reason, even if many things are stated in relation to God, this proves Its absolute simplicity (*S. c. Gent.*, II, 14, 921).

Aquinas’ constant reference to «God», the «God» of Christian theology, was possible because his metaphysical doctrine of participation can be intended as a continuation, even without demonstration, of the dogmatic statement of creation in the Christian sense. This stresses the absolute necessity of the relation between creature and Creator, which stands in opposition to the absolute freedom of the relation between Creator and creature. Conversely, such a metaphysics stands in opposition to the pagan vision of «fate» and/or «destiny», as well as to the modern, Hegelian reading of the same doctrine, where Being and entities are linked by a system of double necessity. This is the scheme of the «necessary constitution» of the Absolute Spirit through the dialectical evolution of nature and history, where, in other words, «God needs the world in order to be God».

¹³⁸ Applied to theology, this means that, in God — the Self-Sustaining Being in metaphysics — the entities making up the universe, as well as their relations, exist «intentionally», as the expression of God’s creative will and thought. The relations that thought-out objects have with one another and with us can be made objective (they exist) in human consciousness (e.g., objects can be useful, hence appetizing, or dangerous and therefore to be avoided in relation to our will, or they can be true or false according to our reason). This is not the case for the relations that we have towards them as subjects, given that these are objects of thought and, therefore, a result of our abilities to which they are intrinsic. Similarly, the relations that entities have with God exist in God, while those that God has with them do not, given that entities making up the universe are «creations» of God’s mind. Because God is the Foundation of the Universe of entities, hence *transcends* it, God is *immanent* to entities, in the sense that these are the result of God’s creative action, the effect of their grounding *themselves* in God.

5.5.5 Infinity of Being vs. Finite nature of entities

Contradiction in the notion of infinity *in act*, while it is possible to speak of actually infinite objects (both in absolute and relative terms)

The preceding discussion highlights two more properties of the Self-Sustaining Being:

- ◆ Its actual infinity or absolute lack of potentiality and negativity;
- ◆ Its absolute simplicity or absolute lack of any internal (quantitative) multiplicity and (qualitative) difference.

This is useful for understanding another fundamental point of Aquinas' thought, relating to actual infinity. For the Philosopher, it is contradictory to speak of objects-collection that are *infinite in act* (*infiniti in actu*), while it is possible to speak of objects that are *actually infinite* (*actu infiniti*), in both relative (*secundum quid*) and absolute (*simpliciter*) terms. Keeping in mind that there is only one object that can be actually infinite in an absolute sense —the Absolute or Self-Sustaining Being— a three-fold distinction of kinds of infinity ensues:

Three substantial types of infinity: 1. In potency; 2. Relatively actual; 3. Absolutely actual

- ◆ Objects that are *infinite in potency*
- ◆ Objects that are *actually infinite* in a *relative* sense (*actu infiniti secundum quid*)
- ◆ Objects that are *actually infinite* in an *absolute* sense (*actu infiniti simpliciter*)

These are all perfectly substantial, contrary to the contradictory notion of

Vs. Infinity in act

- ◆ Objects that are *infinite in act* (*infiniti in actu*)

Enlargement of Aristotle's notion of two types of infinity: in potency and in act

In this way, Aquinas was able to correct, by enlarging it, Aristotle's idea of recognizing, in science, only two types of infinity, the one *in potency* (which is substantial) and the one *in act* (which is insubstantial). More importantly, this distinction challenged the popular pseudo-scientific interpretation according to which Georg Cantor's discovery of set methods to rigorously compare the different types of infinity in mathematics stood in opposition to the Scholastic doctrine that considered contradictory the notion of infinity in actuality except when applied to the divinity (*sic*).

One should recognize that Cantor's contribution, due to both mathematical genius and sound faith, was to initiate, through his doctrine on infinity, a rigorous clarification of the logical and epistemological limits of modern logical and mathematical science, which we dealt with in the First Chapter of this work.

Thanks to Hallett's work, it has become historically certain that Cantor distinguished among three kinds of infinity, strongly approaching Aquinas' distinction recalled above:

Three types of infinity also for Cantor: 1. In potency; 2. Transfinite; 3. Absolute

- ◆ *Infinity in potency*, which is indeterminate and increasable
- ◆ *Transfinite infinity*, which is determinate and increasable
- ◆ *Absolute infinity*, which is determinate and non-increasable and that, for Cantor as for Aquinas, can only be God, the Absolute Being

Cantor's doctrine against two approaches to infinity:

Cantor used this distinction — which, during the author's lifetime, engendered opposition from most scientists, as well as by Catholic and Protestant theologians — to counter two kinds of modern illuministic instrumental uses of science and mathematics, two fundamental principles of which he refuted.

1. Spinoza's principle, which identified nature with a completely determined actual infinity

- ◆ The first is *Spinoza's notion of equivalence between God and Nature (Deus sive Natura)*, which became the basis for modern theoretical atheism. This equivalence was grounded on the presumed possibility of reducing both concepts to the same notion of actual infinity, completely determined as an immanent foundation of both nature and of the necessity and universality of the Galilean-Cartesian explanation of nature (the «new geometric science»).

2. Kant's principle, which understood actual infinity as the limit of potential infinity

- ◆ The second is the principle of the four anti-metaphysical, Kantian antinomies on the idea of world (finitude/infinity, discreteness/continuity, indeterminacy/determinacy, caused/uncaused) that were all grounded on the concept of absolute actual infinity as the limit to the finite (Lombardo–Radice 1981).

Cantor's distinction between *relative actual infinity* (or *transfinite infinity*), as a logical-mathematical notion, and *absolute actual infinity* as a metaphysical and theological notion — attribute typical of the divine nature, absolutely unknowable to mathematical science — went against both principles of the Enlightenment anti-theological and anti-metaphysical philosophy.

Unfortunately, Cantor thought his notion of infinity stood in opposition to the Thomistic one

However, Cantor mistakenly thought that his concept of actual infinity in mathematics should be framed in opposition to the Scholastic doctrine on actual infinity, particularly the Thomist doctrine. This was largely due to the author's scarce knowledge of Aquinas' philosophy, as well as to the insufficient theology of his —even Catholic— interlocutors.

Thomistic distinction between privative infinity (= in potency), for which it is contradictory to think of a termination (= infinite in act), and the notion of negative infinity, as the negation of an object's finite character (e.g., the set of natural numbers)

As a matter of fact, the notion of relative actual infinity, or *infinitum actu secundum quid*, was admitted by Aquinas as non-contradictory and therefore as a possible logical entity, even if the Philosopher did not consider it as an entirely constructive notion (that is, as an *infinitum in actu*), given that to conceive of the complete actualization of the potentially infinite is contradictory. In other words, it is contradictory to understand potential infinity or *privative* infinity — intended as an increasable finite sequence, hence repetitively «lacking» its termination (= infinity in *potency*) — as something that can be determined, or made completely *actual* (= infinity in *act*, *infinitus in actu*). For Aquinas, one can only speak of the actuality of relative infinity as of *negative infinity*, that is, as the non-contradictory negation of the finite nature of a given object, or a given totality, according to its specific modality of existence. As Aquinas' example suggests, to state that the totality of natural numbers is infinite is consistent, since there is no, and there can be no maximum positive whole number in any infinite sequence of numbers.¹³⁹ However, to think that it is possible to construct the infinite totality of natural numbers through a progressive procedure is contradictory.

Hierarchical nature of objects that are actually infinite in a relative sense

Given a similar object, the limit that can be determined and that allows it to be defined as a closed totality — that is, as an *actually infinite* existing being (= *infinitus actu* and not *in actu*) according to that specific modality — implies that that object is contained in another infinite object of a higher order. For example, in the case of numbers and thanks to Cantor, we know that the first transfinite ordinal *W* contains the set of natural numbers within itself. This follows a hierarchy whose limit, which does not belong to the sequence and cannot be derived from it, is «What is Actually Infinite» according to any modality of being.¹⁴⁰

¹³⁹ This is different from the pretence to «construct» such a totality as infinity in act. That is, the pretence to take a large, but still finite, sequence of natural numbers and, by increasing it, to define the whole of the infinite totality step by step. The maximum natural number does not exist.

¹⁴⁰ In the *Quodlibetales* (IX, 1) Aquinas explicitly stated that the notion of a *determinate* physical or mathematical entity (e.g., a body or a line) is consistent with the idea that it has *relative actual infinity* (*secundum quid*) (e.g., the infinity of its length). In this way, the Thomasian concept of *secundum quid* actual infinity — as opposed to that of absolute actual infinity, or «infinity according to all modes of an entity's being and essence» (which is proper only of the Existing Being) — coincides with Cantor's definition of the *transfinite* as *determined but increasable infinity*, which stands in opposition to *God's actual infinity* (which is absolute and non-increasable). This is confirmed by Aquinas' idea that the increasableness of relative actual infinity implies the possibility of distinguishing among different *orders of infinity*. This remains true even if the example used by Aquinas (the infinity of the set of even and odd numbers is «greater» than that of the set of only even numbers) is incorrect — as it often happens with mathematical examples used by the philosopher to justify some of his principles of philosophy of mathematics. Conversely,

It is contradictory to think of actual infinity as the endpoint of a constructive procedure, or as the progressive actualization of a privative infinity

In mathematics, Cantor himself had to realize that his notion of the transfinite was insufficient for a *constructive* and not axiomatic foundation of the existence of the *continuous*, intended as a notion-limit of an ordered sequence of trans-finites. The non-constructive nature of the idea of infinity was the focus of Aquinas' criticism according to which the notion of actual infinity intended as *infinity in act* (*infinitus in actu*, as the constructive or, in Aquinas' words, progressive actualization of a «privative infinity») rather than as *actually infinite* (*infinitus actu*, or «negative infinity»), be that absolute or relative, is contradictory.

It is not contradictory to conceive of actual infinity and of many kinds of actual infinity, some of which are relative and one of which is absolute. However, it is contradictory to conceive of a *constructive* notion of actual infinity, as P.J. Cohen recalled. In other words, it is contradictory

what is considered inconsistent by Aquinas, in relation to the concept of *actual infinity*, is the notion of «entity in act», that is, «brought to actuality» starting from a pre-existing potentiality. The concept of «entity in act» presumes that that entity's existence depends on an *ordering in relation to something other than itself* that, in this case, would be the already constituted infinite sequence. To presume that an entity in act—that is, “constructed” through an ordered sequence of steps (e.g., a recursive procedure)—is actually infinite would imply the necessity that the *constructive completion of an infinite sequence through an absolute ultimate term* is possible, which is clearly non-predicative and, in this case, contradictory. In Aquinas' words: «*necessary (for itself)* infinity conflicts with any entity in act (a progressively constructed entity, *N.o.A.*) because, in such entities, which *necessarily (for themselves)* have an order, it is necessary that the sequence is completed even in relation to its last term (*oportet compleri postremum*), which can only happen in relation to all preceding terms already taken altogether (*nisi per comparationem quodammodo omnium priorum*: non-predicative character!, *N.o.A.*)». As one can see, Aquinas did not exclude the possibility of thinking, hence the mathematical existence, of *relative actual infinities*, as well as the *possibility to arrange them* hierarchically. But he denied the *possibility to construct them*, the possibility of completing them so that they can be defined as “entities (set) in act”. In the words of modern criticism to Cantor's constructivism in set theory: «These issues are by no means trivial. In my opinion, the use of constructivist terminology in arguments relating to set theory is dangerous because it obscures the *realist* nature of the theory. Briefly, it obscures the difference between a unique description of an object *that is taken as already existing* and its constructive definition, that is, the specification of a process through which a *new* object must be constructed (as “entity in act”, *N.o.A.*). As noted by Gödel and Poincaré before him, one should not fear the use of non-predicative definitions (definitions that presume an already constituted set of infinite objects to which the object itself belongs, *N.o.A.*), if the aim of mathematics is intended as the description of, or the establishment of truth upon, already existing objects, «objects that exist independently of our constructions» (Gödel 1944, 136). For example, the proposition «the minimum upper limit of the limited set of points Λ » is innocuous as a *description* of a particular real number, granted that we assume that we are describing already existing objects (...). But if mathematics concerns the construction of *new* objects, the principle of the vicious circle must necessarily come into play. *One cannot* construct objects by using specifications that refer to the object itself or to *the sets to which it belongs*, or that use an unbounded quantification (Hallett 1984, 237).

to think of constructing actual infinity «piece by piece» by bringing to *actuality* an infinite in *potency* (Cf. footnote 24).

Actual infinity must be absolutely simple

The Absolute Being, as actual infinity, cannot contain any difference in itself. Rather, it is *absolutely simple*, contrary to actual infinity as understood in all immanentist metaphysics, starting from Hegel's constructive «wholeness of being» and Severino's non-constructive one. These did not realize that conceiving of the totality of being as actually infinite and at the same time completely diversified in itself — putting together Parmenides and Plato without Aristotle and Aquinas— is an intimately contradictory step (Basti & Perrone 1996).

Thomistic distinction between «infinite totalities made of parts» (e.g., classes) and «totalities without parts» that make up the former (e.g., essences)

In this regard, Aquinas used another distinction. In the construction of any kind (class) of objects, one must distinguish between the «totality made of parts» (*totalitas ex partibus*, e.g., the class to which all men and possible groups of men belong) and the formal principle that makes this collection a coherent totality, or «totality without parts» (*totalitas ante partes*, e.g., «mankind»). Similarly, the «totality made of parts» of the universal collection V , which is made of all objects that share the predicate «being-entity» (that is not the same as the predicate «exist» with no further modalities, as will be explained below) has its constitutive «totality without parts» in the Self-Sustaining Being.

Absolute simplicity of the Self-Sustaining Being is totality without parts

The absolute simplicity of the Self-Sustaining Being must mean not only, as for any form, that it is made of no parts (negation of composition of parts) but also, more radically, that it is not made of *essence* and *being* like all entities (Cf. Aquinas, *In de Div. Nom.* 13; *S. Th.*, I, 18, 4 ad 3; *S. c. Gent.*, II, 14, 921; *In Phys.*, III, xi, 385, etc.).

5.6 Conclusion: metaphysics vs. epistemology

5.6.1 Classical and modern transcendentalism

Meta-logical and metaphysical vs. epistemological foundation of science

This brief *excursus* through some of the fundamental notions of pre-modern classical thought, in relation to what we said in previous chapters, has highlighted a fundamental point for the object of this work — which, we should recall, is the relationship between philosophy of science and philosophy of nature, between the methodological reflections on the logical foundations of the *scientific enterprise* and the metaphysical reflection on the ontological foundation of the *objects of investigation* of the scientific enterprise itself, that is, the entities that make up the physical

universe. The core of the opposition between the notion of *science* and of the more general idea of knowledge in its classical and modern understanding is in the question of the foundation of *predication*. By predication one should understand the act with which a given predicate, at the end of a cognitive act, is attributed to a given subject in the construction of propositions within a given language.¹⁴¹ To use the terminology of contemporary formal logic, the core of the issue relates to the foundation of the *logic of predicates* of the language produced by cognitive activity. Or, in philosophical words, the core of the opposition between classical and modern thought is in the question of *transcendentals*. More precisely, it is in the juxtaposition between

- ◆ *The metaphysical foundation* of scientific or, more generally, of rational language, in the *existence and essence* of the referent of the single proposition (classical approach); and
- ◆ *The epistemological, or «critical» foundation* of scientific or, more generally, rational language, in the *evidence and self-awareness* of the knowing subject (modern approach).



Definition 7

In the philosophy of knowledge, the term «transcendental» indicates the ultimate foundation of the *concepts* of reason, hence of the logical *predicates* in the construction of *simple or atomic propositions* (categorical propositions composed of a subject and a predicate united by the copulative «is») through which the concepts of reason are primarily expressed in a given language.

The notion of «transcendental» as opposed to that of «categorical»

Any concept (predicate) is grounded on other concepts (predicates) that are more fundamental, since they are more «unifying», hence more «generic» than the preceding ones (e.g., «cat» is less general than «animal», which is less general than «living», etc.), leading up to *categories*, or «predicaments» (e.g., «substance», «quality», «quantity» etc.), which are the

¹⁴¹ In the words of the Thomistic doctrine of knowledge, it is the cognitive act of the «second operation» of the intellect, or the act of *formation of judgement* that, linguistically, Aquinas called act of *compositio-divisio*. It is the act, in other words, with which one *affirms* (e.g., when I say «Andy is a man») or *negates* (e.g., when I say «Andy is not a man») the attribution of a given predicate to a given subject.

most universal concepts, in the extensive sense, and therefore the most general predicates. The problem is to find the basis of these general conceptual units and of their attribution (*affirmation* or *composition* through the copulative «is») or non-attribution (*negation* or *division* through the negation of the copulative = «is not») to a given subject. In order to avoid an infinite regression, such a foundation cannot be a universal concept, or genus.

For Aquinas, as well as for any realist philosopher or scientist, the transcendental foundation of categorial «unifications» is the being of the *entity* that represents the referent of the proposition, be that a «substance» (e.g., a tree or a dog) or an «accident» or property of a substance (e.g., a «quality» or a «quantity»).

Classical and
modern
transcendentalism

For modern thinkers, after Descartes and Kant, the transcendental foundation of the logical units in a language (= propositions) is the unifying function of the knowing subject's self-awareness, given that the immediate self-evidence of postulates is the foundation of the evidence as mediated by demonstrations. For this reason, following a use that is typical of some neo-Scholastic philosophy of this century, we have spoken of «objective» (classical, Thomist) transcendentals as opposed to «subjective» transcendentals (modern, Kantian).

In sum:



Definition 7b

In logic, «transcendentals» indicate the most fundamental primitive terms that constitute the basis of any language. Particularly, in formalized languages, the axiomatic propositions and the starting definitions of any demonstrative procedure within a given theory are constructed thanks to these primitive terms. In Aquinas' philosophy, the most fundamental (= «transcendental») primitive terms within a given language are those that state the particular meaning with which the term «entity» and its equivalents should be understood.

The difference between the classical and modern approach to the scientificity of a theory and/or of formalized language can also be expressed differently. While in classical thought the foundation of scientific, or

From the classical
notion of
metaphysics to the
modern notion of
«world visions»

more generally rational, discourse is given by metaphysics, in modern thought it is given by epistemology.

One could state that in modernity, following the «Copernican revolution» and the birth of modern science, metaphysics has been replaced by *world visions*, whose foundation is not in the *world of things and given by the capacity of the mind and language to adapt to it*. Rather, it is *in the ways in which self-aware thought represents the world of things by adjusting it to its pre-understandings*.

These «world visions» will be *rationalist*, *empiricist* or, synthetically, *transcendental* according to their arguments on the ultimate *truth* of concepts and of the logical-mathematical laws of thought. Truth is not grounded on the *being* of things and on the set of their characteristic properties (*essence*) but, respectively:

- ◆ *In rationalism* (Cf. Descartes, Spinoza, Leibniz, etc.) on *reason* and its character of *self-awareness*, intended as the foundation of *conceptual evidence*,
- ◆ *In empiricism* (Cf. Locke, Berkeley, Hume, etc.) on *experience* and its character of *self-awareness*, intended as the foundation of *experimental evidence*,
- ◆ *In transcendentalism* (Cf. Kant and neo-Kantian philosophy) on the *transcendental synthesis* of reason and experience, given that *self-awareness* itself, the «transcendental I-think», is the foundation of both.

In contemporary neo-positivism and logical empiricism the tautological character of pure logical-mathematical forms — hence the underlying self-identity — becomes the linguistic equivalent of self-awareness.

What is the
fundamental
difference between
classical and
modern
philosophical
thought?

Coming back to Maritain's excerpt that we used as a preface to the discussion on the differences and common points between Scholastic and modern thought on science (Chapter Four, Cf. § 4.2.1), the limit of the interpretation of these relations becomes understandable. For a start, there is an uneasy apologetic tone in the following, central claim:

(...) Critical theory of science, intellectualistic or realistic, whose principles were laid down by ancient and medieval metaphysical thinkers, is the only one offering the means to shed light on epistemological problems, which have nowadays become a true chaos.

Clearly, such an affirmation smacks of that «age of world visions» that we are fortunately leaving behind, the detachment from which is still difficult, even over sixty years after the first edition of this work and

over thirty after its sixth edition, which has been quoted here in the Italian translation.

Truth as *adequatio*
and truth as
evidence is the key
difference

Far from being fortuitous, the limit underlined here has deep theoretical roots. The French philosopher seems to overlook the key of the question: the irreducible character of the classical paradigm of scientificity, founded on being and truth as *adequatio*, and of the modern paradigm, founded on self-awareness and truth as *evidence*. Stated otherwise, on the one hand Maritain is correct in stating that the paradigm of scientificity changed between the classical-medieval and the modern era. He is also correct in stating that, for both paradigms, mathematical sciences were a model of scientificity, although it is true that the demonstrative character is what primarily characterizes the scientific enterprise. On the other hand, Maritain is wrong in centring the whole question of scientificity on the principle of evidence. This means that Maritain was influenced by a Cartesian prejudice in his reading of classical thought.

The question of
self-evidence in
mathematical
postulates

While it is true that mathematics was the model of scientificity for both classical and modern thinkers, it was only for philosophers *à la* Descartes or Newton that this was the case, who considered mathematical axioms, particular in geometry, hence physical-mathematical axioms, as endowed with absolute and universal self-evidence. This was no longer the case for modern thinkers after Riemann and the axiomatization of geometries, which demonstrated that there is not only *one* geometry, the Euclidean one of flat space; rather there are an infinity of possible geometries. To give an example taken from physical-mathematical sciences, what is the self-evidence of an axiom defining a 22-dimensional space, as in the so-called cosmological theory of «super-symmetry»?

If, for contemporary epistemology, it is meaningless to speak of self-evidence of mathematical (both theoretical and applied) postulates, the same was true for Aquinas. The latter, while acknowledging the fundamental character of mathematics as demonstrative sciences, firmly denied that this depended on the self-evident character of their postulates. In fact, he used mathematical postulates as examples of propositions that, while being «known in themselves» — propositions whose link between subject and predicate is immediately grounded on necessary implication, rather than being mediated by deductive implication or demonstration — are not immediately knowable *by all*, hence cannot be self-evident (Cf., for example, Aquinas, *In Post. An.* I, v, 50).

Principle of
evidence and
opposed world
visions

Exegetical questions on Aquinas' texts and questions of scientific sensibility taken aside, what is the theoretical problem with assuming that the basis of the scientific enterprise was the principle of evidence, both in classical and modern thought? If this were true, the difference between

the classical and the modern scientific paradigm would all be in the juxtaposition between one type of *immediate evidence*, the metaphysical one — in Maritain's words, the Platonic «intellectualistic-critical» or the «realistic-critical» one of contemporary neo-Scholastic Thomism— and another type of *immediate evidence*, the mathematical or experimental one of modern critical transcendentalism. The whole question would then be reduced to the opposition between two «world visions». However, as long as one takes *evidence* as the foundation of scientificity, one places one kind of *epistemology*, the Cartesian-Kantian one of the so-called critical or transcendental philosophy — which is far from being accepted by all modern epistemologists— before *metaphysics*. This leads us very far from classical and medieval thought, as well as from Aquinas' philosophy.

Evidence and the problem of truth

Evidence is a *state of consciousness*, which goes along with a knowledge to which the subject passively feels he has to *agree with*, for example, because he believes it is true. However, it is not necessarily true. For a schizophrenic it is evident that he is Napoleon, even if he is not. Or, as Aquinas noted in one of his texts criticizing the argument that places thought before being (Cf. Aquinas, *S.Th.*, I, 86, 5, 2c and Basti, 1991, 132ff., for a critical discussion), for someone whose taste is ill, it is evident that honey is bitter, but this does not necessarily mean that honey is bitter. The Modern age is one of uselessly opposed and irreducible world visions, because it is the age of epistemology based on the evidence of things rather than on their being (entity).

Truth as adequation vs. truth as evidence

Indeed, Aquinas argued that the knowledge of truth occurs when we formulate a judgement (= second operation of the intellect), that is, we express our apprehension of the idea or essence of an entity — be that physical, logical-mathematical or imagined— that we want to know (= first operation of the intellect)¹⁴² in the form of a predicative statement. In this way, we subject the judgement we have formulated to a test of consistency, trying to interpret through this the information that is available on the judgement. This information can be an actual sensation, as well as previously acquired empirical or rational knowledge (Cf. Aquinas, I, 16, 2c; *De Ver.*, I, 3; 9; *S. c. Gent.* I, 59; *In Periherm.*, I, iii)¹⁴³.

¹⁴² For the distinction between first and second operation of the intellect in Thomist psychology, Cf. Basti (1995, p. 240ff., esp. p. 252).

¹⁴³ Formally, it is a meta-logical test of consistency of logical propositions, carried out psychologically at the level of the so-called «reason», that is, at the level of the «secondary reflection» of the intellect or self-aware thought (for the distinction between primary and secondary reflection of the intellect and between these and the two operations of the intellect in Thomist psychology, Cf. Basti (1995, p. 240ff.)). It relates to verifying the non-contradictory character of a new proposition against other acquired propositions, be

Therefore, Aquinas denied that the knowledge of truth occurs when we find in our minds the idea (*apprehension of the essence*) — when the perception of evidence usually occurs — that will subsequently be formulated in the judgement.

Epistemological
irrelevance of states
of consciousness

For Aquinas, the conscious state that goes with finding a new idea, whatever its nature, has no epistemological value, nor does it have any meta-logical or metaphysical relevance, unless we opt for *psychologism* in logic, or choose to carry out an investigation on the «inter-subjective» transcendental foundations of consciousness. How many times, in the history of both individuals and of science, has something that was anything but evident to a person or a group showed itself as «true», that is, *adequate to reality* — whatever this expression might mean, at least for the moment?

One thing is evident
because it is true,
not the reverse

Ultimately, one thing *is not true because it is evident*, rather — even if not always — *it is evident because it is true*. What grounds truth in a proposition is not the *conscious certainty* that goes with the cognitive act of the subject that expresses it. Rather, it is the proposition's adequation to reality, whether this «reality» belongs to the physical, logical-mathematical or other realm. «Knowledge is an effect of truth», not the opposite, as Aquinas stated in terms that would be considered outrageous by a modern mentality.



Any knowledge is completed (*perficitur*) by the assimilation of the knower to the known, so that this assimilation is the foundation of knowledge (*causa cognitionis*) (...). As stated, knowledge follows this adequation of the known and the intellect. Hence one thing's entity (*entitas rei*) grounds the content of truth (*praecedat ratio veritatis*), but *knowledge is like an effect (effectus quidam) of truth* (Aquinas, *De Ver.*, I,1c).

The immediate
character of axioms
does not always
imply self-evidence

One clarification becomes necessary at this point. A neo-Scholastic thinker would argue that our opposition to the principle of evidence is not originally from Aquinas, at least as long as metaphysical propositions are concerned. For example, in Aquinas' excerpt cited above, while denying the self-evident nature of mathematical axioms, the Philosopher seemed to state the contrary for metaphysical axioms. Concerning propositions that are taken as *premises* or *principles* of a demonstration, Aquinas stated that while these are all *known in themselves* — that is, they are immediately known, and not apprehended as conclusions of a demonstration — one should distinguish between:

they quantified universally (e.g., «all men are mortal»), particularly (e.g., «Italians are mortal») or singularly (e.g., «Socrates is mortal»).

Negation of the self-evident character of mathematical postulates

- ◆ Those that are *positions* (*positiones*) that is, in modern words, the *hypothetical postulates of a deductive system* or of a system in formulation that, according to Aquinas, are «postulates» because they are «set by» the one who develops the theory, and
- ◆ Those that are *maximal propositions*, or *axioms* (*dignitates*, as Aquinas called them, which is the Latin translation of «axiom»: *akioj* (*axios*) in Greek is *dignus* in Latin). In modern words, these are the meta-linguistic axioms in logic and ordinary language dealing with the formal language that is being formulated. Even those who have not formulated the theory, but are learning it, must share these maximal propositions, as Aquinas noted. Otherwise, they will not be able to understand the necessitating character of the propositions of the formal language that is being dealt with.

Evident character of an entity's primitive and immediate derivation of the p.n.c.

In order to understand this distinction —continued Aquinas— one should know that any proposition whose predicate is necessarily implied in the definition of the subject (*cuius predicatum est in ratione subiecti*: these are not only tautological propositions, as argued by modern thinkers starting from Leibniz, *N.o.A.*) are *known immediately and in themselves*, in relation to themselves.

Now, the *terms* of some propositions are such, that they are known by all, as an *entity*, as *one* together with all the things that relate to the entity as such (the «transcendentals», *N.o.A.*): indeed, the entity is the first notion understood by the intellect. It is then necessary that these propositions are known not just in themselves, but also, *in some way* (*quasi*: note Aquinas' wise caution, *N.o.A.*) *by everybody* (*quoad omnes*). The following proposition and those similar to it are propositions of this kind: *it is impossible that one same thing be and not be at the same time, or the whole is bigger than any of its parts*. These and other similar principles are drawn by all sciences from metaphysics, whose aim is to consider being in the absolute, together with all that is related to it (*In Post. An.*, I, v, 50).

Distinction between meta-logical and metaphysical formulations of the «primary principles»

One should note the close link between *meta-logic* and *metaphysics*. Taking a superficial look, the axioms, for example the p.n.c., are the same. But, in meta-logic, propositions rather than entities are objects of the p.n.c., that is, a particular class of entities, which are *linguistic entities* and, more generally, *logical entities*. In meta-logic (logic of propositions), therefore, the p.n.c. will be formulated as proposed in Bochenski's manual:

It is true that *p* excludes *non-p*. In symbols: $p \mid \sim p$

that is, in equivalent form,

It is not true that *p* and *non-p*. In symbols: $\sim(p \cdot \sim p)$

Where p is a propositional variable (Bochenski 1995, 68).

Other formulations of the p.n.c. in the form of meta-logical laws (Cf. Bochenski 1995, 129) are contained in the *logic of predicates*:

$$\forall x, P \sim (Px \cdot \sim Px)$$

where P is a predicate, and in the *logic of classes*:

$$\forall x, A \sim (x \in A \cdot x \notin A)$$

Where A is a symbol of class. These formulations are logically very strong, because they assume a second order logic, which is quantified not only on the variables of propositional functions, but also on the constants (on the symbols of predicate P and of class A).

The *logic of identities* is one last possible logical formulation of the p.n.c.:

$$\forall x \sim (x \neq x)$$

Aquinas formulated the p.n.c. as a *metaphysical* (or *ontological*) law —thus relating not only to logical-linguistic entities (= meta-logical law), but to any entity as such (= metaphysical law) — very differently:

It is impossible that the same thing is and that it is not

Particular character
of the p.n.c.

In light of the present developments in formal logic, only the p.n.c. among the «maximal propositions» — the meta-logical and metaphysical laws recalled by Aquinas — has remained known in itself as well as *universally essential* for any meta-language, thus maintaining the quality of self-evidence that Aquinas attributed to it. None of the other «maximal propositions» usually recalled by Aquinas and Medieval thinkers can be considered self-evident («principle of the excluded third», «identity principle», «causality principle» and the «totality principle», which was explicitly used by Aquinas in the text cited above). Their use in specific meta-linguistic theories must then be discussed and justified.

Non-self-evident
character of other
meta-linguistic
axioms

For example, there are limits to the universality of the principle of the excluded third in the logic of quantum mechanics and in mathematical logic of an intuitionist kind. Similarly, the totality principle cannot be formulated as Aquinas and the medievals did. It must be used in a weaker and almost tautological form, «*the whole is never identical to one of its parts*», after Cantor's demonstration that, in infinite sets, the whole is not greater than its parts. For instance, the infinite natural numbers that are the square of other numbers are undoubtedly a part of the set of natural numbers. However, they are neither greater nor smaller than the infinite set of natural numbers, given that they are as many as the latter. In other words, these are two «equally powerful» infinite sets, which have the

same «cardinality», that is, the same cardinal number can apply to these two multiplicities. The same can be said for the infinite set of points that make up a straight line segment and those that make up the straight line itself. A totality can never be identical to any of its parts, in the sense that, by definition, a part *is not* the totality to which it belongs (square numbers are not the whole of natural numbers, and a straight line segment is not the whole of the straight line). However, from the point of view of the cardinal number («cardinality» or «potentiality» of a set) of the elements that make up a part and the totality to which it belongs, it is not true that the whole is always greater than the part. Cantor taught us that this is never the case with infinite sets.

Paradoxes of the totality axiom

An infinite totality can be in bi-univocal correspondence with («is equally powerful to») one of its parts. In fact, this is the characteristic of all infinite sets. The points of a line segment can be in bi-univocal correspondence with the totality of the infinite straight line to which the segment belongs; the set of natural numbers — zero and all whole numbers greater than zero — can be in bi-univocal correspondence with the set of rational numbers — finite decimals between zero and one. Even the set including all sets that are *numerable*, which are equally powerful to the set of natural numbers (the set of natural, rational, relative numbers as well as of all other infinite sets that are equally powerful to that of natural numbers) is numerable. On the other hand — and this is obviously evident to all, as it was for ancient thinkers — numbers that are the square of other numbers are neither more nor less than natural numbers: the two infinite totalities are ‘equally powerful’ sets. However, square numbers are a subset of natural numbers: not all natural numbers are the square of other natural numbers. Conversely — and here evidence is insufficient — the set of real numbers (natural, rational, irrational and transcendental numbers) cannot be in bi-univocal correspondence with the set of natural numbers. The former is «more potent»; it has a greater cardinality than the latter.

For all neo-Scholastic enthusiasts of metaphysical evidence, two things should be kept in mind:

For Aquinas, primitives, rather than axioms, are self-evident, starting from the primitive notion *par excellence*: being

- ◆ According to Aquinas, things that can be considered as evident or self-evident in the modern sense are not constructed propositions (be they axioms or principles) but some *primitives* with which the propositions of any language, including the formal language of any scientific theory, are expressed meta-linguistically (for the distinction between «primitive» and «axiom», Cf. *formal language** in the glossary). The most fundamental self-evident primitive is the most universal metaphysical and meta-linguistic semantic term: *being*. It is

true that, today, the term «being» is rarely stated as a primitive in the formalization of any scientific theory. However, this is not because it can be disputed that being is a primitive, given that any object dealt with in any theory, any part of the theory as well as the theory itself are, clearly, «beings». Put simply, beings are not explicitly stated as primitives because this is usually not necessary for the formal correctness of the language, and therefore for the comprehension of the theory. Or, as Aquinas suggested, because some notions are so well-known that they are presumed anywhere (Cf. *In Post. An.*, I, xviii, 158).

- ◆ The principles or axioms that can be reasonably assumed in any scientific theory are meta-logical and/or metaphysical *axioms*. For example, this is the case with the p.n.c.. Even in formal languages that presume exceptions to the p.n.c., the meta-language with which the consistency and/or the truth of such a language-object are expressed or demonstrated must necessarily assume it.

Multiple possible interpretations of the notion of being and non-self-evident character of metaphysics

To state that the notion of «being» is self-evident is therefore acceptable. In this context, the Thomistic argument considering the *ens* as the *primum cognitum* in any epistemological or gnoseological theory is absolutely true (for example, Cf. Aquinas *In de Ver.*, I, 1c). This, however, does not mean —nor Aquinas argued— that the meaning meta-linguistically attributed to the primitive «being», as well as the metaphysical theory underpinning it, are equally true or self-evident. Grounding the truth of any metaphysical theory, particularly the Thomistic one, solely on the self-evidence of the meta-logical notion of being and of the p.n.c. axiom, which is linked to it, would not bear fruitful results. The possible metaphysical theories depending on the different interpretations one might give to the notion of being are theories whose coherence, consistency and truth must be justified and demonstrated, as with any other scientific theory. None of these can exhaust *all truth* or avoid being *perfectible* (Cf. above § 3.1.1).

Limit of the neo-Scholastic interpretation of the relationship between classical and modern visions of science

The key of the relationship between the modern notion of science — linked to the paradigm of mathematical and natural sciences— and the classical-medieval one, particularly Thomistic —linked to the metaphysical paradigm— is to reject Maritain's premise. Science, particularly metaphysics, is not a form of superior knowledge (one should not take into account Maritain's notion of «perfect» knowledge: since Gödel's incompleteness theorems, perfection has become again a quality that is not related to human knowledge, even and above all of a scientific kind). This is because its development is subject to the «constraint of evidence», going from the «immediate» evidence of principles, to the «me-

diated» evidence of conclusions, obtained through a demonstrative procedure.

Birth of the Cartesian-Kantian paradigm of science, grounded on evidence

To the contrary, the difference between the classical-medieval and the modern paradigms of science is linked to the fact that, originally, the modern paradigm was erroneously founded on a particular epistemology of the self-evidence of axioms in geometry first, and in Newtonian mechanics later, rather than on an appropriate meta-logic of scientific propositions. This epistemology, from Descartes on, was extended to philosophy as a whole and to metaphysics, through the principle of «clear and distinct ideas», leading to Kant's notion of the impossibility of metaphysics as a science. This notion, through the so-called modern transcendental method, considered *self-awareness* as the epistemological basis of evidence, the ante-predicative foundation of the formal consistency or coherence (not truth) of scientific propositions in mathematical and natural sciences. It did so by stating that *categories* — the ultimate and most universal predicates — in any type of rational knowledge and of formal language are nothing but *universal modalities of manifestation* of self-awareness and of its *formal function of unification* and *organization* of empirical contents.

In such a context, the very notion of *being* was reduced to the Kantian category of *existence*, as in the proposition « x exists». Being, from being considered the ultimate and transcendental semantic content — that is, always ante-predicative, in any knowledge and in any form of language, scientific or not — was reduced to a universal formal concept (the most general genus), devoid of any semantic content and therefore of any reference to the object of knowledge and/or to the referent in a predicative proposition.

Crisis of the Cartesian-Kantian paradigm of science

Since then, modern epistemological thought has greatly evolved towards an unaware retrieval of the classical paradigm of science. Starting from Kant himself and his *Critique of Judgement*, the *voluntary* character of the principle of evidence, intended as the transcendental foundation of rational and/or scientific thought, has increasingly surfaced, reaching its extreme forms with Schopenhauer's Voluntarism and Nietzsche's Nihilism. At the same time:

- ◆ The discovery of non-Euclidean geometries at the beginning of last century;
- ◆ The limits of the Newtonian paradigm of physical-mathematical science;
- ◆ The ensuing axiomatization of logical, mathematical and natural sciences, which considers axioms as hypothetical (true under given

conditions) rather than self-evident and apodictic (true always and under any condition)

have revealed, in the very sanctuary of «Cartesian» and «Kantian» science, the insubstantiality of the notion of self-evidence for axioms that are not of the kind of the p.n.c.

But, what «kind» of axioms are those like the p.n.c.? In order to answer this question, one should recall another fundamental event: the birth of mathematical logic and its consequent distinction between «language» and «meta-language», that is, between the *language-object* of logical analysis and the *meta-language* of the logical theory used for the analysis. One should then study how this distinction applies to metaphysical, particularly Thomistic, theory (Cf. *infra* § 6.1). Before turning to this, we will briefly summarize Aquinas' notion of the «transcendentals of being» as the foundation of the universality and truth of knowledge, as opposed to the modern notion of «transcendental of thought» (self-awareness).

5.6.2 The Thomist table of transcendentals

Thomistic notion of transcendentals: being and its modalities

The problem of transcendentals relates to the foundation of truth in predications and categories; regarding the human mind, this problem relates to the foundation of truth in concepts. The difference between classical and modern thought, particularly with Kant, on this particular point is fundamental. In classical thought, an *entity's being* and its transcendental determinations, or *an entity's (specific and general) modalities of being* are what are considered as 'transcendental'.

The Kantian notion of transcendental: thinking and its modalities

In modern thought, the self-aware thinking and the different *modalities of thinking*, which Kant called the «I-just-think» (*Ich denke überhaupt*) are the 'transcendental'. For Kant, the unification of experiences in one concept depends on the unifying act of the self-aware thought, which is understood as pure logical-formal structure of reasoning, devoid of any content. This is the «I-think», not the «I-think-something» of the subsequent intensional analysis of the act of consciousness, given by the phenomenological school. For Kant, therefore, the universality of knowledge depends on the fact that there exist some *universal ways of thinking*. This means that there are some universal ways in which one becomes self-aware of sensations, by unifying them in phenomena first, and subsequently in concepts and categories. This was demonstrated by the existence, in scientific thought, of *self-evident* universal laws (= that necessarily impose themselves on awareness) such as the postulates of Euclidean geometry and Newton's three laws of dynamics, from which modern

science originally derived all its predictive and descriptive universalizing power.

Crisis of the principle of evidence: it led to discover that will, rather than thought, is the real modern transcendental

As long as the logical-mathematical way of thinking of modern science remained *exclusive* (exclusiveness of Euclidean geometry and of Newtonian physics), Kant's «Copernican revolution» —considering a subject's way of thinking, rather than an object's way of being, as the basis of categories— seemed able to guarantee logical universality and necessity. However, thanks to the axiomatization of logic and mathematics —the discovery that it is possible to build consistent logical-formal systems by changing axioms— which multiplied the modern «scientific» ways of thinking, it became clear that Schopenhauer's and Nietzsche's ways to deal with the transcendental were correct. If one discards the classical approach, and the object (the entity under study within a given axiomatic system) is not taken as the basis of truth, while at the same time re-discovering, in logic, the *analytical method* as a set of rules for the definition and construction of axioms, then the modern transcendental is not the *I-just-think*, but the *I-just-want*. The choice of axioms becomes *arbitrary*, thus bringing Western thought back to the Sophists and to the times before Plato and Aristotle, as Carnap and Neurath's neo-Positivist manifesto pointed out.

If we do not take the being of an essence as the foundation of an axiom's truth, the choice of the axiom becomes arbitrary

Once established that the basis of scientific thought —the axiom— is not a truth that imposes itself on an individual's self-awareness, and that it depends on a subject's arbitrary choice, a fundamental conclusion appeared clear. The real modern transcendental is not the way of thinking, but of wanting; it is not the «I-just-think», but the «I-just-want». If it were true that the different, consistent ways of thinking depend on the unification operated *a priori* by awareness through the definition of axioms, rather than on *transcendental unity* (the irreducible individuality of an entity), then the different ways of thinking would ultimately depend on the will that arbitrarily determines the axioms and therefore the different «starting points» of the different scientific and philosophical formulations.

The consequence is «weak thought», enshrined in present Voluntarism and Nihilism

For this reason, modern transcendentalism directly leads to today's Voluntarism and Nihilism, the so-called «weak thought». It then becomes of utmost importance to verify whether the historical and theoretical reasons that led Western culture to relinquish classical transcendentalism are still valid.

Fundamental illustration of the Thomistic transcendentals in the *De Veritate*

Following Aquinas' explanation of the transcendentals, as contained in the *Quaestiones Disputatae De Veritate* (I,1), the key is establishing whether the notion of «being» identifies itself with that of «true». Aquinas' answer to this was *negative*. If being identified itself with being-true, given that

being-true depends on the intellect and the latter's action depends on an act of will (*voluntas vult intellectum intelligere*), then we would have the same problem encountered by modern thought. We would ground truth and being itself on the constitutive power of the «transcendental-I», of an act of consciousness that — as Heidegger already pointed out — is reduced to the nihilist voluntarism of a sterile «wanting to want oneself». *Voluntas vult se velle*, «will wants to want itself», as Aquinas stressed in his metaphysical psychology of the act of will.

Being is the first notion of the intellect to which predicates cannot add anything. Being is not a logical genus

Being is the starting point of any knowledge, hence the transcendental foundation of any concept — including categories — as well as the intensional content of any conceptual and voluntary act. This is a content that is *always already formed* in relation to the willing and intelligent subject, not one that is constructed by an act of the latter. As Aquinas stated, any knowledge, definition or concept of the intellect can be reduced to the elementary verbal form «it is... » through the simple addition of a *predicate* (e.g., tree, man, house...). Yet, these predicates are not «added» to being as «specific differences» (e.g., «rational»), but are added to a «genus» (e.g., to «animal» in the definition of man as «rational animal»). Nothing can be added or removed from being, because it contains everything. Stated otherwise, *being is not a genus*; it is not the most generic concept that contains all other concepts as its specifications.¹⁴⁴

The different specifications of being add nothing to it, rather they articulate different modalities of being

Conversely, the specifications added to being *simply articulate different «modalities of being»*, just as the «form» articulates different modalities of the *being-one* of the material parts of a physical entity's essence. These modalities of being make up the so-called *Thomistic table of the transcendentals of being*. These are unique, self-evident notions that express different *equivalent*, but not *identical* meanings of the word «being», intended as *primitive* content, which is ante-predicative in relation to any other predicative or categorial distinction with which one can differentiate entities. In transcendentals, Aquinas identified an equivalent number of *primitive terms* that are common to any language, starting from those formalized in the various sciences.

Transcendentals as the primitive terms of any language

The following is the beginning of the *De Veritate* (1,1), where Aquinas listed the transcendentals and where it appears clear that these are, for the Philosopher, the most fundamental «primitive terms» in any lan-

¹⁴⁴ The identification of being with one genus has been at the roots of all forms of *rationalism*, both materialist and idealist, throughout the *whole* of Western thought. The demonstration of the logical insubstantiality of such an idea, which characterizes contemporary thought, is at the origins of *Nihilism* and of the belief in the impossibility of a given *metaphysics*. Nihilism is not the death of metaphysics, but the *death of a specific way of reasoning in metaphysics*, the *rationalist* one.

guage. This paragraph comments on the remaining parts of this article in the *De Veritate*. As Aquinas stated:



In demonstrable propositions, one must operate a reduction to some principle that is known in itself to the intellect. One must do the same when searching for the «what is» (*quidditas*, the definition) of a given thing, otherwise, in both cases, we would go back *ad infinitum*, and the science and knowledge of things would be lost (that is, the semantic function of languages, starting from formalized ones, would be lost, *N.o.A.*). Now, what the intellect apprehends as the best-known thing of all, as the thing in which it subsumes all concepts, is being (the most primitive notion in any language, *N.o.A.*) (...) so, it is necessary that all other concepts of the intellect be obtained by means of additions to being. But nothing can be added to being as something external, the same way that the species is added to the genus or the accident to the substance, because any nature is 'being' predicated as an essence (*essentialiter*: as the predicate of an individual subject, *N.o.A.*), as the Philosopher demonstrates in Book III of the *Metaphysics*. We in fact say that some things add something to being *in the sense that they express a modality of being itself that is not rendered with its name*. And this happens in two ways. With the first, *the expressed modality is a particular modality of being*. There are indeed different degrees of entity (*entitas*)¹⁴⁵ according to which the different modalities of being (existing, *N.o.A.*) are drawn, *and according to these, the different types of genus of things are drawn*. For example, 'substance' does not add a difference denoting some nature added to being. The name of substance simply expresses a particular modality of being (existing, *N.o.A.*), that is, the being-for-itself, and the same is for all other types of genus. With the second, the expressed modality is a *general modality of being*...

We will treat in the next chapter the fundamental Thomistic distinction between being as «entity» (*entitas*) and being as «existence» (*existentia*). Here we will deal with the distinction between the «particular» and «general» ways of stating being.

Equivalence but not identity among transcendentals

One should recall that these notions are all *equivalent**, in the sense that the class of objects to which one of these notions is applied is the same to which any of the remaining notions apply. This, however, does not mean that these notions are *identical**. They have different meanings, each of them giving different connotations to the unique term «being». For instance, when I say that each entity is «one», I am saying that it is «undivided in itself», a notion that cannot be rendered with the simple formula «being entity». Similarly, when I say that each entity is «a thing» I

¹⁴⁵ This intensive graduality of the content of an entity's being relative to its essence depends on the richness of property of the being of an entity's essence compared to another's. For example, the being of a man's essence (his human entity) is much richer than that of any animal (its animal entity). As well, the way of expressing this *intensive* graduality has remained in common language, for example, in phrases such as 'the being of the damage was greater (than the being) of available resources', and 'the being of the prize', of a sum of money, etc.

am saying that it «has a given essence», a notion that is not rendered in the simple formula «being entity», etc.

The following is a schematic illustration of the table of transcendentals, based on the section of *De Veritate*, partly quoted above.

Transcendental notions are equivalent but not identical to the notion of being

There are some terms that are equivalent to the notion of «being», that are as self-evident as this notion, but that express some «modalities of stating an entity's being» that the notion of being cannot express by itself. There are two fundamental modalities:

- ◆ *A particular modality of being*, that is, according to *each thing's* specific *being* or intensive degree of being (e.g., being-man, being-horse, being-living, being-number, etc.; cf. fn 40). This is the most important and fundamental distinction. Synthetically, what Aquinas argues is that any language must explicitly state what kind of entities it generally deals with. For example, biology deals with «living-entities», arithmetic with «numerical-entities», logic with «logical-entities», physics with «physical entities», etc.
- ◆ *General modalities of being*, common to all entities. Any language, whatever the type of entity it deals with, will deal with entities that are «things», that are all «units in themselves», that are to some degree «true», that are all to some degree «endowed with value», etc. More precisely:

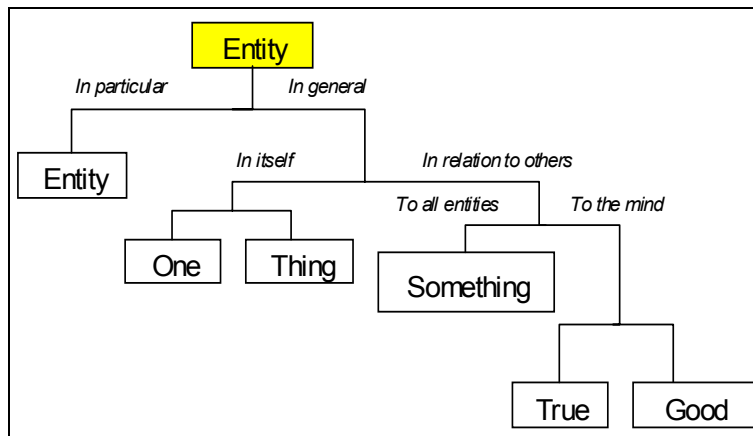


Table IV Summary diagram of the transcendentals of being according to Thomas Aquinas.

- *In relation to itself* (= in itself), an entity is:

- ◆ In affirmative terms, a generic **thing**, that is, generically endowed with some essence or nature;
 - ◆ In negative terms, undivided, **one**, a *transcendental unit* or *individuality* (the «transcendental one» as the foundation of the «formal unity» and the «quantitative unity» of an entity)
- *In relation to something other than itself* (= to other entities), each entity:
- ◆ *In relation to any other entity*, it is **something**, that is, a qualitatively distinct, «qualified thing». ¹⁴⁶
 - ◆ In relation to an entity that can enter into relation with any other entity—that is, compared to the (human or other) mind—each entity:
 - Is (more or less) **true**, as the object of different degrees and modalities of knowledge on the side of the intellect
 - Is (more or less) **good**, that is, endowed with a given *value*, given that it presents itself in different ways in relation to the ‘will’

An entity’s being, therefore, is the basis of both scientific truth and ethical value, but it identifies itself neither with «truth» (=rationalism), nor with «value» (= voluntarism, nihilism) ¹⁴⁷.

¹⁴⁶ «In such a way, as any entity is defined as ‘one’ because it is undivided in itself, so it is defined as ‘something’ because it is distinct from something other than itself» (*In De Ver.* 1,1, resp.). This comment by Aquinas is fundamental. Indeed, the limit of rationalism consists in identifying, in a Platonic sense, the foundation of the unity or individuality of a really existing entity (*this* tree, *this* man, etc.) with its *formal unity*. This, however, entails the simultaneous presence of *all* other entities compared to which the entity in question is a different *iniqu* thing. In so doing, one entity’s individuality would never be certain, unless the human mind is capable of knowing the absolute totality. Aquinas’ fundamental discovery was to link an entity’s *individuality* to its *self*, rather than to its *being in relation to something else*. This is *fundamental in anthropology*: personal individuality is not linked to the *relation with others*, otherwise neither an embryo, nor a person in a coma would be individuals. Any entity’s individuality, including that of human beings, is linked to its *being-in-itself*, hence, as we will show, to its *act of being*. Relations are based on the individuality in itself of a substance, not the opposite. Only in the Holy Trinity the contrary is true: the divine Persons are *self-sustaining relations*, but one should not confuse the natural with the supernatural realm. The so-called *dialogue* is a property, a faculty of the personal being of a human individual, but it does not ground this being, at most it tries to express it. Nevertheless, human persons are characterized by a *radical impossibility of expressing their deep being*, or ‘being-in-itself’ and this is the origin of their *mystery, dignity and unreachable beauty*!

«True» does not identify itself with «being» even if these are equivalent notions

Hence, *truth is not identified with being*, because truth is not simply being, but it is being in relation to the intellect. If being identified itself with truth, then the being of one thing would depend on its relation to the human intellect that *knows* it and, even more radically, to the intellect that *wants* to know it. If this were the case, however, we would fall back into the absurdity of modern transcendentalism that, by trying to make consciousness the modern «creator» of being—first with the notion of a self-aware intellect, later with will—ended up with the «nothingness» of nihilism. Metaphysics, as a science, requires that mistaken formulations be corrected.

Transcendentals as the ante-predicative foundation of predicates and of categories

In sum, **being**; *being* a **thing** and *being* **one**; *being* **something**; *being* **true** and *being* **good** are the «general» *transcendental determinations* of any entity. They are «general» in the sense that they apply to any entity, be that a substance or accident. For example, «being-horse» and «being-tree» are two distinct and *specific* ways of being substance, two distinct «entities». Each of them, however, is *generally* «being-thing», «being-one», «being-something» etc. Transcendentals, therefore, are not *ways of thinking or of wanting* (= modern transcendentalism) but are *ways of being* (= classical transcendentalism). These are the basis of the ways of thinking and defining an entity according to conceptual categories in various languages, as well as the basis of the attribution of value to an entity, so that the latter becomes object of desire (or rejection) on the part of will.

5.7 Summary of chapter five



In this chapter we have presented a brief account of the metaphysical problematic encountered in classical thought, which could help identify the novelty of Aquinas' metaphysical notion of being-as-act as well as the continuities of his thought with the highest representatives of Greek philosophy, Plato and Aristotle. This aimed at identifying the fundamental turn brought on by the birth of modern philosophy on issues of meta-logic and metaphysics, specifically in relation to the notions of truth and being, as well as to their logical and ontological foundations (§ 5.1.1).

In (§ 5.1.12) we dealt with the origins of western philosophical thought in the workings of the so-called **fusikoi**, the «physical» philosophers of the school of Miletus in Asia Minor (Thales, Anaximenes, Anaximander) who searched for the **archē**, the «beginning», in elements of the physical realm. In § 5.1.1.3, we briefly recalled how this rational reflec-

¹⁴⁷ We should recall that, as rightly noted by Heidegger, the reduction of being to value, the reduction of «what is» to «what *I want it to be*» is the origin of Nihilism.

tion on the nature of things moved to Greater Greece, merging in the arithmetical metaphysics of Pythagoras and his school. Thanks to these, the first form of Western mathematical science was born.

In § 5.1.4, we briefly dealt with the first real Western metaphysician, Parmenides of Elea, who focussed on metaphysical reflections on being and on its complex but fundamental relation with logical laws, particularly the principle of non contradiction (p.n.c.). On the other hand, Parmenides' failure in distinguishing between *logical entities* and *physical entities*, and the consequent *univocal interpretation* of the notion of «being» —that is, the rationalist formulation of his metaphysics— led him to three paradoxical consequences in his ontology: the contradictory character of *quantitative multiplicity*, *qualitative diversity* and *change*. Reactions to these three provocations became the *leit motiv* of the highest metaphysical reflections in Greek thought.

In § 5.2 we treated Democritus' metaphysical atomism. In § 5.2.1 we saw how the philosopher's notions of «matter» and «void» could fully solve the first problem posed by Parmenides —demonstrating the non-contradictory character of quantitative multiplicity— and partially answer the third —demonstrating the non-contradictory character of a particular kind of change, «local motion». Taken to absolute extremes, these two answers —particularly the fact of reducing qualitative difference to quantitative multiplicity, and all forms of change to only local motion— led to *metaphysical atomism*, of which Democritus was the first important representative in the history of Western thought (§ 5.2.2).

We owe to Plato's philosophy the justification of a metaphysics of form and quality (§ 5.3). In particular, we owe to him the discovery of the necessarily formal and immaterial nature of logical universals, or *ideas* —based on their *unrepeatable uniqueness* (= self-referentiality)— as opposed to the multiplicity and possibility of repetition characteristic of quantity that, as Democritus argued, are grounded in matter (§ 5.3.1). Plato, however, equated *ideas* with entities' *essences*, once again missing the distinction between *logical* and *physical entities*. The two other «weak points» of his metaphysics were due to this fundamental neo-Parmenidean mistake. These are the notion of «formal participation» (§ 5.3.3), used to justify the passage from the formal unity of ideas to the multiplicity of physical entities that share the same specific essence; as well as the «dualist» formulation of his metaphysical anthropology (§ 5.3.4), which led to losing the being of man's psychophysical unity. In any case, thanks to the Platonic distinction between «absolute» and «relative» being and non-being, Western thought found the theoretical key to overcome the paradoxes of rationalist Parmenidean metaphysics, par-

ticularly the second one, related to the presumed contradictory nature of qualitative difference. This distinction entails that if an entity A is-not an entity B this does not imply the total negation of B's being but only of its form, the same way that Democritus' «void» is only a negation of an entity's matter, not of the entity as a whole. Through the distinction between *absolute* and *relative* being and non-being, Plato discovered that «being» and «entity» (and, correspondingly, «non-being» and «nothing») are not univocal notions, but *analogous*; they are not *absolute*, but *relative* notions (§ 5.3.5).

This discovery («being is stated in many ways») was the starting point of Aristotelian logic and metaphysics, with which Greek thought reached its highest point of maturity (§ 5.4). Aristotle applied this notion to the distinction between «logical entity» and «physical entity», specifically between «ideas» or «logical universals» —which are necessarily immaterial or purely formal, since they are self-referential, existing only in the mind that produces them— and «essences» or «natures» of physical entities —which are composed of matter and form and do not exist in themselves (they are not substances) but rather in the many, even infinite individuals that belong to one same species (§ 5.4.1).

Thanks to the stress put on the «hylomorphic» nature and the non-self-sustaining character of physical entities, Aristotle was able to operate the further fundamental distinction in metaphysics between «primary substance» or self-sustaining individual —metaphysical subject of all properties or «accidents» that characterize it— and «secondary substance» or «essence», which is not self-sustaining, but exists in the various individuals that share the same essence. From this derives the additional distinction between that which a primary substance (= individual subject) *is* (= secondary substance, or essence) and the other properties or attributes that a primary substance *has* (= accidents). This distinction was the metaphysical basis of Aristotle's doctrine of the «ten categories», which was the ontological foundation of his logic of predicates and of syllogistic techniques (literally: «techniques for combining terms») to construct propositions. These techniques allowed combining subjects and predicates, with different degrees of necessity, through inductive inferences (inductive syllogism and analytical methods of searching for middle terms) and deductive ones (deductive syllogism, «demonstrative» or scientific, both apodictic and hypothetical, in its various forms and shapes) (§ 5.4.2).

On the other hand, thanks to his doctrine of change, Aristotle was also able to solve the third Parmenidean paradox. Aristotle understood change not as the passage from being to non-being and/or the reverse that, as Parmenides had correctly pointed out, would be contradictory.

Rather, he intended it as the passage from «being *in potency*» to «being *in act*» of a form in a material substratum. Aristotle proved the non-contradictory character of qualitative and quantitative change, and not only of local motion, which Democritus had already demonstrated. This allowed overcoming the *ontological reductionism* (quality is reduced to quantity and change to local motion) implicit in Democritus' metaphysical atomism (§ 5.4.3).

Finally, in § 5.4.4, we dealt with another fundamental point of Aristotelian metaphysics, the *causal explanation of physical entities' essence*, which is extremely important for philosophy of nature. This explanation was based on a hierarchical ordering of the levels of physical causality (unchangeable motions of celestial bodies *vs.* unstable motions of earthly bodies) which is structurally similar — even if phenomenologically different — to that in physics of materials explaining the physical-chemical properties of chemical elements. The latter is grounded on the notion of stability of an atom's nucleus on which, at the low levels of energy typical of chemical transformations, the dynamics of electrons in the external «shell» have no influence. This electronic shell, on which chemical interactions typical of a given type of material depend (at both atomic and molecular levels) displays the characteristics of *stability*, *identity* and *specificity* that make atoms of a given element independent from external interventions upon them. Indeed, the possible undulatory configurations in the electronic shell — in both the fundamental and the activated state (when an atom is subject to external stimulation) — are always the same, given that they only depend on the attraction to the nucleus and on the electrical effects of interactions with other electrons in the atom.

The Thomistic metaphysics of being-as-act surfaced at this point, solving a fundamental dilemma found in Aristotle's thought (§ 5.5). The Greek philosopher's causal explanation of essences created the problem of justifying the existence of an individual substratum — the «primary» substantiality of being in any individual that possesses a given set of properties. This led many followers of Aristotelian thought — starting with the Stoics, the Epicureans and including commentators such as Alexander of Aphrodisia — to give a materialistic interpretation of Aristotle's metaphysics. This meant to confuse the *substratum of form* (matter) with the *individual substratum of accidents* (*primary substance*). In so doing, however, the very core of Aristotle's metaphysics of nature broke down, that is, the distinction between the being-in-potency of matter and the being-in-act of primary substances (actually existing individual entities), which created an unsolvable antinomy (§ 5.5.1).

In order to solve this antinomy, Aquinas formulated his metaphysical theory of *being-as-act*, where the duality act-potency applied by Aristotle to form-matter to causally explain essences—the secondary substantial nature of entities capable of self-sustaining—was extended to essence-being. In so doing, Aquinas was able to explain causally the being of an entity as a whole, including its «primary» substantiality. If the nature or essence of entities that are subject to transformations («earthly» physical entities) is ultimately the effect of other entities' causal action—as implied by Aristotle's scheme of «double» («earthly»/«celestial») causality—then the nature or essence of entities is a «potential» principle in relation to «being», similarly to matter in the constitution of essence. «Being», in turn—similarly to form in the constitution of essence—becomes an «actual» principle, an «act»—being as «act»—that, like form-act, refers to some causal «agent».

Each physical entity has, at the metaphysical level, a double composition of act and potency: in relation to *essence*, because it is made of form and matter; in relation to its *absolute being* (*existence* and *entity*) because it is made of essence and of act of being. This double composition is related to a *double causal concurrence*: a) *physical*, with which entities' material forms are educed from the potentiality of matter through causal action by physical entities; b) *metaphysical*, given that any existing body or physical entity (substance) is contingent, because it depends on the causal action of other entities to be able to exist. On the other hand, a universe only made up of contingent entities that are caused and that cause would have no substantiality, unless one assumes the «primary causality» of a unique un-caused Agent that does not belong to the universe of contingent entities.

This Principle, with one single act and at once, grounds the metaphysical substantiality of the causal network of entities that make up the universe and of each «intersection» of this network—that is, the «primary» substantial nature of any existing physical body. In order to describe this *metaphysical foundation* of being—of all being, essence and existence—to which the notion of «causality» can be attributed only by analogy, Aquinas formulated the notion of «participation of *being*» (*vs.* Plato's participation of *form*).

The Self-Sustaining Being «outside» the universe participates being to the entire material universe with a single act, even if the different types of physical entities can come to exist, at different points of time and space, on the basis of the essence (or composite of form and matter) that is made available by the concurrence of physical causes. In Scholastic philosophy of nature, the classical Thomistic formula of essence as

really (causally) *distinct* from being in the ontological construction of entities, which therefore participate in being (as actuality) in reason of their essence (as potency) (§ 5.5.2), must be intended in this sense of an implicit reference to a double and complementary grounding process of being (primary substantial nature) and of essence (secondary substantial nature) of the individual (simple and complex) bodies that compose the physical universe.

In order to understand the metaphysical specificity of the «participation of being» in relation to physical causality, Aquinas used the meta-logical analogy of the reference of a symbol to the entity that it denotes — in the case of names —and/or connotes — in the case of definitional propositions. A reference is an asymmetrical relation in which the proposition necessarily refers to an entity in order to exist as a logical entity (= to be true). The physical being of the entity, however, while grounding the logical being of the proposition, does not depend on what the proposition states. The same is true in metaphysics.

All contingent entities necessarily refer to the Self-Sustaining Being in order to exist as physical entities, but the being of the Self-Sustaining Being is not affected by this reference. In short, the relation of «participation» is a causal relation only in an analogical sense, given that, compared to common causal relations (which Aquinas called «real» relations), it does not display a double and symmetrical necessary link between caused and cause. This necessary link — as in the (meta)-logical relation of referentiality, of constitution of semantic truth (= adequation) — only goes in one direction, so that the relation is asymmetrical: $(aRb) \neq (bRa)$. Aquinas' constant reference to «God», the «God» of Christian theology, in his writings dealing with the «participation of being» is justifiable, because this metaphysical doctrine of participation can be put in continuity with the dogmatic notion of creation in the Christian sense, clearly without demonstrating it. This notion states the *necessity* of the relation between creature and Creator, which stands in opposition to the *freedom* of the relation between Creator and creature.

Conversely, this metaphysics is opposed not only to the pagan vision of «fate» and/or «destiny», but also to the modern Hegelian interpretation of Christian doctrine itself. Hegelian metaphysics argued for the double necessary link between Being and entity. This is typical of any deterministic metaphysics, according to the scheme of «necessary constitution» of the Absolute Spirit through the dialectical evolution of nature and history. In this scheme, in other words, «God needs the world in order to be God» (§ 5.5.3).

Finally, the absolute uniqueness of the relation between the actual Infinity of the Self-Sustaining Being and the potential infinity, *in fieri*, in construction (privative infinity) of the universe of contingent entities stresses how the Absolute Actual Infinity of the Self-Sustaining Being (negative infinity) means the absolute simplicity, with no difference and/or diversity, of the Pure Act. Hence Aquinas' distinction between *three types of infinity*—as opposed to the two kinds found by Aristotle—*potential infinity* (privative infinity) and *actual infinity* (negative infinity), given the insubstantial nature of the notion of *infinity in act*—the presence of a constructive notion of actual infinity that starts from potential infinity. Aquinas divided actual infinity into Absolute Actual Infinity (*simpliciter*) and relative actual infinity (*secundum quid*), or *virtual infinity*, on the basis of which even in the realm of finite entities different degrees and types of infinity can be discerned. This doctrine can be put in continuity, while avoiding confusion, with the modern distinction operated by Cantor between, respectively, potential infinity (indeterminate and incrementable: Aquinas' and Aristotle's privative infinity), Absolute Infinity (determinate and non-incrementable that, for Cantor as well, could only be God) and transfinite infinity (determinate and incrementable) (§ 5.5.5).

In §5.6.1, by way of conclusion, we introduced the fundamental difference between classical and modern epistemology concerning the foundation of predication in the construction of true propositions, an issue that was dealt with at the end of Chapter Two. In classical philosophy, this foundation was defined as the *transcendental*, which stands in opposition to the *categorical* of predicates. In classical thought and, more generally, in any realist approach to science, the *being of entities* is the transcendental, hence the foundation of truth in propositions. In modern thought, instead, it is *self-awareness*, or the «transcendental I-think», as Kant named it. In modern philosophy, *evidence* and the presumed *self-evidence* of postulates in «exact» sciences were the foundation of truth. In Aquinas' realist approach, instead, being was the foundation of truth, but not the being of existence.

The being that grounds predication, hence the truth in propositions, is not existence, but the *being of entities*, the one that relates to each entity according to the diversity of its essence. In his *table of transcendentals* (§5.6.2) Aquinas identified different ways to state an entity's being, which are *equivalent but not identical*. Some of these are *specific* to each thing (= *entity*) while others are *general* (= *thing, one, something, true, good*). Clearly, these transcendental or pre-categorical notions, particularly the one of entity, are *primitives* in any meaningful language, be that ordinary, scientific or formalized. In any formalized language, as well as in any interpre-

tation or model of formal systems, the *entity* is the first thing that must be explicitly stated. This means stating what type of entities are the focus of this language, hence making it clear how the being of the entity or entities (e.g., physical, biological, logical entities, etc.) dealt with in that language should be understood. From this derives the need to delve deeper into the Thomasian notion of *being*, both in logical and ontological terms.



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6. Towards a metaphysics of physical being

The metaphysics of physical being, in light of the principles of Thomistic metaphysics of being as act, presented within the contemporary debate about the foundation of logical sciences (meta-logic) and physical sciences (metaphysics).

6.1 Meta-logic and metaphysics

6.1.1 Meta-logic of metaphysics

The modern distinction between logic and meta-logic, and the classical theory of *suppositio*

As a consequence of the scientific revolutions of the last two hundreds years, presented in the First Part of this work, the debate about such semantic notions as “meaning”, “consistency” and “truth”, deriving from the rise – following Frege – of formal logic as *symbolic logic*, i.e., as *calculation of predicates** (i.e., of classes) and of *propositions**, drove modern thought about the foundations of sciences to revive a fundamental distinction of classical formal Scholastic logic. This resulted from the distinction between *language* (- *object*) and *meta-language*, introduced by David Hilbert with reference to mathematical language alone (mathematics and meta-mathematics), and it can be expressed in the distinction between *formal* and *material presupposition* of a term and/or a proposition within its logical analysis (see Bochenski 1956, II, 380). In short, when we are to analyse a statement (e.g., «Tony runs») we need to distinguish always between (see Aquinas, *In Periherm.*, I, v, 57):

The *suppositio materialis* and the syntactic analysis of language

- ◆ The purely linguistic object, i.e., the considered language statement, taken in itself as an object of analysis. This applies, for instance, when we say that ‘runs’ within the statement ‘Tony runs’ is a verbal predicate, or also when we say that ‘equals four’ within the statement ‘two plus two equals four’ is a nominal predicate¹⁴⁸.

¹⁴⁸ Consider these two statements, (i) the former refers to an object belonging to the collection of natural entities, (ii) the latter refers to an entity belonging to the collection of logical-mathematical entities. This helps in underlining that the meta-logical discourse, of a semantic kind, about the *existence* of the referents of statements and about the *truth* of the statements themselves has nothing to do with the epistemological reference to experience, that is, to the so-called “sensible datum”. In other words, what constitutes Aris-

In this sense, in compliance with Scholastic logic, we used to say that the considered statement was taken into account according to *suppositio materialis*. In modern logic, we, rather, say that the meta-linguistical analysis of a given language-object statement is situated at a *syntactic* level.

The *suppositio formalis* and the semantic analysis of language

- ◆ The referential object to which the considered language statement refers, as, for instance, when we say that ‘runs’ within the statement ‘Tony runs’ denotes (i.e., means) the action that Tony is now performing, or, in brief, that it is true that ‘Tony runs’; or, like when we say that ‘equals four’, within the statement ‘two plus two equals four’, denotes (i.e., means) the exact result of the arithmetical formula ‘ $2 + 2 = 4$ ’, or that it is true that ‘two plus two equals four’. In this case, Scholastic logic use to say that the considered statement was taken into account in terms of *suppositio formalis*. In modern logic, we say, rather, that the meta-linguistical analysis of a given language-object statement is situated at a *semantic* level.

Syntactic and semantic function of the verb within the proposition

Now, as Aquinas maintains (see numbers 58 and following.), when we analyse logically the function of a verb, we need to make a distinction between its syntactic and semantic analysis. From a syntactic perspective, on the one hand (*in suppositio materialis*), a verb with a *predicative* function can be defined as what *formally* determines the indeterminacy of a name, which stands as the *subject* of the proposition. From a semantic

totalian and Thomistic semantic is not the *epistemological* reference to experience, but rather the *meta-logical* referent of a statement. The fact that such entity, representing the referent, is a natural object of experience, or a logical entity and a product of reason is utterly insignificant, both for metaphysics and for Aristotelian and Thomistic meta-logic. It is important to bear that in mind since modern transcendentalism, following Locke, seeks to give an empiricist interpretation of the fact that the constitution of the consistency and truth of a statement depends on the reference to the entity, which is the object of the predicative statement (referent). In other words, building on the *faux* presupposition that according to Aquinas and Scholastics the foundation of the truth of statements is of an epistemological and not either a metaphysical or a meta-logical kind, Locke and many modern Neo-Scholastics, unfortunately, tried to extend to Scholastic and, in particular, Thomistic logic the Lockean empiricist axiom *nihil est in intellectu quod prius non fuerit in sensu*. As it is well known, this axiom was used by Locke to criticise Cartesian innatism of ideas, so as to support the claim that the “clear and distinct”, or “self-evident” ideas that lie as the epistemological foundation of true statements, have a sensible – and not rational – origin. That Aristotelian-Thomistic epistemology is against the innatist theory of knowledge is a fact, for: according to Aristotle and Aquinas all concepts and principles – including the most abstract ones, or the p.n.c. – are certainly not innate, but are originated by induction. However, to conclude from this that they are thus founded on sensible intuition – as the Empiricists argue – is a historical and theoretical error, bearing serious cultural consequences. This interpretative prejudice is one of the reasons why many modern logicians and mathematicians ignored Aristotelian-Thomistic logic, like any other that is grounded in an empiricist theory of foundation.

perspective, on the other hand (*in suppositio formalis*), a verb with a predicative function can be defined as *referring* either to a particular action (*actio*, like, for instance, in “Tony runs”) or to a property (*passio*, like, for instance, in “Tony is white”)¹⁴⁹ of the *referent* of the subject of the proposition.

6.1.2 Meta-logical analysis of the copula “is” within statements

Centrality of the copula «is» in the construction of every proposition

From this perspective, as Aquinas underscores (see numbers 71 and following), particular attention is to be paid to the verb “to be”. Since every predicate, or verb, of any proposition can be built associating a given predicative name with the copula “is” (e.g., “x runs” → «x is running»), it can be said that every proposition is syntactically built as the sum of a *predicative name* and the universal formula *subject + copula*: «x is». In Aquinas’ words, “a thing is” (e.g., “John is a man”, “Fury is a horse”, “the electron is an elementary particle”, “Tony is running” (i.e., runs), etc.). We are now faced with the meta-logical correlate of what we have previously maintained in saying that the entity is *the first to be known* (*ens primum cognitum*) in every cognitive act.

Double meaning of the copula «is»

If we analyse meta-logically the semantics of the predicate “is” in the universal formula “x is”, or, in Aquinas’ words, in the formula “a thing is” (*rem esse*), which underpins any predicative form of language, it is clear that it has two meanings: a primary meaning – i.e., *being an entity* – and an indirect one – i.e., *being something*.

¹⁴⁹ The definition of a certain property, or quality, of a given entity as a sort of “suffered action” (*passio*) is tightly linked to the theory of the form as an “act”, or, better, as the outcome of an action on a passive substratum. For instance, the property “to be white” of a given material entity is a consequence of a series of causal actions, which have determined the fact that that entity is actually white. Accordingly, “to be white” means the same as “to have been made white”. For instance, consider a man with white skin: this means that the sun has not been strong enough to make his skin tanned. Or consider a case of an illusion in which someone, at a traffic light, sees a green light instead of a red one: the way the property “to have been made green” of the traffic-light is perceived depends on the visual problem of this color-blind person. Therefore, it is clear that the degree and the way in which a physical entity participates in the property of whiteness depend on a particular reciprocal determination between the activity on a given agent, and the passivity of a given patient that is constituted by matter. In the case of logical and/or mathematical entities the “efficient cause” of a considered property – e.g., the property of the geometrical entity “triangle” to “have the sum of the internal angles that equals two right angles” – are the postulates of Euclidean geometry and, specifically, of the Fifth Postulate.

Primary meaning of
«is» as «to be an
entity» and its
insignificance in
terms of existence

- ◆ *In the first place*, taking the verb “is” as it is, without adding anything on its right hand side, it means to be an entity, i.e., “to be what is”. Here “entity” is the predicative name of the verb “to be”, like “running”, i.e., “what runs”, is as such for the verb “to run”, in the sense of “what participates in the action of running (or in the ‘run’, as an abstraction of ‘to run’)”. Similarly, the predicative name “white” can be considered “what participates in the whiteness”, as an abstraction of the “passion” of “having been made white” by a given causal action. Now, if it is true that the predicative name “entity” literally means “what is”, “what participates in the being”, the expression “something is what is” does not mean the same as “something exists”.

Similarly, to define someone, e.g., a “runner”, as “running”, as an “entity that runs”, as “being what runs” is by no means the same as saying that “someone runs”, i.e., that someone is actually running. In sum, in the expression “what is” (*quod est*) – meaning an “entity” – the referent that is directly pointed out by “entity”, according to Aquinas, is not at all the “is” (*est*), but rather the “WHAT” (*quod*). In other words, if we accept that “an entity is what is”, it does not follow that such a tautological formula means the same as the other meta-linguistical expression “what is exists”.

Thus, when we say that anything is an “entity”, we are saying that “anything is WHAT is”, yet we are not certainly saying that “this ‘what is’ EXISTS”. But we are not equally saying that the expression through which we define this “what is”, e.g., “the Arabian phoenix” of some mythological story, exists in view of the mere fact of its “being an entity” – which is indubitable as all tautologies are.

Put another way, from the identity principle in its most general formulation “an entity is what is”, which we could define – following Parmenides – as the most fundamental tautology of thought, does not certainly derive, as Parmenides believed, the existence of an entity. The tautology “the entity is what is” is not sufficient on its own to eliminate the non-existence of a certain entity. To say “the Arabian phoenix is an entity”, i.e., “the Arabian phoenix is what is” has no meaning regarding the existence of that entity. In order to justify the existence of an entity we need thus to determine the way in which the existence corresponds to that entity. To determine this means precisely to somehow connote the *thing* of that entity (= the first of the transcendentals reported in Table IV, § 5.6.2). The determination of the thing of that entity thus seems to be essential to affirm in a truthful way its existence, or non-existence.

Secondary meaning of «is» as «to be something» and necessity of its combination with a predicative noun

- ◆ *In the second place*, accordingly, in Aquinas' view, the predicate "is" within the universal schematic formula " x is", "a thing is", co-means (*consignificat*) to be something (*esse aliquid*). In other words, it refers necessarily to the explicit definition of an essence. In view of what we have explained so far, so as to make the schematic formula "a thing is" somehow "co-mean" the "IS", the "EXISTS" of the "what IS" it is necessary that such formula is "combined", or "divided", with a certain explicit – non-tautological – connotation of the being of the entity, i.e., with a certain definition of its *essence*, by making explicit some property which corresponds *per se* to that entity. Put another way, the meaning immediately expressed by the formula "a thing is" – i.e., "to be an entity", "to be what is" – transcendently indicates only the left term of a predicative formula, the "what" expressed by the grammatical subject of the copula "is".

Only after knowing the other term of the copula "is", the right predicative term that defines the "to be something" of an entity – i.e., some connotation of the thing of that entity – we will be able to decide whether we should attribute to the referent of the entire predicative statement the meta-predicate "exists" or "does not exist". Moreover, only after this first decision, we will be able to decide about the other meta-predicate "is true" or "is false", which now does not refer to the consistency of the predicative formula by which to connote the object, but rather to the statement itself, in its assertive capacity either of connoting and denoting an existing object, or to deny that a certain predicative connotation can denote an existing object. In Aquinas' words:



The combination in which the truth or falsity of a formula is grounded cannot be understood if not in light of its connecting the terms of the proposition itself. (...) Therefore, it cannot be understood without its components, because its comprehension depends on the terms. If these are not considered, there cannot be any full comprehension of the combination so as to predicate either its truth, or its falsity (Thomas Aquinas, *In Periberm.*, I, v, 71, 72).

The meta-logical predicate «exists» as referred to the referent, and «is true» referred to the proposition as a consequence of combination/division

Therefore, when Aquinas spoke about the psychological foundations of reasoning, he maintained that the being is known by the intellect not when it "learns" the essence of a given entity – i.e., when it builds the predicative connotation of the thing of the object – but, rather, when it "formulates the judgement" – i.e., when it builds the statement in terms of a denotative (either affirmative or negative) assertion (see Part Four). To say that the predication "exists" / "does not exist" and, therefore, "is true" / "is false" can be related to a proposition only after we have set

the two terms of the copula “is” have been placed as opposites, thus affirming, following Aquinas’ medieval logical language, that the predicates:

- ◆ “exists” / “does not exist”;
- ◆ “is true” / “is false”

are meta-linguistical predicates, or meta-predicates. These are predicates that have as their domain not only terms, like linguistic predicates, but full propositions. They are respectively referred to:

- ◆ *the entity*, which represents the *extra-linguistic referent* of a simple proposition, complete both at the right hand and the left hand side of the copula “is”;
- ◆ the *proposition* itself, or *linguistic referent*, as far as the thing expressed by the name that completes the copula “is” *at its right hand side* correctly denotes the way in which the existence corresponds to the entity indicated by the name which completes the copula “is” *at its left hand side*.

An example of the meta-linguistical use of the predicate “exists”

Accordingly, to say “the Arabian phoenix is the bird resurrecting from its ashes”—and not simply say that “the Arabian phoenix is an entity”—immediately allows us to decide about the existence / non-existence of the referent of such a statement, and thus of its “truth / falsity”. Indeed, once we have defined the Arabian phoenix — as we have just done — it follows immediately that the referent of such definition *cannot exist*, and, accordingly, that the definition is false. This is due to the fact that by “bird” we indicate an animal of which — at least in finite times, beyond any ergodic hypothesis — we cannot affirm the possibility of resurrection, in accordance with empirical data and physical laws.

Conversely, as soon as we build that definition using, as the other term, the predicate “mythological (fantastic) bird resurrecting from its ashes”, we also immediately remove the contradiction, because we thus define the conditions of its “existence”, as a “product of someone’s imagination”, and not as “something produced by natural causes”, as we, instead, were doing when we defined it as an organism belonging to the genus of birds.

Meta-logical nature of the predicate “exists” and overcoming of Parmenidean rationalism

At the same time, against what Parmenides and his modern followers — such as Emanuele Severino — maintain, we can state that the fact that an entity, i.e., a “what is”, does not exist does not violate the p.n.c. Indeed, to say “what is does not exist” is not contradictory. It is, rather, *equivocal* simply because *exists* is a meta-predicate that has to indicate a full formula, which the expression “what is” fails to be. This is only a formula

used to express the subject of a proposition with no predicate. When we say, “*x* is what is” a question follows immediately, namely “*what is* ‘what is’?”. Put another way, when we say the equivalent formula “*x* is an entity”, the question which immediately arises is “what kind of entity?”.

A first logical distinction between the being of the essence, or entity, and the being of existence

When we complete the formula, which will be the focus of our meta-linguistical analysis, like when we say “the Arabian phoenix *is* the bird that resurrects from its ashes”, we can immediately build the correct meta-linguistical implication: “if the Arabian phoenix *is* the bird that resurrects from its ashes, then it *does not exist*”. Now, if we bear in mind the Scholastic doctrine of *suppositio* and, accordingly, the modern distinction between language and meta-language, it appears to be immediately evident that we are not caught up in a contradiction, since the above-mentioned expression does not equal that according to which “if the Arabian phoenix is, then it is not”. The “exists” of the second part, or apodosis, of our implication does not equal the “is” of the first part, or protasis. In order to have a logical contradiction it is necessary that the affirmed and denied predicates be identical. But the “is” of the copula in the protasis, being a formula completed both at the right hand and at the left hand of the copula, here denotes the *being of the essence* of an entity, that is precisely its *entitas*, i.e., the first transcendental of the Thomistic table (see Table IV). On the contrary, the “is”, or the “exists” of the apodosis denotes the *being of the existence* of that entity, *qua* determined according to the essence defined in the protasis, i.e., — from a logical perspective — *qua* meta-predicate of a predicative statement.

Mind that modern *formal ontology* has at last rediscovered this essential distinction between *entitas* (the thing) and *existence*, as two different meanings of the copula “is”. However, having lost the intensity of Medieval ontology, and being slave of Kantian reduction to the *mere existence*, this promising and fascinating discipline has to make use of such paradoxical formulas as the expression used by Alexius Meinong (1853-1920) “*non-existing objects*”, connoting by this ambiguous phrase:

- ◆ Sometimes those objects that, according to Aristotle and the Scholastic tradition, are mere *logical entities* (e.g., when one speaks about the concept of “nothing”) — almost suggesting that only physical entities exist, so that the logical entity is connoted as a “non-existing object”, or “abstract object”;

- ◆ Some other times, the mere *entitas* (the thing) of an, either logical or physical, entity, which does not at all correspond to its existence, as I hope it has now become clear¹⁵⁰.

Difference between
epistemological and
meta-logical
foundation of
science

At this point we have everything we need in order to understand the Thomistic difference between the notion of *being*, in its several meanings, and the notion of *existence*, that is the nucleus of Thomistic metaphysics. In order to conclude this sub-section, let me add a further remark. We said that Thomistic metaphysical and meta-logical analysis of the foundations of scientific language was free from all epistemological references to the act of knowing and the act of conscience. I believe that the analysis conducted so far has been enough to underscore the correctness of what we have maintained, through the opposition of the meta-logical and metaphysical foundation of science to the modern epistemological one. The entire analysis is not at all linked to the cognitive act, or to the act of conscience. Conversely, however lacking an appropriate formalization, it has developed at a theoretical level, without any reference to – either individual or transcendental – “consciences” of some kind.

6.2 The being-essence distinction in meta-logic

6.2.1 Essence, nature and *quidditas*

A first definition of
the notion of
essence

In light of what we have said, the core of Thomistic metaphysics, meta-logic and, implicitly, mathematics, expressed in the famous doctrine of *real distinction between being and essence*, seems now to be clear. Aristotle, following Plato, had already highlighted that “there are several ways to say being” and that the principle ones are:

- ◆ “being *per se*”, or “being *per essence*”, or *being necessarily*;

¹⁵⁰ Such a revival of Meinong’s thought is due primarily to E. Zalta (Zalta 1980; 1988), who established at Stanford University, more than twenty years ago, a Metaphysics Research Laboratory (<http://mally.stanford.edu//index.html>) in order to support research about these issues. For an introduction see K.J. Perszyk, “Non-existent Objects: Meinong and contemporary philosophy” (Perszyk 1993); for an updated general framework of the current debate see the collection of essays edited by Everett & Hofweber (2000). In order to distinguish the two abovementioned meanings of “to be” (logical entity) *qua* different from “to exist” two diverse areas of study developed from formal ontology, namely *meontology* (from the Greek *mē* meaning “not”) – in the sense of an ontology of non-existing objects, i.e., the entities of reason; and *tinontology* (from the Greek *ti* meaning “what”), in the sense of an ontology of essences (literally of *quidditates*, see *infra*, p. 423423) which, as actualized in a given existent, constitute its *entity* (see www.formalontology.it).

- ◆ “being *per accident*”, or *being contingently* (see (*Met.*, V, 1017a, 9)).

Since Greek thought, as the first expression of scientific thought in the West, met the fundamental problem of building an *adequate classification* of the entities that are objects of rational investigation, the first issue to tackle was the characterization, in the connotation of the entities that were objects of rational investigation, of the fundamental, or *essential* properties of a given entity. These are those properties that allow us to distinguish between different entities through their allocation to diverse species, and are opposed to those unessential, or *accidental* properties that, within certain limits, could vary, and even be gained and lost without implying any consequence for the properties of the former kind.

These are those properties that are necessary to identify an individual in itself and, as far as these (or, some of them) are shared by other individuals, to make it belong to one single species. The “being together”, or formal unity of the properties that identify an entity with itself, and distinguish it from what is other than itself, is what is indicated by the term “essence”.



Definition 8

By “essence”, or “nature”, of an entity we mean, metaphysically, the *formal* principle constitutive of something, i.e., what makes it identical to itself and distinguishes it from what is different.

Essence and causality: two deeply connected notions

At the end of the previous sub-section, when we underscored the meta-logical nature of the predicate “to exist”, we have seen that the *being of the essence*, or *entitas* (the thing), is relative to the *sufficient* (but not *necessary*) determinations to make that entity exist, according to its own mode of existence (e.g., the “Arabian phoenix” was seen as an imaginary entity, thus *produced* by imagination, and not as a natural entity (a bird) *produced* by natural causes). Hence derives what we can term as the *individual essence*, i.e., the essence of an entity taken in its individuality of something existing under certain conditions, namely a *contingent entity*¹⁵¹. This is an entity which does not exist *per se*, but that receives its existence from something external to it.

¹⁵¹ Spiritual entities cannot be either individual or contingent because they are not made of matter.



Definition 8b

As far as we consider the (physical, logical, spiritual, etc.) entity in its *individuality* of a *contingent* existent – i.e., something that, in order to exist, needs a set of causes, or sufficient conditions to make it exist – we can define the *essence* as the *unity* of the set of all those *determinations* that participate in the *constitution* of an entity as *different* from all the other entities of the universe. In this sense, we can speak about the “individual essence” of a given entity.

If we do not consider the individual essence of an entity, but rather some of its properties that are shared by other entities and that can thus be defined as belonging to the same *species* or *class* of entities, then the whole of these properties will represent the *specific* essence of that entity, that is what Aristotle defined as the “secondary substance” (Cf. § 5.4.2).



Definition 8c

By “specific essence” (secondary substance) of an entity we mean the whole of all those properties or determinations of an entity that “assign it to a certain species of entities and, at the same time, distinguish it from all the other species” (Mondin 1991, 228).

In Aquinas’ words,

The (*specific*) *essence* comprises only what is included in the definition of a species. Accordingly, *humanity* embraces only what is included in the definition of “man”. (...) The term *humanity* indicates this: that for which a man is a man (Thomas Aquinas, *S.Th.*, I, 3, 3c).

Thus, we need to distinguish between what is the *essence in itself* of an entity (= individual essence), or of a collection of entities of the same species (=specific essence), and what is the knowledge of that essence, and its connotation *qua* expressed in a simple proposition, constructed as a combination of subject and predicate (categorical proposition).

The essence and its knowability: unknowability of individual essence

In other words, we need to distinguish between the essence in itself of an entity and its *quidditas* (literally, its *whatness*), i.e., what of that essence can be known and expressed by means of statements constructed as affirmations or negations (combinations/separations) of certain predicates in relation to a given subject. What needs to be born in mind is thus that the denotation of an essence can *never* concern the essence of a *single* entity, taken in its irreducible *individuality*, but only of a class, or “species” of entities.

Perfectibility and temporal development in the definition of essences

The impossibility to define the essence of a single entity, taken as single, follows the fact that human knowledge has an intrinsic *limit*. Now, since the essence is constituted by the whole of the “determinations”¹⁵² of something, no human being can aspire to know the whole of all factors that concurred in the determination of an existing entity, taken in its irreducible *individuality*. In order to achieve this, we should come to a full knowledge of the entire universe and of the relations linking the various entities that are part of it in the past, present and future. The knowledge of the *individual essence* of a *single* entity (e.g., of the single man, Gianfranco Basti), just like – for the same reason – the *full* knowledge of any *specific essence* of a class of entities (e.g., the essence of the man with all the determinations that can characterize it in comparison with any other past, present and future entity of the universe), is something reserved exclusively to a form of absolute knowledge which is far from being that of human beings. Aquinas asserts:



there are several determinations (*quod quid est*¹⁵³) of the same thing: some of these can be highlighted, some other simply supposed (...). Aristotle says that we can come to know the being of every thing without having to *perfectly* know its essence (...), e.g., we may understand the being of man in light of his being “rational”, not yet knowing all those other determinations which complete its essence (*In Post.An.*, II, vii, 472, 475).

Essence, *quidditas* and nature: three different facets of the notion of essence

Certainly, the fact that a man cannot arrive at a full knowledge of an essence if an element of radical progress in knowledge is introduced – in order to reach a more and more accurate connotation of a considered entity or species – does not prevent us from achieving a *true* knowledge of that essence. What matters is that men can always reach – by means of the abstractive capacity of the intellect – that *specific difference* which distinguishes a certain species of entities from all those that belong to the same genre across different contexts (i.e., across the different possible

¹⁵² By “determination” we mean here the sense of *quod quid est* (literally “what is”), i.e., a classical Thomistic expression that was mainly used by Aquinas as a synonymous of “essence”, that is – in Aristotle’s words – the “secondary” substance.

¹⁵³ Literally, “that which (*quod*) is the ‘what is’ (*quid est*)” of a certain entity.

worlds), so as to always be able to have a *sufficient*, although never complete, knowledge of the determinations of an essence. Thus, the issue of the relation between the essence, in terms of its being *ontologically* constitutive of a given entity, and its *knowability* / *connotability* leads us to a further fundamental distinction, which is explained in Aquinas' words:



[By *nature* I mean] the essence of the thing in accordance with which it has a relation to its own operation, because nothing fails its characteristic operations [e.g., from the action of attracting iron I can recognize the nature of a magnet]. The *quidditas* of something signifies the essence as a principle of definition of something; accordingly, the *essence* is defined as that by means of which, and in which, an entity has its being (Thomas Aquinas, *De Ente*, 1, 3).

In this sense, Aquinas introduces a fundamental *equivalence** (which is not to be confused with *identity**)¹⁵⁴ between three different meanings of the term “essence” which applies to all entities: this is the equivalence between the *nature*, the *quidditas* and the proper *essence* of an entity.

1. What is meant by “nature” of an entity

- ◆ By *nature*, following Aquinas, we mean the essence of an entity *qua* ontological principle of the operations that characterize a certain species, or class of entities. It is from the *actions* that are typical of a physical entity (or, from its *reactions* facing certain actions of a physical kind that are operated on it) that we can get to the essence of that entity – in terms of knowledge – since it is from the being of an entity that its actions in the real world come: *agere sequitur esse*, “the actions are a consequence of the being”. Here the considered being is not that of the existence, but clearly that of the essence, i.e., *the entitas*. Accordingly, the epistemological order (of knowledge) and the ontological order (of reality) follow two opposite directions, as it is sketched in the following Table V:

¹⁵⁴ According to the *logic of predicates**, two predicates are synonymous, and their referents are identical, if they indicate the same thing and share the same meaning (e.g., *acqua* and *water*). Conversely, if they indicate the same thing, but they do *not* share the same meaning, these will be *equivalent**, but not identical – e.g., “to be water” and “to be H_2O ” indicate the same thing, i.e., the same *sort* of entities (in short, they are predicates that are defined for the same “domain”, or set of objects), but they do not share the same meaning. “ H_2O ” has a more technical, physical and chemical, meaning, which, however, does not cover all possible meanings that “water” can be thought to have, e.g., religious meanings, poetical meanings, etc.

Ontological Order	Logical Order
Nature → Actions	Actions → Essence Denotation (<i>quidditas</i>)
Nature ≡ Essence	

Table V. The sign «≡» means equivalence, as distinguished from identity «=». This suggests that everything having an essence also has a nature, although nature and essence in themselves are not totally identifiable because they do not share the same meaning.

2. What is meant by “quidditas” of an entity



- ◆ *Quidditas* is another term that is equivalent to essence. “In fact, the *quidditas* represents the answer to the question *quid est?* (i.e., what is it?) that, clearly, is a question addressed in order to understand the essence of a thing” (Mondin 1991, 511). Accordingly, Aquinas defines *quidditas* in his fundamental treatise, *De Ente et Essentia*, from which we are quoting in order to provide some definitions that are essential for our work.

Since that through which a thing [e.g., a man] is constituted in its proper genus [e.g., the genus of animal entities] or species [e.g., the species of rational entities] is what is signified by the definition indicating what the thing is, philosophers introduced the term *quidditas* to mean the same as the term essence (*De Ente*, I, 3).

In other words, the *quidditas* is the essence *qua* known and expressed through an appropriate linguistic connotation within given contexts. Clearly, what is expressed in the connotation is always far less than what the essence is. In order to connote in full the essence of an entity (throughout all possible worlds), we should know what makes it different from *the infinity of all the past, present and future entities of the universe*. But this not only is *impossible* for the human intellect, but it is also totally *useless* for it, given its actually finite (although potentially infinite) capacities.

3. What is meant by “essence” of an entity, as distinct from, but equivalent to its nature and *quidditas*



- ◆ By *essence* of an entity, which is not reduced either to its “nature” or its “*quidditas*”, Aquinas means the “relation of the entity, constituted by its essence, to being” (Tyn 1991, 401). Put another way, “the *essence* is what by means of which, and in which, an entity has its being”. In a nutshell, “things cannot be distinguished one from the other on the basis of being, which is shared by all of them”, i.e., the being of their *existence* (Mondin 1991, 230). Therefore,

things are not distinguished from one another in so far as they all have being, because in this they all agree. If therefore things do differ from one another, either ‘being’ itself must be specified by

certain added differences, so that different things have a different specific being; or things must differ in this: that 'being' itself attaches to specifically different natures. The first alternative is impossible, because no addition can be made to 'being', in the way that differences are added to genus [e.g., in the case of man, the specific difference "rational" to the genus "animal"]. It remains therefore that *things differ in that they have different natures, to which 'being' accrues differently* (Thomas Aquinas, *S.c.Gent.*, 26, 239).

6.2.2 Entity and existence

Difference between being and essence: from meta-logic to metaphysics

Through the study of the notion of essence we have reached the core of Aquinas' metaphysical doctrine, namely the idea of the difference between *being* and *essence*. As we have already seen in the previous subsection, such difference emerges at the level of the meta-logical analysis of the predicate "to be", *qua* constitutive of every other predicate.

The being of the essence, or *entitas*

There, we have also seen that being, which has the function of copula in every predicative statement, indicates *the being of the essence*, or *entitas* (the thing) of an entity, i.e., the being that somehow defines the *specificity* of an entity through assigning it to one or more species, or classes of entities – until the class containing only one element, i.e., the considered entity, and that contains it as an individual – thus becoming the real ultimate, or *transcendental*, foundation of all *true predications* that can be given to that entity.

The being of "existence", or *esse ipsum*, 'being itself'

Conversely, *the being of the existence* of an entity – i.e., Mondin's being shared by all entities, or Aquinas' *esse ipsum*, or "being itself" – is fundamentally a meta-predicate judging the consistency of the referent of a considered predicative statement, with that series of logical (laws) and ontological (causes) conditions that characterize an entity, or the mode of existence of an entity – or of a species of entities – in accordance with the denotation given by the statement. Accordingly, if the denotation that the statement gives of an entity contradicts some logical laws (e.g., mathematical laws, as in the case of the statement: "in Euclidean geometry, the sum of the internal angles of a triangle is 190°") and/or ontological ones (e.g., physical laws, as in the case of the statement "the Arabian phoenix is a bird resurrecting from its own ashes"), it is clear that the considered statement is *false* because the logical or natural entity to which it makes reference either *does not exist*, or *cannot exist*, in accordance with the mode of existence, or *entitas*, expressed by the considered statement. Nothing forbids that, once their *entitas*, or mode of existence, is appropriately defined in a true way (i.e., as a triangle existing in a non-Euclidean geometry, or as a bird only existing in books about mythology) those entities can exist, or do actually exist.

In brief, we can suggest the following definitions:



Definition 9

By “entitas”, or “being of the essence” of a physical entity, we mean the concrete essence of a single entity *qua* existent, i.e., the whole of its properties *qua* effects of causal (both physical and metaphysical) actions, which led that entity to exist, and to exist *in that particular way*, with those properties.



Definition 9a

In metaphysical terms, by “entitas” of something we mean the set of topical elements, or perfections, which correspond to it as a result of its participation in being-as-act in virtue of its essence.



Definition 9b

In meta-logical terms, by “entitas” of something we mean the real anti-predicative, or transcendental, foundation of all predicative terms that can be attributed truthfully to the name that denotes the thing, across the different contexts in which it is defined.



Definition 10

By “existence”, or “common being”, of an entity we mean the act of existing that is shared by all entities, since each of them is the concrete realization of an essence, within a given causal context.

Every essential definition is a short form of the logical and ontological conditions that determine the being of an entity

From this analysis it clearly emerges that every *essential definition* is a sort of short form of that series of logical and ontological conditions (causes) that determine the existence of an entity. The same thought also emerges from the consideration of the emphasis that we placed in the last quotation from Aquinas on what he said about the essence, or nature, of a given entity in terms of its ultimately indicating the *proper mode in which that entity acquires its existence*.

6.2.3 *Per se* and *per accidens*

Per se and *per accidens* predication: Thomistic solution to the fundamental problem of the justification of universality and necessity in logic

At this stage we need to turn to the problem we introduced at the beginning of this sub-section with the Aristotelian distinction between *being per se* and *being per accidens*, as the two fundamental modes of every predication. In the light of Aquinas' teaching, we can say now that these two modes of predication concern the indication of certain properties of being among which only the fundamental ones are *essential*, i.e., necessarily (i.e., *per se*) belonging to the denotation of an entity because they, either explicitly or implicitly, denote the *mode of existence* of the considered entity. From a metaphysical viewpoint, they derive, indeed, from that causal concurrence through which a given entity was and is maintained existing — at least for a certain amount of time. This is why we cannot state that being without naming, at least implicitly, its essential properties. The others, instead, are only *accidentally* inherent to the denotation of being, since they derive from a causal concurrence on which the existence of the entity does not depend, if only in a marginal way.

Distinction between essential and accidental properties of an entity

Let me give an example. When denoting an entity through the attribution of certain properties (e.g., denoting a man, say Beethoven, as a "musician", or a triangle painted on a piece of paper as "red"), it is clear that such properties do not *necessarily* determine the existence of that entity. Without these, indeed, both of these entities could well go on existing, respectively as a man and a triangle, although they will not be the same they were before at an individual level, in particular in the case of the former. Therefore, although these properties factually belong to the two considered entities — and, thus, the referent of the statement which describes them exists, and the statement itself is true — their loss, however, does not prevent the man Beethoven from continuing to exist as a man, and the triangle from continuing to exist as a triangle: they would simply be a *different* man and triangle.

Causal foundation of essential properties: these specify an entity according to the set of causes and/or conditions that make it exist in a particular way

Conversely, when we state that certain properties *necessarily* belong, or belong *per se*, to an entity, or to the *reason to be* (*ratio essendi*) of that entity, we are actually saying that these properties are those without which an entity would cease to be as such, because these properties indicate the modes through which that entity has access to existence in its order. For instance, regarding the man Beethoven, his definition as an “organism of an animal kind which possesses in itself the capacity to perform intelligent and free operations”, or, regarding the triangle sketched on a piece of paper, its definition as a “three sided polygon whose sum of internal angles is 180°”, implies a specification of those entities in line with the whole of natural causes and/or logical conditions that concur to determine them as existent, either in the domain of physical entities, or in that of mathematical ones. A non-musician man can go on existing, and do it as a man. Conversely, a man without a body, as a consequence of a series of physical causal actions that made him exist and go on preserving him to exist in the physical world, could certainly no longer go on existing as a man. The same applies to a man without his spiritual soul, as a consequence of another causality in the metaphysical – and thus psychological, moral, social, etc. – order.

Similarly, a triangle sketched on a piece of paper could certainly go on existing as a triangular object, despite its being red, or yellow, or any other color. But it could no longer exist as a triangle if we added a side (if we removed a side it could no longer even be a polygon). Clearly, these series of properties are *essential* to the two entities that we have made reference to as examples. In other words, they are properties *necessary to the existence* of each of those entities (being *per se*), and are not merely *accessory or accidental for their existence* (being *per accidens*).

Novelty of Thomistic proposal: the above-mentioned distinction is based on that between being and essence

In short, Aquinas, following Aristotle, pointed us in an utterly original direction in order to understand the sense of the typical Greek philosophical distinction between *essential properties* – as the object of the *necessary* predication of an entity – and *accidental properties* – as the object of the *non-necessary*, or *contingent*, predication for every entity. He did this by means of the emphasis placed on the *intimate link* – which does not mean identity, but rather *difference* of a *real*, and not merely logical, kind – between essence and being of an entity, and thus between *being of the essence*, or *entitas*, and *being of the existence*, or *common being* of the same entity. Some properties are essential for the connotation of an entity and are, accordingly, *universally* and *necessarily* bound to its definition, because they connote the proper mode of existence, or mode of existence *per se*, of an entity. They are, indeed, the effect of a set of causes thanks to which the entity itself has come to existence and is maintained in it with those properties. Other properties are *non-necessary*, or “contingent” for the

definition of an entity, and are thus logically *non universally* and *necessarily bound* to its definition, because they do not indicate the mode of existence of that entity *in itself*, but rather of some events (or “accidents”, in line with the literal translation from the Latin word *accidens*) that are inherent to it, in accordance with different causal situations of its existence. These latter properties, unlike the former, can vary in number and not only in their dimension as the former; namely they can be either gained or lost in time, without involving any change in the existence of a given entity according to its own mode, i.e., without involving its “essence”, or “nature”. In view of this, being and essence are reciprocally bound in the determination of the existence of an entity, according to its own mode, or essence.

The proposition “to be an entity” is underspecified because the notion of being is over-determined

In light of this, let us recall the meta-logical analysis of the function of the copula “is” in the proposition that we have introduced in the previous sub-section. There, when we maintained that to define something as an “entity”, i.e., as “what is”, leaves the proposition underspecified until the right hand side of the copula is defined – this being the essence, or the actual mode of existence of that entity – that was not because is *indeterminate*. This was not because the “being” is to be conceived as a sort of “very general genus”, or an abstract concept at the highest possible degree. Quite the reverse, the sentence remains indeterminate because *the being, taken in absolute terms, is an over-determined notion that contains in itself the principle of any other determination, or formal act*. This is the notion of being as act, i.e., as ‘act’ of every formal act and, accordingly, original act from which any other formal actuality (the being *this* or *that*) of each entity ultimately derives.

Therefore, the notion of “being” in light of its absolute *positive nature* that cannot include any negativity and, accordingly, any *difference* is inapt for limited intelligences like ours to define the existence of an entity which is limited in turn, and determined according to its specific *mode of existence*, i.e., according to its essence, which makes it different from the other entities that participate in the same being in accordance with other modes. To use the well-known remark introduced by Plato in his *7th Letter*, the expression “being an entity” is underspecified for our limited intellects, which aspire to know being, but are doomed to know it “in flashes” through the qualities. They are compelled, in other words, to know it in a predicative form through the construction and testing of assertions, and not in an intuitive form, as it could be done by an actually infinite intellect — and not only a potentially infinite one like ours is; an intellect which could embrace, in a single act, all of being and all entities with their reciprocal, infinite relations that mutually specify them.

Aquinas thus summarises the core of his metaphysical doctrine of act and potency applied to the real distinction between *being* and *essence*, for all physical, logical or spiritual entities:



It is thus clear that what I call *existence* is the actuality of all acts [therefore also of every form] (...). Nor should it be understood that something more formal could be added to that which I call existence, determining it, as act determines potency. *Nothing extraneous can be added to it*, since nothing other than non-being is extraneous to it, which can be neither form nor matter. Thus existence is not determined by another as potency is by act, but rather as *act is by potency* (*De Pot.*, VII, 2, ad 9, my emphasis).

Being is the most perfect of all: it relates to everything as *act*. Nothing has actuality if not *inasmuch as* it is: therefore being is what actualises all things and forms (*S.Th.*, I, 4, 1, ad 3, my emphasis).

It is clear that the First Entity (...), is infinite act, i.e., it has in itself the fullness of being, *insubordinate to any nature either of genus or of species* (...). Therefore, every entity that is after the first entity, *since it is not its being* [but only the being in its essence, i.e., the being of existence comes after the being of its essence and, accordingly, being and essence are not the same thing], has the being received in something [namely, in the essence with its both generic and specific components] *by means of which the being itself is reduced*: thus in every created entity, the nature of what participates in being is different from the participated being (...). Therefore, it is necessary that the participated being relates to the participating nature like act to potency (*Q. de Spir. Cr.*, 1. My emphasis).

We can find in all things two principles of which one is complementary to the other: the *relation (proportio)* between them is like that between potency and act, nothing can indeed be completed if not through its own act (*S.c.Gent.*, II,53,1283. My emphasis).

The Absolute Being as foundation does not include any difference

Therefore, if we cannot make a distinction between being and essence, we cannot but conceive the Absolute Being as what actually *contains in itself* every determination and, thus, also every difference, as if it were an absurd actual diversified infinity. This is the metaphysical nucleus of the doctrine of the *universal determinism* of matter in natural sciences; but also, in metaphysics, it is the core of such Neo-Parmenidean doctrines of being as static infinite actuality of every determination, as in the doctrine of being in Severino's metaphysics.

However, since every difference implies *negativity*, these doctrines seem to be contradictory, leading to the ultimate identification of being and *nothing* (see Basti & Perrone 1996). In short, they lead to the "absence of foundation" as the ultimate sceptical result of the modern "theory of

foundations” (see Giorello & Palombi 1997). Conversely, as it is well-known, the Absolute Actual Infinite, to be what it is, can be only Absolutely Simple, *qua* Subsistent Being, with no negativity, or potency (See § 5.5.5). The separation of being from essence, in the justification of the existence of every entity and, accordingly, in the justification of its *relative necessity to exist* —that is relative to the set of determinations that necessarily justify its limited existence — is the contribution that Thomism can offer to the modern crisis of foundations.

6.3 The being-essence distinction in metaphysics

6.3.1 Metaphysical foundation of causality

Necessity of a post-modern, new foundation of causality

In light of the historical presentation of Aristotelian and Thomistic thought that we have outlined in Chapter 5, we have seen that these metaphysics are characterized by a causal foundation both of the *essences* (Aristotle), and of *being* (Aquinas) in its double composition of *being of the essence* (*entitas*) and *being of existence* (common being, or being itself). The doctrine of the *real difference* between being and essence means, indeed, that — at least in the constitution of physical being — *essence and being are relative to two different causal orders*, namely physical and mathematical orders, reciprocally ordered in the sense that the former is included and founded in the latter. A post-modern re-reading of such metaphysics makes little sense if we do not also take into consideration a post-modern metaphysical re-foundation of the principle of causality.

Looking carefully, in the Preface to the *Prolegomena to Any Future Metaphysics*, Kant invited “all those who believe it worthwhile to study metaphysics to temporarily suspend their efforts” (Kant 1781, 3), until a solution, different from his, was found to Hume’s problem of an *a-priori* foundation of causal necessity¹⁵⁵. Now this time has come. The Kantian solution seems to be utterly unsuitable, beginning with its two chief ideas, i.e., formal logic and theoretical physics.

¹⁵⁵ Actually, Kant did not suggest suspending metaphysical work and research *tout court*, but rather he invited philosophers to abandon classical “dogmatic” metaphysics so as to embrace a modern “critical” one, which is reduced to an aprioristic foundation of the three metaphysical ideas of the *soul*, the *world* and *God*, as postulates of morality, or practical reason. We all well know what has been the sad story of these three ideas within Kantian modern metaphysics. Think only, for instance, of the “three masters of suspicion”, i.e., Marx, Nietzsche and Freud, who had only one goal, namely to denounce the alienating power of Kantian ethical formalism that was concealed behind the religious thought of a certain protestant Christianity in last century Germany — especially after its re-interpretation by the Hegelian right.

- Triple motivation: The need of a post-modern re-foundation of causality can be seen from three main converging perspectives:
1. From the need in physics to develop an adequate causal scheme after the theory of complex systems
 - ◆ First of all, from the viewpoint of the ontology of physical systems, in their paradigmatic manifestation – for modern physical sciences – as *dynamical systems*, we have seen that Humean and Kantian schematism of the principle of causality in physics — temporal antecedent–consequent + necessitating logical link between the two — is utterly unsuitable to causally justify the existence of ordered structures within physical processes characterised by *dynamical instability*. These are those processes in which the system in the long run “has lost memory” of the initial conditions, although in the short run for many of them a differential causal law holds – i.e., the following step exclusively depends on the previous one –, according to the original Newtonian idea, which lies at the basis of the modern scientific revolution (non-integratable and/or complex dynamical systems: Cf. 2.7.2).
 2. From the need in logic of an intensional foundation of logical necessity
 - ◆ In the second place, from a *logical-epistemological* viewpoint, we have seen that an adequate foundation of logical necessity in hypothetical-deductive demonstrative procedures – typical of modern mathematical and physical-mathematical sciences – after the demonstration, following Gödel’s theorems, of the impossibility of a formalist foundation of logical necessity (see what Quine said about this, § 4.4.1), requires a non-formalist, content-based foundation that is *intensional* (with an “*i*”) and not purely *extensional*. This is constitutive of logical necessity itself and, accordingly, of the notion of “being”, which cannot be reduced to the mere extensional notion of “existence”, that is to say to the “being” reduced to the mere copula of categorical statements — to say it in Kant’s words — i.e., to the “being” reduced to the pure “belonging to a logical class”, to say the same thing in Frege’s and Quine’s words, which are the words of logical empiricism and of so-called contemporary analytical philosophy (see § 4.5.2).
 3. From the historical need to better characterize Thomistic metaphysics, logic and epistemology in contrast with Platonic and Aristotelian ones from which they originate
 - ◆ In the third place, from a historical viewpoint, in Chapter Five, we found the synthesis between the two previous perspectives in Aristotelian *metaphysics and meta-logic*, and in its fullest realization by Thomas Aquinas. Indeed, the causal scheme *act-potency* applied to the pair *form-matter* (*forma ut actus*, i.e., “form as act”), which replaces the *proprium* of Aristotelian ontology against Platonic ontology, is designated as the natural substitute of the Humean-Kantian scheme of causality in the ontology of complex systems. Conversely, the attempt to make such Aristotelian ontology coherent by means of the

application of the same scheme of the pair *being-essence* (*esse ut actus*, i.e., “being as act”), which represents the *proprium* of Thomistic metaphysics against Platonic and Aristotelian ones, provides the missing *trait-d’union* between the causal ontology of essences in philosophy of nature (Aristotle), and the content-based, intensional and not extensional foundation of being in metaphysics and meta-logic (Plato).

Consequences of
such re-
examination for
philosophy of nature

Hence the need to investigate whether it is possible to establish a more systematic link, on the one hand, between the Aristotelian act-potency (form-matter) causality doctrine and the theory of complex systems in physical ontology; and, on the other hand, between the Thomistic act-potency (being-essence) doctrine and the principles of quantum-relativistic cosmology regarding the origin and the evolution of the universe in metaphysics. In a nutshell, we need to start asking more systematically the question about whether the principles of Aristotelian-Thomistic philosophy of nature, despite their having been *in the past* poles apart in early-modern physical-mathematical science, can put forward *nowadays* as valuable suggestions to the formal ontology of the physical being, which is being constructed building on 20th century conceptual revolutions in natural and logical-mathematical sciences. Here, particular attention should be paid to the *causality-determinism* issue in the metaphysics of physical being, and to the *truth-realism* issue in the meta-logic and epistemology of mathematical-physical sciences.

This last closing part of the work will be devoted to the most general issues. They will provide the background for a more detailed analysis of the same issues in relation to the three main domains of contemporary natural sciences – namely, *physical, biological and cognitive* sciences — and their objects — namely, *inorganic, organic and human* physical entities — which will be dealt with in the second volume of this work.

6.3.2 Aristotelian model of physical causality

6.3.2.1 The ‘four causes’ model

Some details about
the sense of the
analysis of the
consistency of the
Aristotelian-
Thomistic doctrine
of causality

Before proceeding further with this preliminary analysis of the consistency of the Aristotelian-Thomistic “act-potency” causal model, introducing the results obtained by contemporary physical-mathematical and logical-mathematical sciences, let me add an important consideration. In doing this, we do not intend to perform a defensive or culturally reactionary operation, as if to justify a return to a pre-modern philosophy of nature. In other words, we do not disregard the modern distinction between *philosophy of nature* and *sciences of nature* (Cf. § 1.1.) and, consequently — at the level of their respective objects:

- ◆ Between what physical being is and can be explained to be by means of the definition of certain causal factors, which determine both its being (essence and existence) and its becoming;
- ◆ And what empirically appears through the measurements of the related quantities, and can be explained by means of the definition of quantitative, mathematically formalizable, laws that govern the variations of such quantities and their relations.

For the same reason, we do not intend by any means to question the other fundamental modern idea of the *hypothetical-deductive method*, as “the method” of logical-mathematical and physical-mathematical modern sciences, against the illusion of the apodicticity of the early-modern principle of evidence (see § 4.2.2).

In general terms, what we want to deny is here merely the presumed arbitrariness of hypothesis in scientific demonstrations

What we want to confute of modern hypothetical-deductive epistemology is the presumed *arbitrariness* of hypotheses in scientific demonstrative procedures. Such a thesis is shared both by Neo-positivistic epistemology and by Popperian and Post-Popperian falsificationism. Moreover, this thesis — as we noticed in the Introduction (Cf. § 0.2.7.1) —, with a typical irony of history, links the “weak thought” of contemporary epistemology with the most fundamentalist position of Medieval Aristotelianism, in the interpretation of the hypothesis of physical-mathematical science with “mental fictions meant to save phenomena”, without any real foundation. Historically, this reduction ignores the fact that logic is not only the *logic of proof*, and thus axiomatic method, but also the *logic of invention / constitution* of hypotheses, and thus also analytical method. Furthermore, it ignores that, after Gödel, it is nonsense to go on conceiving logical systems as exclusively formal systems, i.e., as “closed” systems. The partial, and thus modular, interconnected, nature, which is open to the continuously changing contexts of the objects to which logical constructions are applied (Cellucci 1998), requires a post-modern (neither archaeological or apologetic) reconsideration of the Aristotelian-Thomistic epistemology of *adaequatio* (Cf. § 4.4).

Non-arbitrariness of the formulation of hypotheses:

According to this epistemology, logical links within and between propositions, with their different modes, or degrees, and types of necessity — therefore, with an integration between extensional and intensional logic — have a *real foundation* — i.e., are logical entities *cum fundamento in re*, according to classical terminology, differently from such imaginary entities as our “Arabian phoenix” — in the causal links between the entities that they denote. Accordingly, the formulation of hypotheses is not arbitrary in a three-fold sense:

1. Because it follows a logical method	◆ Because it follows rules and has its logical method, namely the analytical method, related to its typical forms of non-demonstrative inference (such as induction, abstraction, analogy, generalization, etc.).
2. Because it expands knowledge	◆ Because although the results it obtains with its inferences are never absolutely certain – and, after Gödel, this applies also to those of the axiomatic method and of deduction – they have some certainty only in relation to the <i>contexts of their application</i> , and are thus <i>capable of expanding knowledge</i> , unlike those deriving from the deductive inferences of formal systems.
3. Because it is founded on the being of objects	◆ Because such a formulation has a <i>veritative foundation</i> in the being of objects and of their reciprocal <i>real</i> (= causal) <i>relations</i> , which the hypotheses are meant to denote and/or connote in terms of <i>logical relations</i> .

Subject-predicate relation in language and substance-accident in reality

In view of this, in this context, what we seek to highlight is the continuity between the *ontology* and the *phenomenology* of physical being according to the Aristotelian-Thomistic scheme of (primary and secondary) *substance* and *accident* and its *causal foundation*, both at the level of the secondary substance (or “essence”, according to Aristotle) and at the level of primary substance (or “individual”, according to Aquinas). In other words, what is empirically evident of an individual (physical entity) is the *accident* of an *individual substance* (primary substance) and/or of a *species of substances* (secondary substance) and, according to the link of causal necessity that binds them, its *more or less essential properties*. In short, the logical-formal link between subject and predicate in the definition of a given scientific theory is the abstract expression of an underlying *ontological bond* between a metaphysical subject and one of its properties, i.e., between an individual substance (subsistent individual physical entity) and one of its events (accident), that is the outcome of a given causal action exercised on that physical entity by other physical entities.

In particular, in the light of our previous analysis:

Real foundation of the distinction between essential and accidental properties

- ◆ A property appears to be either *essential* (*per se*) or *accidental* (*per accidens*) for a given entity in accordance with its deriving from a combination of causes on which, in turn, the beginning of its existence, and its (relative) duration in the existence, either *depends* or *does not depend*.

Real foundation of the distinction between specific and individual properties

- ◆ A property is *specific* (i.e., shared by all the agents of the same species), or *individual*, according to whether, respectively, it derives or not from a *hierarchical* combination of causes, that is capable of making (or not making) the structural properties of a given entity inde-

pendent of its *intervening* history, and, thus, *indefinitely replicable* for a multiplicity of entities that share the same conditions.

Example of the specific properties of chemical elements and their compounds

This can be understood in view of the explanation of the properties of *identity/stability/specificity* of atoms and of molecular compounds — i.e., the bodies of our ordinary experience — as a modern exemplification of the *causal foundation of the stability of the* (not only, and not chiefly biological¹⁵⁶) *species*, i.e., of the several physical-chemical substances (secondary substances) that constitute the nature of the bodies (primary substances) of our ordinary experience, and that is the *proprium* of the Aristotelian philosophy of nature (Cf. above § 5.4.4)¹⁵⁷.

Necessity of overcoming the mechanistic scheme of physical causality

At the same time, both of these causal foundations underscore that they cannot be expressed through a model of causality based on the “dictatorship” of the initial conditions, as for the mechanistic model of Newtonian-Laplacian physics. In both cases, the – constitutive and substantive – causal action of certain structures cannot be limited to the beginning of the process, but it must be somehow *simultaneous* to the process itself, so as to appear to be fully intelligible to the human observer *only at the end*. We shall see shortly that it is precisely this overcoming of the mechanistic (Democritean) causal scheme that expresses the meaning of the Aristotelian doctrine of the four causes. At the same time, this explains the reason why many contemporary philosophers of science, subjected to the modern identification between *physical causality* and its *mechanistic model*, speak about a “violation of the causality principle” both in quantum physics and in the physics of complex systems.

Logical relations (laws) grounded in real relations (causes)

In light of this we can, accordingly, maintain that, metaphysically, the (*relatively*) *necessary logical law*, which governs the different predicative attributions in the deductive procedures typical of a certain physical theory, is an expression of an underpinning *causal process* among physical entities, that reveals itself at different ontological levels and according to different modes.

¹⁵⁶ This is due to the fact that, at a biological level, we need to take into account further factors that are linked to complex systems, about which there is no clarity concerning their relation to quantum laws.

¹⁵⁷ The statement according to which quantum mechanics of the atom represents the basis for a re-discovery of the Aristotelian theory of the *specificity* of bodies is shared by all 20th century Neo-scholastic philosophers of nature (see Wallace 1996; Artigas & Sanguinetti 1989; Selvaggi 1985; Masi 1960; Hoenen 1956).

Overcoming of nihilism and of relativism through the integration between the analytical and the axiomatic hypothetical-deductive methods, between intensional and extensional logic

Consequently, against the risks of epistemological irrationalism and metaphysical nihilism, following the collapse of the myth of the aprioristic apodicticity of early modern science and of its mechanistic background, we claim the need to integrate the hypothetical-deductive method, typical of the demonstrative character of modern science, with an adequate development of the analytical method (induction, analogy, abstraction), as the method for the *discovery* and *definition* of scientific hypotheses, that are suitable for the specific objects of research of the different disciplines, i.e., that are *true* within contexts and modes, which can certainly be defined, even though they are never absolute.

In this context, precisely because the *constitutivity of the logical languages of theories* is at stake, a purely extensional logical approach is unsatisfactory. We need to justify the domains of predicates. Therefore, the hypothetical-deductive *intensional* foundations of extensional** demonstrative procedures, including logical necessity, is brought into question at a *content-based*, intensional and pre-formal level of the meta-logical analysis of scientific theories.

With this important consideration in place, we can now proceed with the presentation of the Aristotelian doctrine of the four causes, typical of his philosophy of nature, through the elaboration of a definition of the notion of “cause”, that is apt for our ontological aims and, at the same time, that is the broadest possible one in order not to exclude *a priori* any possibility of research.



Definition 1

By *cause* we mean everything that determines the being – essence and existence – of the caused entity.

Generic definition of cause

This, very broad, definition of “cause” that is inspired by Aquinas’ – “all that influences the determination of the being of the caused, i.e., the cause” (see Thomas Aquinas, *In Met.*, V, i, 751) – is enough in order to introduce a discussion about the close relation between the notion of “cause” and that of “being”.

Formal ordering
cause-effect has to
be causally founded

The qualifying point made by the Aristotelian doctrine of the four causes in the ontology of the physical entities is that the *necessary formal ordering in a deterministic causal law* (i.e., double implication, or necessary and sufficient condition) of a cause (i.e., “efficient” entity/event) to its effect (produced entity/event), in general, has to be *causally founded* in turn. To say that the effect of a physical process is “in potency” in its initial causes¹⁵⁸ (i.e., “in active potency” in its efficient cause, and in “passive potency” in its material cause”) means precisely this.

Act-Potency: the effect is not implicit in the cause like a theorem in the potentiality axiom. The effect, i.e., the final state of affairs coming out of the process, has to be ordered to its causes; it is not “implicit”, i.e., ordered in a way that is necessary to them, as a theorem is necessarily ordered to the axioms from which it is deduced. This may sound awkward to a modern scholar, who, following Newton, is used to conceiving the differential causal law as the “the other side” of the formally integratable nature of every dynamical system. However, nowadays,

Its topicality after
Gödel's theorems,
Von Neumann's
quantum logic,
KAM's theorem

- ◆ After the discovery of Gödel's theorem in logic, and the relativization of logical necessity within demonstrative procedures, and, at the same time,
- ◆ After the demonstration offered by John Von Neumann that the logic of formalization in quantum physics necessarily implicates a violation of the principle of excluded third;
- ◆ After the discovery of the non-integratable nature of almost all dynamical systems of classical mechanics, despite their being differentiable, in light of KAM theory, i.e., after the discovery of the role of

¹⁵⁸ The essential epistemological and ontological difference between the “initial causes” of Aristotelian philosophy of nature and the “initial conditions” of modern physics-mathematics is the following. According to Aristotelianism, initial causes are entities (substances) and events (accidents) of the physical world, in which we need to ground *a posteriori* the necessary relation to the event and/or entity that is produced as an effect of its cause, thus abstracting (= inductive-abstractive analytical method) the logical law – which defines the hypothetical relation between initial and final conditions of the subsequent demonstrative procedure and/or calculation (= hypothetical-deductive method) – from the causal process itself. Accordingly, the initial causes are the “real foundation” of initial conditions. Put another way, the initial conditions (quantities) are the result of a “reductive abstraction” (measurements) of the initial causes (entities/physical events). The core of the integration between the (classical) analytical and the (modern) hypothetical method is that the unit of measurement depends in turn on the very process that is to be measured. This means that it is not fixed *a priori*, but that it is rather continuously adjusted so as to give to the functional relation – that is purely formal and defines the law – a semantic interpretation, or a “model”, that is constantly updated against the time series of the data that are to be represented through the model itself, as the calculation (i.e., demonstration) proceeds.

dynamic instabilities within complex physical, chemical and biological systems; in short, after it has become clear that in the behaviour of most dynamical systems, the structure of the final state of affairs of the process is not predicable from its initial conditions,

the Aristotelian scheme “act-potency” seems to be extremely interesting in order to define a new epistemology of physical cause, that is more comprehensive than the early-modern Humean-Kantian one.

Cause is not a category

The bond existing between the notion of cause and “act-potency” relation explains why, according to Aristotle, the notion of cause *is not a category*, i.e., it is not a first and irreducible predicate. It is, rather, the outcome of the composition between the categories of *relation*, *act* and *potency* taken from Aristotle’s table of categories (Cf. § 5.4.2).

The extension of Aristotelian *physica* corresponds to that of modern sciences, ‘intelligence’ excluded

Thus, we have presented the typical Aristotelian doctrine of the *four causes* (namely, efficient, or agent, material, final formal) that characterizes the Aristotelian-Thomistic epistemology of natural sciences, grouped under the generic label of *physica* or *philosophia naturalis*. However, it needs to be born in mind that, according to Aristotle, *physica*, as the “science of the entities that are capable of becoming, and thus are made of matter”, was a container embracing a number of disciplines including, in modern terms, cosmology, mechanics, thermodynamics, chemistry, biology and also neuro-physiology, thus only excluding the *operations of the intellect*⁵⁹. In light of their *immateriality*, these latter were studied by metaphysics (see Aristotle, *Phys.* II, 2, 194b, 15).

Cause does not equal efficient cause: four causes

What are then, in Aristotle’s view, the above-mentioned four causes? Whereas for the common sense of modern men, after Newton, “cause” has become a synonymous of “force”, or, more precisely, of “action”, i.e., of the application of a force for a certain amount of time so as to change the inertial status, either of rest or of motion, of a certain body, according to Aristotle (*Phys.*, II, 7, 198a, 14), the four causes represents four “*why*” (*dioti*) to which the natural philosopher has to answer in order to achieve a sufficient determination of the essence and of the being of a given natural entity, and /or of its becoming, whether that natural entity’s being is either a substance or an accident.

Aristotle’s causes are Aquinas’ “second causes”

From a Thomistic viewpoint, of course, such causes are merely “second causes”, since, in view of his immanentism, Aristotle was not aware of the notion of the “act of being” and, accordingly, was not aware of the need to make reference to the “First Cause”, as ultimate constitutive

⁵⁹ We can say that each of these subjects was studied by Aristotle in his naturalistic works, although it would be pretentious to seek to establish a bi-univocal correspondence between such Aristotelian works and the modern disciplines we have mentioned.

principle and, thus, as the “ultimate and fundamental *why*”, of the being of every entity (Cf. 5.5). Anyhow, let me present what Aristotle meant by the above-mentioned four causes, that are necessary to obtain a complete determination of physical processes¹⁶⁰, namely *agent* cause, *material* cause, *formal* cause and *final* cause.

The agent, or efficient, cause

Triple meaning of
the agent, or
efficient, cause

The *agent cause* can also be defined as “*efficient cause* of” (= “a cause having as an effect”):

- ◆ *Local motion* (= moving cause), thus being conceivable as the efficient cause of all the other forms of becoming of physical bodies, which have their foundation in the local motion of elementary particles that constitute the material substrate of a body, but which cannot be reduced to it, and are:
- ◆ *the intensive alteration* of qualitative properties of the material substrate of bodies and that correspond, in substance, to the *active-passive dispositions* of elementary particles constituting bodies to dynamically interact with the particles that constitute other physical bodies (= active-passive qualities, among which heat is the main one)¹⁶¹;
- ◆ *the extensive modification*¹⁶², or increase in the quantitative dimensions of bodies.

¹⁶⁰ By “process” we mean here any not-ordered collection of physical events, which are not yet sorted in terms of a law, or of a *mathematical function* through which they can be represented in an *ordered* way, and can be predicted (deduced) in their temporal development.

¹⁶¹ From this viewpoint, the “active-passive qualities” of bodies, from a phenomenological perspective, look very similar to the *charges* of the four fundamental (electromagnetic, gravitational, strong nuclear and weak nuclear) forces of the current physics of particles, which we shall study in Part Three. A “charge” can indeed be defined as “the disposition to interact” of the correspondent particle with other particles that share the same kind of charge, either in an electromagnetic, gravitational, strong or weak form. Moreover the similarities are even more evident if we consider that, according to the second principle of thermodynamics, all four forces can be reduced to heat. According to Aristotle, indeed, the sensible qualities of bodies (i.e., those that can be perceived by men) are considered those deriving from the four active-passive fundamental qualities (warm, cold (=active), wet and dry (= passive)). Coupling them according to different combinations they characterize the four fundamental elements (air, water, earth and fire) constituting the material substrate of all “earthly” physical bodies, since the “celestial ones” are only made up of the fifth element, i.e., ether, which fills in the cosmic space.

¹⁶² By means of the distinction between *extension* and *intension*, we refer here to two different kinds of quantification: the former is related to *cardinal* numbers (1, 2, 3...), through which we identify all continuous extensive quantities (= that can be infinitely divided: e.g., lines, spaces, volumes, but also weights, time extensions, etc.). The latter is relative to *ordinal numbers* (first, second, third, etc.) by which we define discrete scales of

Thus the agent cause, if taken in the former meaning of “efficient cause of a local motion”, or “moving cause”, corresponds, from a certain perspective, to the modern concept of “mechanical force”, or “locomotive force”, i.e., a force that is capable of provoking a local motion in the bodies that are subjected to it. What is different between these two notions of “moving cause” and “force” is that, while according to modern science force is essentially a quantity measuring the intensity of an “action” (which corresponds to the Aristotelian idea of *virtus*), according to Aristotelianism, the agent cause is *the active subject entity of the causal action*, i.e., the entity that has the necessary energy to exercise the above-mentioned force in time, thus determining an action.

The material cause

Meaning of the
material cause

The *material cause* can be defined as the *passive* material substrate of the action performed by the agent causality. Since it was considered by Aristotle, together with the moving cause, the second *initial cause* of a local motion of bodies, its modern equivalent is another particular quantity, i.e., the *position*, that varies in relation to the variations of the first quantity, namely the “force”. More precisely, in the equations of classical mechanics, *position* and *quantity of motion* (that is tightly bound to kinetic energy) constitute the so-called *canonical variables* of functions that represent the local motion of a body.

Moreover, if by “material cause” we mean, more generally, the “matter”, it is important to remember that the Aristotelian concept of matter does not have an appropriate equivalent in modern physics. In particular, it would be seriously mistaken to identify the Aristotelian notion of matter with such notions as that of *mass* – according to Newtonian mechanics – since in Aristotle’s view the “matter”, *qua* passive substrate of action from an agent cause, was represented by the *everlasting* movement of elements, which are constitutive of the material substrate of bodies in development (Cf. §5.4.1). In this everlasting motion, the external action of the agent cause brings about, in the first place, a certain *instability*, thus “corrupting” the previous “form” in which the motion of elements was previously ordered and, thus, also produces a *substantial, or accidental,*

quantities (= that cannot be infinitely divided, but in which there is always a *maximum* and a *minimum*, e.g., the ranking of a competition, a scale of temperatures, etc.). One of the emerging features of intensive quantities is that they do not enjoy the property of summation, i.e., they are not arithmetical quantities. Accordingly, whereas two extensive quantities can be added (e.g., a segment that is 2 *m*. long plus one that is 2 *cm* long equals a segment that is 2,02 *m* long), two intensive quantities cannot be added (e.g., a glass containing water at 10°C plus one containing water at 15°C equals two glasses of water at 12.5°C, rather than 25°C).

change of a given body, i.e., the “generation” of a new (substantial, or accidental) form of ordering of matter.

Eduction of form out of matter	Let me analyse further the Aristotelian principle of the <i>eduction</i> of a (material) form from a material substrate of elements in motion. The motion of elements, on which the agent cause acts in order to “educe”, i.e., to pull out, from such a material substrate a new form, may be:
Substantial form	◆ Either that which constitutes the material substrate of a physical body (primary substance). In this case, the form that orders it is the <i>substantial form</i> of that body, so that the substitution of this form by another one implies the shift from one substance to another one. Think, for instance, of the molecules in a piece of wood. Changing the substantial form of it (e.g., burning it, i.e., changing the atomical composition of its molecules), all the other physical-chemical properties (own accidents) will change (e.g., the piece of wood becomes coal). This is a <i>substantial</i> change.
Accidental form	◆ Or that which was the material substrate of a given physical state (accident) of a physical body (substance), or a set of them. In this case, the form that gave it an order was an <i>accidental form</i> of that body, or collection of bodies. Think, for instance, of the motion of molecules, respectively at a gaseous state (almost absent intermolecular links, molecular uncorrelated motions) or at a liquid state (weak intermolecular links, better correlated molecular motions), or at a solid state (strong intermolecular links, coherent molecular motions) of a substance like water. The change provoked either cooling down or warming up water is, accordingly, an accidental change of the same substance, since replacing an accidental form of the motion of the molecules of water with another accidental form (e.g., that related to the liquid state with that related to the gaseous state) the atomic constitution of water molecules does not change.
Function of heat and link to thermodynamics	But what is, according to Aristotle, the physical mechanism underpinning such changes? According to Aristotle, the particles that constitute the material substrate of bodies are endowed with “forces”, i.e., with “active-passive qualities” (essentially reducible to “heat”) through which they reciprocally interact. In this way, the <i>external</i> action performed by an agent cause, was capable, according to Aristotle, of inducing an <i>irreversible process</i> , having a <i>preferential direction</i> ¹⁶³ , towards a new stability, that can be

¹⁶³ The key difference between Aristotelian science of motions and their causes (dynamics) and Newtonian dynamics lies precisely here. Aristotelian dynamics is essentially thermodynamics because of the irreducible presence of heat within every mechanical process. Accordingly, the inertia principle does not apply to Aristotelian dynamics: frictions are irreducible, hence every physical process is characterized by an exchange of

more or less ordered in comparison with the previous one, so that the new (substantial or accidental) *form*, thus “educated” from the material substrate, could be defined by Aristotle as an *act* (*entelecheia*), i.e., literally, an “intrinsic end” of the motion of elements.

Material form as act and termination of an irreversible process of the material substrate. Link to the ideas of attractor and dissipative structure

In other words, it could be defined as the final stability effect of the motions of the substrate, irreversibly inducted from the action of an agent cause on the material substrate itself. The irreversibility of such a process of “generation” of a form from a substrate of elements in unstable motion is related to the constitutive role of heat in each of these processes. Moreover, the fact that the considered “form” appears thus to be a sort of “limit” (*περα*), or “threshold” (*ουδο*) intrinsic to the motions of the substrate (Cf. Aristotle, *Met.*, VIII, 2, 1042b, 25-27), which emerges as such only at the end of the process, justifies Aristotle in his definition of the *form* as a (non-intentional) “end of matter” in its becoming, as we shall see shortly. The link between Aristotelian *material form* to the notion of *attractor* in a thermodynamic, or “open”, system and, accordingly, to the notion of a *dissipative structure* is thus crystal clear (Cf. § 2.5.3).

At this stage we should be ready to analyse the other two causes, namely the *formal* and the *final* ones, which – as we shall see in detail in what follows – are not *de facto* different between themselves as to their *physical processes*, but only as to the *intentional* ones.

The formal–final cause

Formal and final cause

The *formal* and the *final causes* are certainly the two farthest causes from mechanicism and from the mathematical-analytical formalism of modern physics. But for this reason they are the most interesting for us, especially from an epistemological perspective, so as to understand the Thomistic notion of *abstraction* of the axioms, on which the demonstra-

heat. Therefore, as it is well-known – and as Aristotle constantly underscores in his works about physics, especially in his cosmology treatise, *De Caelo et de Mundo*, and in his chemistry-thermodynamics work, *De Generatione et Corruptione* – since heat always generates a preferential direction in the motions of bodies (i.e., the centrifugal motion towards the extremes of the volume in which the bodies are contained), all physical processes have a sense and a direction that cannot be modified. In other words, all bodies “irreversibly tend” towards something, i.e., towards a new stability (in mathematical terms, Aristotelian physical space is *a-isotropic*: all directions of motions are *not* equivalent, and accordingly, it is thus different from a geometrical space). This is the physical foundation of *finalism*, characterizing Aristotelian physics, which is not linked to any theological *a priori*. Conversely, Newtonian dynamics is based on the inertia principle and thus eliminates heat as the fundamental cause for the explanation of all physical processes. It eliminates preferential directions from the physical space, thus making physical space *isotropic* (all directions of motion are equivalent) and fully analogous to *geometrical space*.

tive procedure of a science is based. In light of this, we need to deal with them.

Formal and final causes are distinct only in the intentional order

To begin with, let me put forward a consideration that, if it had been kept in mind in the discussions between Scholastic philosophy and modern science, would have avoided many misunderstandings. Under the pressure of justifying a “providentialistic” view of nature, many scholars forgot to explain that, in Aristotle’s and Aquinas’ views, the formal and the final causes are *de facto* separate only within intentional – and not physical – processes. Indeed, only within an intentional order (e.g., when I want to grasp a certain object), the ‘end’ temporally precedes, in the human subject’s mind, the formal and the agent cause, since, in its being intentionally desired by the will, this *irreversibly orders* the sequence of physical actions that a human subject is supposed to perform so as to achieve the *intentionally desired* object (e.g., to stretch the hand and move the fingers, according to an ordered sequence, in order to reach and grab the desired object). Put another way, *finalism* is explicative only within an intentional order. As we shall see, this is a crucial point in order to verify the logical consistency of the Aristotelian doctrine of causality, in its difference from, say, the Neo-platonic one (Cf. § 6.3.2.3.).

This mainly applies to metaphysical finality in relation to God’s mind

Accordingly, considering the “mind of God”, i.e., in metaphysics, or theology, it is certainly correct to speak about an *intentional* finality, or *teleology*, of the universe – after having affirmed, in other ways (e.g., by means of a metaphysical demonstration, or through a postulate of faith in theology), the existence of a creating God – since God thought of it for a certain purpose (Galvan 1992b; 2000).

However, metaphysical intentional finalism cannot be used in physics to justify finalism at an ontic level

But if we used such a principle to justify in physics some form of explicative finalism, we would be guilty of a “shift of category”, besides denying ourselves the possibility of using a correct non-mechanicistic approach in metaphysics to the causality of the physical order — that Aristotle exemplified through the notion of formal–final cause, as distinct from the two “mechanical” causes, i.e., agent and material causes — as a *way* (i.e., the well-known Aquinas’ fifth way, namely that of the finalistic ordering of the universe, in relation to divine intentionality) to demonstrate God’s existence.

Three components of the notion of formal–final cause

Going back to the doctrine of the four causes, by formal–final cause — because, let me repeat following Aquinas, there is no distinction between final and formal causes within the physical order — Aristotle essentially meant two things:

1. Totality principle,
irreducible to the
sum of the parts

- ◆ The *intrinsic principle of ordering* which makes it possible for the *totality of the parts* in a body (i.e., physical entity), or in a proposition (logical entity), to be different from the mere *sum of the parts* (*Met.*, VII, 7, 1041b, 10-13)¹⁶⁴. To use an example given by Aristotle, the pronunciation of a word, e.g., “who”, is more than the mere sum of the pronunciation of its individual letters “double u – eich – o”. To say that a body made up of elements is the mere sum of its components means to say that the composing elements (e.g., atoms) remain independent individuals (= atomism) even after they have become a material component of a more “complex” entity (e.g., a molecule) and, accordingly, more “ontologically perfect”. But if the new more complex physical entity is “transcendentally” one (despite its being a substance or an accident), i.e., if it is a “new” irreducible *individual*, its individual uniqueness is strictly related to the formal unity characterizing many of its components and that makes it possible for them in turn not to be “many” anymore, but “one”¹⁶⁵.

Accordingly, the *transcendental unity* (Cf. § 5.6.2) of the existing individual is something that also modifies the *quantitative unities* through which quantifying the object.

Thus, if the new individual is characterised by the new “way of being together” of the parts that constitute it, the *form* is, accordingly, that intrinsic principle of totality, and of ordering of the parts, which *distinguishes* it from other entities and makes it *irreducible* to the sum of the composing parts, just like the word “who” is different from the sum of the composing letters “double u – eich – o”.

Conversely, in order to understand the difference between this Aristotelian principle and the basic principles of modern physics, it is necessary to pay attention to the hypothesis of the ultimate linearity of the fundamental laws of Newtonian mechanics, and thus the integrability of the dynamic systems that are objects of study, which leads to conclude precisely the reverse of what Aristotle maintains.

¹⁶⁴ Building on this simple consideration we can understand that the connectives of Aristotelian propositional logic are not merely the truth-functional connectives of extensional logic.

¹⁶⁵ In the philosophical essay *De Mixtione Elementorum*, Aquinas explains such formal unity of compounds by underscoring that that which characterizes them is that 1) all composing elements *instantaneously modify* their characteristic quantitative properties in accordance with their belonging to the totality; 2) in view of this, the elements within the compound are characterized by a “weak objectivity”, i.e., they *virtually* exist in compounds, and, specifically, are not univocally identifiable within them. The similarities with the phenomenology of the quantum states of the atom, or of matter, are again impressive: think, for instance, of (1) quantum *entanglement* and of (2) quantum non-locality.

Indeed, modern mechanistic atomism has — let me repeat once more— its own precise mathematical foundation, and is thus not at all ideological in its origin, but it may be as such in the philosophical speculation that derived from it.

2. Stable final state
of an irreversible
dynamic process

- ◆ The *final state*, that can be more or less ordered in comparison with that of the beginning, to which a physical process is irreversibly oriented.

The natural form is an end [*telos*]: only in this sense it is an “end”, as non-intentional end of a physical process] and a that-because-of-which (*to\ou#efeka*). Indeed, since the movement (of elements) is *continuous*, we also need an end and a that-in-cause-of-which such end is achieved [i.e., we need an ordering principle for the whole of agent causes towards that end: in this sense we can speak about a strictly bound “formal-final cause”] (Aristotle, *Phys.* II, 2, 194a, 27-29).

Aristotle's example
of the drop of water

The example that Aristotle gives elsewhere to explain what he is saying, namely the example of the drop of water, is very effective, not only in itself, but also for us, considering that the phenomena of turbulence in fluids and the transitions of phase (e.g., the changes of state, like gas state – liquid state) are two points where modern Newtonian physics reaches its intrinsic limits in its capacity for determining phenomena. Think, as Aristotle asks us to do, of the particles of water in steam: they move frenetically and without “ends” that contain them, trying to occupy all available space (e.g., on the surface of the glass of a window, i.e., the system follows the ergodic hypothesis. Cf. above § 2.7.2).

As soon as another appropriate causal action is performed (e.g., the temperature is diminished), these particles get quickly condensed into a few drops (ergodicity is broken). However, Aristotle invites us to be careful, since within the drop, the particles of water keep on moving. But, although all individual particles composing the drop keep on moving frenetically, such motions *reciprocally limit themselves* (see Aristotle, *Phys.*, IV, 5, 212a, 34 and Aquinas' commentary, *In Phys.*, IV, vii, 479, 482).

A “limit” (*peras*, “an irreversible ending”) thus *irreversibly* emerges from the process itself, that is a *telos*, of the ongoing motion of particles, which confines it within the regular volume of the drop. In the terms of the dynamics of complex systems, in the phase space of the system, several *attractors* (chaotic attractors) of the dynamics got structured (Cf. §2.6.2). The “form” is not only an ordered state, end of the motions of the material constituent, but also an end irreversibly ordering (towards itself, as if it were a “goal” of an intentional kind) the physical process that generated it.

3. Catastrophic effect non-proportioned to the cause

- ◆ The “catastrophic” effect of a *small change* of the initial agent and material causes. This confirms that the Aristotelian concept of “formal cause” was deeply bound, as well as to its necessary condition, to what we now define in physics as dynamic instability, typical of “non-linear systems”. The characterization in modern physics of dynamic instability is precisely the following: a small change in the initial conditions of dynamics is able to produce absolutely unpredictable catastrophic effects (Cf. 2.6.1). This is what Aristotle says about this:

Small changes cause larger ones, not in themselves, but when they occur with a change of ‘principle’. Indeed, principles are small in their dimensions, but large for their potentialities and, accordingly, this is what makes them principles: being the cause of several things and having nothing higher than themselves (Aristotle, *De Gen. An.*, V, 788a, 10-15).

Form as the intrinsic end and limit of an irreversible process, that is unpredictable from initial causes

The natural form of a physical entity, or event (= bodily form or material form), is thus an *end*, an intrinsic *limit* (this is the real meaning of *entelechy* in Aristotle, to which the Latin term *actus*, i.e., the act of the form, corresponds) of the motions of the unstable material substrate of entities and, at the same time, “that-in-cause-of-which” such end is reached by motions. This is the Aristotelian formal-final cause in physics; it is thus striking to notice the difference between the “form” of the Aristotelian drop of water and the ordered and geometrically predictable motions of the planetary orbits in modern physics, or between this “form” and the “cold” geometries of a snowflake. In Aristotelian physics, such “form” is not deducible from any universal law that can fully determine *a priori* the process (e.g., who would be able to make the steam on a window re-condense in the same way, notwithstanding any attempt to exactly reproduce the initial conditions?); it emerges only as the *necessary end of an irreversible process*.

In other words, formal cause as non-intentional end, or “natural end” of a process

Therefore, the formal cause is an end, or, more precisely, the *non-intentional end of a process*, because it consists of a concurrent set of factors which *irreversibly* orders the causal process of the initial agent causes (i.e., the initial external agent cause, and the following causal interactions of material particles through the “active-passive qualities” which bind them) and material causes towards the necessary achievement of the effect, namely of a final (more or less) ordered stability¹⁶⁶.

¹⁶⁶ In order to produce a certain effect it is not enough to focus on a set of agent and material causes, but rather it is necessary to consider the *ordering* of such causes. For instance, in order to produce a fire it is not enough to have a match, some straw and a puff of air from my mouth. If the puff of air precedes the touching of the match to the straw,

The formal-final cause does not produce anything, it is not an efficient cause

However, such form does not produce anything: it is the efficient cause of nothing, unlike Platonic forms (e.g., human soul) which exercise a ‘motor’ action (i.e., they move particles) on matter. It expresses merely the need that a set of factors, that is different from the initial causes, produces the final effect. The connections between these principles of Aristotelian physics and the so-called “complex” physical processes and the principles of non-linear thermodynamics have been noted by several scholars and appear to be well grounded.

For an adequate understanding of the logical-epistemological consistency of the Aristotelian formal-final cause, a point needs to be made. The order is a “form” that emerges from the “end”, not from the “beginning” of the causal process, building on the initial action of the agent on a material substrate. Accordingly, this will be an *a posteriori* order of the physical process, and not an *a priori* one.

Cause-effect a *posteriori* ordering, foundation of the logical antecedent-consequent implication that expresses the causal relation in terms of a law

Hence a qualifying point of the Aristotelian theory of the four causes, which – as we shall see – makes the logical scheme underpinning his theory of final causality totally different from the Neo-platonic one, in particular from Plotinus’. While in Plotinus’ theory the final cause is that which *also produces the effect* of the existence of the entity (here *efficient cause and final cause are identical*) — think of the theory of the *necessary* emanation–return of the finite entity from-to the One — in Aristotelian theory the efficient cause and the final cause are separate. The final state, or “effect”, *exists* thanks to the initial agent–efficient cause, but the *order*, or the necessary link between an efficient cause and its effect depends on the effect, i.e., on the final state. This is the meaning of the well-known Aristotelian and Thomistic doctrine, placing the final cause as the meta-physical foundation of the *necessity* of the causal relation between the agent and the effect that is produced by its action. This necessity is expressed by the subsequent causal law. In short, if the agent cause grounds the existence of the final state, it is only building on the final state that the *formality of the causal implication* (*ratio causalitatis*) can be grounded. Let me report Aquinas’ words on this:



It needs to be noted that there is a certain order among the four causes, according to which a clear correspondence is established between material and formal cause, on the one hand, and efficient and final cause, on the other. Indeed, there is a correspondence between the efficient and final cause, because the efficient cause

I shall blow out the match and the only thing I produce is a blown-out match. If the puff of air *follows* the touching of the match to the straw, a fire will start. After having discovered this order-relation, nothing prevents me from formalizing it in terms of a “universal law of fires”, building on two *initial* agent causes (i.e., the match and puff of air) and one material cause (i.e., the straw).

starts the ‘becoming’ and the final cause ends it. Equally, there is a correspondence between matter and form: indeed, the form gives being (*dat esse*) to matter [for instance, it makes it possible that chlorine and sodium *are* sodium chloride, if the motions of the constituent particles – in particular, of the so-called valence electrons– are ordered in such a way that makes them “turn” around the nuclei of the two atoms, instead of one alone] and matter receives it [this means that the order that emerges from the physical process determines the material particles in motion to be the substrate of some given entity, and not of some other one]. The efficient cause is the cause of the final cause in relation to being [*quantum ad esse*: i.e., *makes the final cause be*; since it emerges during and at the end of a process started by the agent cause, the existence of the formal-final cause depends on the agent cause], indeed, by its action, the efficient cause makes the end real. Conversely, the end is not the cause of the agent cause in terms of being, but only in terms of the relation of causal implication [the ordering of the whole of agent causes makes it possible that, at the end of the process, a certain effect – and not any other – is produced from the agent causes themselves: in other words, it is the final cause that makes the agent cause an “efficient” cause, i.e., that makes it capable of *necessarily* producing a certain effect, and not any other]. Indeed, the agent is cause inasmuch as it acts, but it acts [producing a certain effect] only thanks to the end. Therefore, the agent is made “efficient” [i.e., capable of *necessarily* producing a given effect] only by the final cause (Thomas Aquinas, *De Pot.*, 5, 1).

6.3.2.2 Analysis of the model

Relevance of
Aristotelian doctrine
for the revision of
modern causal
scheme

Let me try to further explain this fundamental point in Aristotelian-Thomistic epistemology, in which the alternative to the Humean-Kantian scheme of causality is concealed. Such an alternative has nowadays become urgent in a two-fold sense:

- ◆ In the sense that the causal antecedent-consequent temporal scheme cannot be applied to the causal explanation of complex systems, considering that the “ordered” final state does not univocally depend on the initial conditions.
- ◆ In the sense that the necessitating logical link between antecedent and consequent characterizing that scheme has to be grounded in turn, considering that, after Gödel, logical necessity is not capable of grounding anything, at least in terms of that which is ‘finite’ – and all physical systems are finite.

The problem of the foundation of logical necessity was in place also in past times

Especially in this last sense, current issues are very similar to what in past times was known, also by Aquinas, as the problem of the Megaric historical critique of the validity of Aristotelian syllogistics in natural sciences. The issue at stake is the *justification of the necessity of the causal link* cause–effect in natural sciences.

Thomistic analysis of Aristotelian doctrine and distinction in physics between stable and unstable processes

This issue is presented by Aquinas in a well-known part of his commentary on Aristotle's *Physics* (see Aristotle, *Phys.*, II, 7, 198a, 14-26 and Thomas Aquinas, *In Phys.*, II, xi, 242-248) in the following way, thus essentially anticipating the modern distinction between stable processes in equilibrium and stable processes outside equilibrium, such as dissipative structures.

Literally following Aristotle, Aquinas maintains that in order to fully determinate a process in physics, we do not always need all the four causes, namely agent, material, formal and final cause.

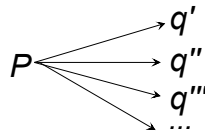
The case of stable systems and the apodictic nature of physical laws that concern these

In fact, in some cases the two *initial causes* (i.e., agent and material cause) of a physical process are enough to *univocally* determine the final state of the process itself. Here, there is no need to make reference to a further formal–final cause in order to justify the deterministic link, but, rather, the four causes are essentially reduced to two of them. Clearly, these are the two initial causes of (Democritean) mechanicism, i.e., moving and material cause, on the basis of which, as Aquinas notes, the demonstrative procedure in physics takes the shape of a perfectly *deductive* procedure from *apodictic* premises. We can say that the causal model of all stable physical systems can be individuated in this former case. These are the models of classical mechanics in particular, and, accordingly, we can find here the whole “dream” of early-modern Newtonian science, which aspired to reduce all dynamical systems to integrable systems.

Characterization of complex physical systems in Aquinas

However, as Aquinas maintains, in most of the cases studied by natural sciences this is not true: the same multiplicity of initial (agent-material) causes does not univocally determine a single final state, but it can, rather, determine a multiplicity of final *possible* states, that cannot be *a priori* predicted, each of which is, accordingly, produced by those initial causes with a certain *frequency* (*frequenter*). Moreover, Aquinas notes, such ambiguity of the relation cause-effect, if considered as to the initial causes alone, is removed by the physical process itself, in the sense that anything happens, *when it has happened* – i.e., *a posteriori* – it will have *necessarily* happened, as something causally necessitated. Put another way, this means that if a certain multiplicity of initial causes, say **P**, will be able to produce not only an effect *q*, but a multiplicity **Q** of effects, then it will be the physical process itself, on the basis of the produced effect, to specify the two multiplicities **P** and **Q** and their *relations*.

In modern terms, the process itself specifies the two multiplicities, changing them, from *generic non-ordered collections* $\mathbf{P}$ and $\mathbf{Q}$, into two *ordered systems* $\mathbf{P}(\preceq)$, $\mathbf{Q}(\preceq)$ with their reciprocal relations, that can be mathematically formalized as functions. Therefore, it is the actually produced final state that has a *specifying* and *ordering* effect on the two collections of causes-effects that are to be made correspondent. This is the work done by the final-formal cause, that is to be added to the two initial causes in order to make the process perfectly *decidable*, or univocally determined. In modern terms, it is like the physical process itself determines, in relation to the successor actually produced by the process itself, the *functional relation* $y = f(x)$ which is to represent it logically in a necessitating, and thus *univocal*, or at most, *bi-conditional* way. In short, the causal relation:



will be *ambiguous* if and only if we consider $\mathbf{P}$ a sub-specified collection of initial causes, which can be specified and ordered – i.e., changed into an ordered set – only *a posteriori*, in relation to the collection of effects $\mathbf{Q}$. In other words, this would be the case if we redefined every p belonging to $\mathbf{P}$, i.e., $p \in \mathbf{P}$, in relation to every $q \in \mathbf{Q}$ and the other way around, i.e., if we demonstrated their reciprocal *convenience*, so as to obtain that *bi-conditionality* of the relation ($p \equiv q : \ll q$ if and only if $p \gg$) that determines every p in relation to q and the other way around, as is expected of a *necessary and bi-univocal* (= double implication) causal relation of a deterministic kind.

Necessarily
hypothetical nature
of related physical
laws

Hence Aquinas' response to the Stoics' criticism of Aristotle. In Aquinas' view, in this kind of physical processes, the demonstrative procedure of natural sciences does not take the *apodictic* shape of the deductive demonstration used in the previous case, in which the two initial causes were enough to univocally determine the final state. It should, rather, take the *hypothetical* form "if p then q " ($p \supset q$), where p and q are two descriptive statements referring, respectively, to the initial and the final state of a physical process. The problem is that I cannot know *a priori* which q a certain p necessarily implies, and/or, conversely, from which p a certain q was produced. A chaotic system is a perfect example of this double indecidability.

Difference between
Thomasian and
Leibnizian
foundation of the
double causal
necessitation
(necessary and
sufficient condition)

The core of Aquinas' argument is the following: whereas in logic the truth of the consequence cannot ground the truth of the premise (= *falsehood of the consequent*; Cf. § 4.2.2.3) because what is true can be implied from what is false, in physics the existence of the effect *necessarily supposes* the existence of the cause, so that the relation of material implication $p \supset q$ becomes reversible. Such relation is established between the categorical statements p and q , respectively connoting the initial and final state of the process, so as to inductively justify, in relation to a *model applied to an experimental case* of a general hypothetical law, the desired bi-conditional relation: $p \equiv q$. The *ontological necessity* (= cause) grounds the *logical necessity* (= law), without supposing any *mathematical law of symmetry* given *a priori*, like in modern epistemology from Leibniz onwards, in compliance with the Aristotelian motto according to which necessity is greater than logical necessity:

What is necessary has a greater extension than simply that of a syllogism: indeed, every syllogism is necessary, but a syllogism does not fully represent what is necessary (*An. Pr.*, 47a, 33-35).

To accrue the above-mentioned specification, it is necessary, *in the first place*, that the *a posteriori* observation of the process grounds a peculiar form of *hypothetical* reasoning, say $(p^* \supset q^*)$, where the *protasis*, or the *antecedent*, p^* , is not the descriptive statement of the set of the initial causes of a certain process, and the *apodosis*, or the *consequent*, q^* , is not the descriptive statement of the final state of the process, but rather the other way around. Since in such processes the logical constitutivity (necessity-universality) of the connective of the causal law that is to be defined ($p \supset q$) essentially depends on the final state that is actually achieved by the process, it is clear that the two demonstrative forms of the hypothetical procedure, the so-called *modus ponendo ponens* and *modus tollendo tollens* of Stoic propositional logic, can be validly applied if and only if we shall consider as antecedent, i.e., as the *sufficient* condition, the description of the *observed* final state, p^* , of the physical process, and as consequent, i.e., as the *necessary* condition, the description of the initial causes q^* . The “physical” evidence of a causal relation that is always experimentally achievable – e.g., that according to which, in a single causal process, when we deny the cause we also deny the effect – has in this case the logically valid form of the *modus tollens*, namely “every effect p^* implies that its cause q^* happened. The absence of the cause q^* implies the absence of the effect p^* ”. In symbols:

$$((p^* \supset q^*) \cdot \sim q^*) \supset \sim p^*$$

And, consequently, the assertion that q^* is the cause of p^* has the logically valid form of the *modus ponens*, namely “every effect p^* implies that

its cause q^* happened. The presence of the effect p^* implies the presence of the cause q^* . In symbols:

$$((p^* \supset q^*) \cdot p^*) \supset q^*$$

Therefore, in all non-univocal, or “probabilistic” causal processes – i.e., those in which a multiplicity of initial causes *frequenter* produces certain effects – the hypothetical law that represents them in physical science necessarily has to have, at a first stage, as its protasis (i.e., premise) the description of the final state, actually achieved by the process; and as its apodosis (i.e., consequence) the description of the initial causes that necessarily produced it. Thus, the collection of initial causes, that was previously under-specified, receives its ultimate determination from the process itself, and from its final outcome. In a nutshell, the final state establishes *a posteriori* the *constitutivity* of the logical relation of implication in the definition of a physical hypothetical law, which, only *after* the application of the *modus tollens* - *modus ponens* in inverted terms, can take its *a priori* “normal” shape (i.e., one where the initial causes are the protasis, and the produced effect is the apodosis).

Relation to
abstraction in
epistemology and
constitutive
induction in logic

In other words, what is logically *a priori* in the demonstrative procedure of a science can only derive *a posteriori* from the observation of the physical process, so as to make it universally definable and reproducible. The bi-conditionality of the relation that grounds the *necessity* of the causal law ($p \equiv q$: “ q if and only if p ”) can be founded only through the reference to the single real process R : $p \equiv_R q$. Such a constitution of the

logical connective of the law on the single *res*, which is to be extended to other similar cases, is what characterises, in epistemology, the Aristotelian – Thomistic notion of *abstraction*, and, in logic, the procedure of *constitutive induction*. *Hypotheses*, far from being arbitrarily decided so as to apply them to the real world through a subsequent *act of interpretation* of the abstract formal relation as *models* of that reality, are built on the reality itself.

As it can be noticed, and as we have anticipated in the Introduction, Aquinas was one of the first critics to confute the ideological interpretation of Aristotelian philosophy of nature, which aims to turn hypotheses, in physical-mathematical sciences, into pure “fictions of the mind meant to save phenomena”. In a nutshell, the *esse* of the universal logical entity (statement) is *constitutively* grounded in the *esse (entitas)* of the individual *res* (event). The intellect is not the measure of things, as Sophists

(in past times) and Neo-positivists (in modern times) contend, but the things are the measure of intellect¹⁶⁷.

In Aquinas' words:



In axiomatic sciences the necessary is constituted *a priori*, like when we say that if the definition of a right angle is such, it is necessary that the triangle is such, i.e., that it has three angles that equal two right angles. Indeed, from what comes first (*ex illo ergo priori*) and is assumed as a principle, the conclusion necessarily derives [= if the premise is true, the conclusion is also true: *modus ponens*].

But from this we cannot derive the opposite, namely that if the conclusion is (true), then the principle is also true [= fallacy of the consequent], since sometimes from false premises we can infer a true conclusion [= the material implication of the Stoic logic of hypothetical reasoning in its most “disturbing” aspect]. However, this does not change the fact that if the conclusion is false, the premise is necessarily false, since what is false can only be inferred from what is also false [= *modus tollens*].

However, in those things that happen because of something (*propter aliquid*), either according to technology or according to nature, the above-mentioned opposite case may apply: since if the final state is, or will be, it is necessary that what comes before the final state is, or has been. Indeed, if what comes before the final state is not, the final state is not as well: the same applies to demonstrative propositions, i.e., if there is no conclusion, there is no principle [= *modus tollens*, but applied to the inversion antecedent/consequent that characterizes the demonstration *propter aliquid*, i.e., by means of the final cause].

In other words, it is clear that in what happens because of something the *final state has the same order that applies to the principle in demonstrative procedures*. This is the case because the end is in fact a principle, namely not a principle of action, but of reasoning. Indeed, *building on the end we start reasoning about the things that are related to the end* [= procedure for the formulation of the law, by means of the reciprocal adaptation premise/conclusion in order to constitute their reciprocal *convenientia*, and, thus, the logical truth of the implication] and in demonstrative procedures we are not interested in action, but rather in reasoning, since in demonstrative procedures there are no actions, but only reasoning. Accordingly, it is convenient that the end, in those things that happen in rela-

¹⁶⁷ “Things relate to our intellect *qua causes*, because our intellect depends on them. Thus our science does not ‘measure’ things [i.e., does not endow them with criteria], but is measured by them, like the Philosopher argues in the tenth book of his *Metaphysics* (1, 1053a, 31 -1053b, 3; 6, 1057a, 8-11)” (Thomas Aquinas, *In I Sent.*, XIX, 5, 2, ad secundum).

tion to a final state (*propter finem*), occupies the place of the principle in the consequent demonstrative procedures. Therefore, the similarity (between natural processes and demonstrative procedures) applies to both sides, although there is an inversion in the relation between the two of them, since the end is the last in action, but not in demonstration (*In Phys.*, II,xv,273).

The formal and material cause, intrinsic to every physical entity (body), constitute its essence as made up of form and matter

In brief, according to Aquinas, the necessity of the relation cause-effect in scientific determination, in terms of a hypothetical law of a physical process, can be grounded, in principle, only *a posteriori*, building on the irreversible achieved effect, in relation to which only a certain multiplicity of the initial events of a process properly receives the determination of initial “agent causes” of that process, by which the final effect is *necessarily* reached. Therefore, as a consequence, the nature, or the essence, of the contingent physical entity/event, deterministically produced by those “causes”, is constituted by a material substrate and a form, i.e., by a material cause and a formal cause. Accordingly, these are the *internal* causes of the existence of that entity, just like the agent cause and the material cause are the *initial* causes of a physical process, i.e., the causes that have brought about the existence of that entity, as their final effect.

In other words, the existence of a certain *material form* depends on the action of an agent on matter. Such material form can be expressed in terms of a principle of internal organization of the material substrate (= the stability of the motions of the substrate) of the entity produced at the end of the process of “eduction” of this form from the substrate. Therefore, it is the form itself, *qua* actually produced by the same initial causes of the process, that *univocally* – i.e., necessarily – determines the causal *link* between the effect and the initial agent cause, so as to make it possible to universally define a *law* that represents the process itself in *inferential* (i.e., hypothetical-deductive) terms.

6.3.2.3 Consistency of the model

Final causality of Neo-Platonism and strong version of the anthropic principle

In light of the analysis so far, the logical and ontological consistency of the Aristotelian model of causality is confirmed by the results of the study of the consistency of final causality in classical and modern thought conducted by one of the best Italian logicians, Sergio Galvan (Galvan 1992a,b; 1997). The results of this study have been recently reported by Galvan in an essay (Galvan 2000) published in a book that we have already mentioned several times, i.e., *Determinismo e complessità* (Arecchi 2000). This essay is particularly useful for our purposes, since it proposes a critical analysis of the causality principle in light of the results of the theory of complex systems and dynamic instabilities — applied,

in particular, to the so-called “strong version” of the *anthropic principle* (Cf. 4.1.) —, which is the same context in which this work is situated.

Character of
efficient causality of
the end within
Plotinian doctrine as
paradigm of every
ontic, non-
intentional, finality

The core of Galvan’s analysis is the critique of the Neo-Platonic – in particular Plotinian – notion of “final cause”, according to which through the doctrine of the *necessary emanation* of all entities from the One and of the *necessary return* of all entities to the One *qua* Good, the efficient cause and the final cause are *de facto* equated. The Good, i.e., the ‘Should Be’, according to Plotinus, has an original explanatory force, because it takes from being everything that is worthy of being. Thus, final causality acquires an explanatory value at an *ontic* level. Put another way, on the basis of the analysis of such a causal scheme made by J. Leslie (Leslie 1989), in the approach of Plotinian cosmology, like in that of the “*strong anthropic principle*” in quantum-relativistic cosmology, what Leslie terms the “ethical creationistic requisite” (ECR) is in force. Such a scheme in the two cosmologies is thus formalized by Galvan:



This is a genuinely finalistic explicit argumentation, which, presented in full and applied to our world, becomes: 1) our universe is a positive world, since it contains intelligent forms of life; 2) what is positive must exist; 3) what is positive does exist, therefore 4) our universe exists. Moreover, (and here we move from Plotinus to Barrow): 5) the existence of our world implies the occurrence, in the initial moments of the universe, of anthropic coincidences; therefore 6) it is clear that in the initial moments of the universe such anthropic coincidences occurred (Galvan 2000, 227).

According to the
Neo-platonic model,
the ordering to the
end is a sufficient
condition so as to
produce an effect.

Accordingly, the core of every finalistic explanation is *the attribution of efficacy* “to the order towards the end, so as to find in the end the cause of the event” (*Ibid.*). In other words, the core of the explanatory use of finality is the equivalence between the efficient cause (i.e., the capacity for producing events) and the final cause (i.e., the order towards the end). As Galvan promptly notes, this is absent in the doctrine of Aristotelian four causes, in which the efficient cause is separate from the final cause. In short, in the finalistic scheme that attributes an efficient causality to the end, a “principle of deontic reflectivity” is at work:

$$OA \supset A$$

Logical
inconsistency of this
model

Where O is a modal operator of deontic necessity which, when applied to its subject, determines its “should be”, and the reflectivity of the relation expressed by the principle is given by the fact that it is enough to apply the operator to the subject in order to obtain as a result the necessary existence of the subject itself. The serious logical mistake that is concealed in the way of reasoning proposed by ECR is the confusion of

axiological positivity (the “should be” of a value that thus acquires the role of an end) with *ontic necessity* (i.e., the “necessity to exist” of a given entity/event). Indeed, at an ontic level, it is not only formally possible that the best does not come to existence, but it is also highly unlikely that it happens without the interference of other factors. Statistically speaking, in the context of the application in physics of such theories:

- ◆ that which expresses the *high improbability* of the occurrence of an event (i.e., what determines its “should be”, or final causality)
- ◆ cannot be also that which *makes it probable* (i.e., what determines its “necessity to exist”, or efficient causality).

Violation of Hume's law of “facts/values” dichotomy

In philosophical terms, the “principle of deontic reflectivity” operates an consequential confusion between facts and values, thus going against the fundamental “law of Hume” about the facts-values dichotomy, according to which it is impossible to logically derive normative propositions (containing *deontic* modes, i.e., deontic logic) from descriptive propositions (containing *alethic* modes, i.e., alethic logic) and the other way around, as it is in this case.

Consistency of the generating mechanism of a chaotic attractor with the separation between causal efficient and formal factors (of ordering)

If we apply Galvan's scheme to our considerations, it becomes clear, in chaotic systems, the presence, at the level of the statistics of the system dynamic evolution, of the distinction between:

- ◆ causal factors that make it unlikely to obtain a final state that is ordered, or attractor of the dynamics, i.e., the principle of mechanics that lies at the basis of the phenomenon of dynamic instability (like in the metaphor of the “transformation of the baker” – Cf. § 2.6.2 – this is the “stretching” of the space of phase with the consequent divergence of trajectories);
- ◆ the factors that make it likely to achieve an ordered final state (strange attractor) that is totally unpredictable, i.e., the thermodynamic principle that lies at the basis of the dissipative phenomenon of the ‘coming back’ of the trajectories on a further dimension of the space of phase (in the metaphor of the baker, this is the “withdraw into itself” of the space of phase). This is a separate dimension from those in relation to which the trajectories themselves go further away.

Causality in complex systems

Accordingly, as Galvan correctly notes concerning complex phenomena, we should be careful not to mix up these new causal models imposed by the study of complexities – that are different from the

Humean-Kantian-Hempel model¹⁶⁸, which identifies the efficient cause in the initial conditions alone – with a finalistic explanation, in which the two above-mentioned orders of factors are systematically confused, if not made identical. At most, according to Galvan, what is at work in the study of complex phenomena is a principle of “backwards-causation”. Namely, let me add, a principle of ‘necessitation to exist’ of an ordered final state (attractor), or a principle ‘disrupting the ergodicity of the system’ (Cf. 2.7.2), that is linked to factors that are independent from the initial conditions — this is the dissipative mechanism for the ‘coming back’ of the initial trajectories, according to an irreversible path, totally unpredictable by relying only on the initial conditions.

An example not of final causality, but of backwards-causation

In other words, within complex systems, the mechanistic model of causality is overcome. However, this has nothing to do with a Neo-Platonic model of finalistic causality. This confirms again that the Aristotelian model of causality – especially according to the Thomistic analysis we have just presented (see, in particular, the quotation at p. 453) – constitutes a good metaphysical antecedent to the current physical theory of complex systems.

Explicative finality only at an intentional level, since the efficient cause is not the end, but rather the intentional subject

Conversely, it is at an *intentional level* that finality acquires an explanatory value. By means of the influence of an *epistemic factor*, connected to the conscious knowledge of a subject, the end intended by that subject acquires an explanatory value. But again, as in the previous case, we are not faced here with a genuine agent causality of the end, since the element of efficient causality (i.e., the conscious agent subject) and the element of order towards the end (i.e., the awareness of the positive nature of a certain desired event) are well individuated and connected only through the epistemic factor. As long as a certain event is consciously intended by a subject as the end towards which ordering a given sequence of actions that productively depends on the agent subject, the intended end acquires an explanatory value. The sequence of events leading to the final event is not produced by the end, but rather by the agent subject which remains the sole efficient cause.

Formally, the scheme of an intentional action is very similar to the ECR scheme:

$$IA \supset A$$

Intentional finality scheme

The intention of A produces A: this is the scheme of any kind of intentional causality. The peculiarity is that in this scheme it is not the ‘should

¹⁶⁸ In fact, Galvan explicitly refers to an Hempelian model of causality — (Carl Gustav Hempel (1905–) — which, to our ends, we can consider an axiomatization of the Kantian – Humean model, and hence substantially equivalent to it.

be' of the end that determines its realization, but the fact that it is thus intended by the agent subject. The end certainly plays an explanatory role, but only because it is intended as such by the agent subject, who is thus the sole efficient cause. The explanatory role of the end is thus mediated through the epistemic, cognitive, factor, without originating any confusion between the axiological and the ontic order, like in the case of Neo-platonic finality.

Consistency of
Aristotelian model

Let me apply what we have just said to the Aristotelian scheme of the four causes. It seems to be perfectly able to satisfy the requisites of Galvan's analysis, showing again that Renaissance Aristotelians were biased by heavy Neo-platonic influences in the interpretation of Aristotle's final causality, thus exposing it to Hume's critique:

- ◆ In the first place, in this doctrine a clear separation between efficient and final cause is in use.
- ◆ In the second place, an autonomous explanatory worth is attached to the final cause only in the context of intentional operations (= epistemic *a posteriori* foundation of logical necessity in causal physical processes).
- ◆ In the third place, according to Aristotle and Aquinas, at an ontic level, the final causality does not exercise any efficient causality. However, we need to make a distinction about this point:
 - As to those causal processes, whose final state does not *univocally* depend on the initial causes and that are represented nowadays by complex systems, in Aristotle's and Aquinas' views Galvan's principle applies. We must be careful not to misinterpret the necessity of extending the explanatory causal principle from the mere initial causes to a set of events, that are subsequent in time to the event-effect (backwards-causation). What in non-intentional physical processes, characterized by a dynamic instability – in which, that is to say, the final state is not predictable from the initial conditions – Aristotle and Aquinas define as the *formal cause* nothing but a principle of order for the causal chain and the sequence of events that emerge during the process, and, at most, at the end of it (*forma finis materiae*). However, no final causality is at work in terms of the addition of an efficient function, producing effects, to the mere "should be" of an entity/event.
 - As to causal processes of the same kind, but that relate to *intentional behaviours* of animal subjects that can perform them, final states could exercise a function of efficient causality, not as such, but rather *qua* final states that are represented in the mind and/or brain

of the intentional agent subject, so that the efficient cause is again exclusively contained in a complete way in the agent cause.

- ◆ As deeply connected to the previous argument, we can consider the *epistemic* worth that Aquinas attaches to the final states of such processes, in which the necessitating logical link, which characterizes hypothetical physical laws that formalize the causal relation of these processes, emerges from the end of the process, and not from its beginning. The backwards-causation has only an epistemic – non ontic – value¹⁶⁹.

Difference from the
epistemic context of
intentional finality

We are faced here with a further case of *explanatory* finality, having no ontic worth (since the final state of the process does not ontically cause anything, i.e., does not make anything exist at a physical level), but rather an *epistemic* value, that is always bound, in other words, to the cognitive act of a non-agent, however *knowing*, subject. Here, the conscious subject is not called upon to produce an external event by means of an intentional behaviour, but rather to produce an event inside his/her mind: i.e., an increase of knowledge.

Relation to the
constitutive
induction process of
syllogistic *inventio
medii*

Accordingly, in Aquinas' words, we are faced here with an exemplification of a process of constitutive induction of the hypothesis of a demonstrative hypothetical-deductive procedure, which we addressed in Chapter Four, in relation to contemporary epistemology. This is a wide and complex context related to the use of a syllogistic procedure of invention/discovery (*inventio medi*) for the constitution of hypothetical-deductive procedures of proof, that are typical of modern propositional logic.

6.3.3 Thomistic model of metaphysical causality

6.3.3.1 Cosmological significance of the model

The distinction
being-essence from
meta-logic to
metaphysics

So far, we have seen the importance of the Thomasian *being-essence* distinction in the theory of the foundations of logic, i.e., for that part of metaphysics that studies the foundations of *logical being*, namely meta-logic. We need to consider now the same distinction also from the point of view of the other main part of metaphysics, namely that which studies the foundations of *physical being*. This is metaphysics proper, which is sometimes defined by Aquinas as *trans-physicam*. As Aristotle reminds us from the First Book of his *Metaphysics*, this discipline is tightly related to

¹⁶⁹ Similar considerations about the critique of the “strong version” of the anthropic principle, and the acceptance of Aristotelian and Thomistic positions can be found in Castagnino & Sanguineti (2000, 159ff.).

cosmology. In other words, it is related to that part of physical science that studies the structure and the origin of the cosmos, i.e., of the physical universe, in its entirety¹⁷⁰.

Relation between
the theory of the
foundations of
physics and
cosmology, and
metaphysics of the
notion of causality

In our time, the study of the foundations of physics, in its being linked to quantum-relativistic cosmology, is one of the most interesting, profound and fascinating fields of theoretical physics. Essentially, it engages with the problem that captured the interest of Albert Einstein in his last forty years of research, although an acceptable solution – i.e., a *unified theory* for relativistic and quantum mechanics – has not yet been found. Beyond the most technical and physical-mathematical issues, that we cannot even touch upon here, there is a background question that can be tackled by a metaphysician.

This is the question about the foundation itself of the principle of causality in physics, which has excited those who study quantum mechanics ever since it came about (Cf. § 4.1). More generally and radically, the problem is now formulated as that of the *ultimate foundation of determinism of physical laws and causes within the universe*, given that, as Einstein liked repeating, “God does not play dice”; or, in other words, given that real scientific research presupposes the existence of an objective order of the cosmos that needs an equally reasonable justification, beyond any religious belief, and/or any more or less politically correct ideological view, in accordance with times and contingencies.

Hawking's
standpoint

We can express this issue in the terms of one of the most famous theoretical physicians of our century, who, being religiously agnostic, cannot be accused of being “ideologically biased” either in a theological, or apologetic sense. In one of his latest books, Hawking answered a BBC interviewer, Sue Lawley, who asked him whether by his research about the foundations of cosmology he “managed to do without God”:

¹⁷⁰ Such a statement should not astonish the reader who is used to thinking that Aristotle lacks any evolutionary idea of the cosmos, since he believed in its being eternal. This statement is only partly true: according to Aristotle, only that part of the physical cosmos concerning celestial physical bodies was eternal and unchangeable. Earthly physical bodies were rather subjected to an unceasing evolutionary ‘becoming’. Precisely, the earth itself at the beginning was a chaotic *mass* of elements, from which, subsequently, began the separation (as a consequence of a progressive cooling) of a cold set of mineral bodies – that went to form the planet Earth – and then an intermediate stratum – between the Earth and celestial bodies – i.e., the living bodies, or organisms, that represent what we now call the *biosphere*. This is the fundamental thesis that Aristotle presents in his cosmology (*De Caelo*) and “non-linear” thermodynamics (*De Generatione et Corruptione*) treatises.



Hawking's critique
of the "strong"
version of the
anthropic principle

What my work has shown is that we cannot say that the way in which the universe began was God's personal whim. The question that remains is the following: *why does universe exist? If you like, you can say that God is the answer to this question.* (Hawking 1993, 204).

In synthesis, Hawking is trying to say that his criticism is addressed towards those who, by their defence of a "strong" version of the "anthropic principle" in cosmology (Cf. § 4.1), refer to the Creator as the "*deus ex machina*" who establishes arbitrarily the initial conditions of our universe so that they can eventually lead to us. In a way, his stance corresponds to Galvan's logical critique of the "strong" anthropic principle, so as to confirm the universality of logical laws for those who believe and those who do not¹⁷¹.

In short, the problem is the following. According to the "classical" conception of relativity theory, the *big-bang* is viewed as a "singularity" of the space-time curve of general relativity. The problem emerges when we try to go back from today to the initial conditions of the universe, following the *reversible* laws of general relativity. When we approach the supposed initial singularity — i.e., when we seek to formally reconstruct the state of the universe, a few seconds after the big-bang, when it was rather small — and the laws of general relativity do not apply any more, but quantum laws come into play, it is not possible to univocally define the initial conditions, building on the current state of the universe. The universe behaves as if it were an unstable system, in which, from the final state, it is not possible to rediscover unique initial conditions. Such conditions need to be considered "arbitrary" and, thus, they must have been chosen with an almost absolute necessity by an omniscient "Someone", so as to allow for the evolution of the universe in a way that made the appearance of life and human beings possible.

Hawking's theory of
foundations and
Feynman's
formalism of the
'sum-over-histories'

Hawking's theory builds on another perfectly consistent hypothesis, that seeks to avoid arbitrariness. Indeed, it maintains that any "Grand Unification theory" between quantum and relativistic mechanics cannot avoid an essential element of quantum formalism, namely, Feynman's "sum-over-histories" or *path integrals* in order to describe the evolution

¹⁷¹ From this viewpoint, in his writings, Hawking likes repeating that the "weak version" of the anthropic principle — i.e., the high unlikelihood of life, and of intelligent life in the universe — is anything except scientifically baseless and is, rather, "what is to be explained". The easiest way to do this is to suppose the existence of infinite universes: this supposition underpins several cosmological models — including Hawking's — alternative to the "strong" version of the anthropic principle. In short, what is highly unlikely within one *single* universe ceases to be as such in a distributive function, mediated on infinite universes.

of quantum particles. Such integrals are defined on a time described by means of imaginary numbers.

According to Hawking *big-bang* and *big-crunch* are no longer singularities. The history of every universe, each of which with its own internal time, develops within a "circular" eternal time

If we insert the formalism of the "sum-over-histories", defined on an imaginary time, in a geometry of the space-time curve of general relativity, and if we suppose that the space-time curve of relativity has "Euclidean" properties, i.e., considers the reversible time direction in terms of the three space relations (the irreversible always increasing time, from the past towards the future, i.e., that is described for real numbers, is "internal" to each universe), we will no longer have to define the *big-bang* (and the possible *big-crunch* of the final gravitational collapse) as singularities "at the border" of the space-time geometry of the "global" universe (a sort of meta-universe).

They could be represented as two (of the many possible) opposed poles, belonging to the same sphere of imaginary time, in which it is possible to define the path integrals of the stories of all possible, infinite universes. In other words, considering all the possible infinite stories that the universe(s) could experience, each of which with their internal, *always increasing*, time defined for real numbers, the most likely are those that resemble the surface of the earth (= *geodesics** of imaginary time). Accordingly, global time within this cosmological model would be *eternal*. Within Hawking's space-time curve, every universe would start from a minimum area (i.e., pole) of the time-space sphere, and would evolve along its geodesics, expanding until reaching the limit (equator) of the space-time sphere, to collapse then into a minimum area (opposed pole) of the space-time sphere (Cf. Figure 6-1).

There is a possibility that space-time is finite and, nonetheless, has no singularities that may form a limit, or a border. The space-time will then be as the surface of the earth, with the difference alone that it has four dimensions, instead of two. The surface of the earth is finite, but it does not have not a limit, or a border. If we sailed towards the West, we would not come up against any singularity, neither would we fall off the earth (Hawking 1988, 160).

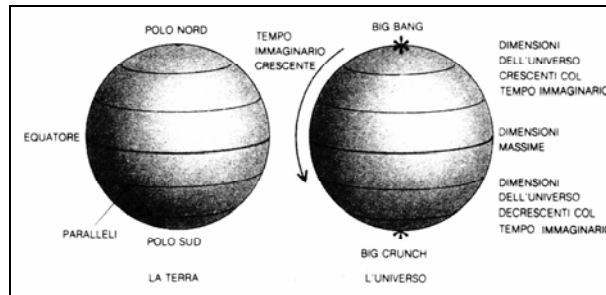


Figure 6-1. Visual representation of Hawking's hypothesis (Hawking 1988, 162).

Hypothetical value
of Hawking's theory

According to the interpretation given by Hawking, such a theory does not aspire to be apodictic – and how could it be nowadays? – but only *hypothetical*, although consistent with the main laws of relativity and quantum mechanics. Similarly, a hypothesis consistent with the same laws is that which characterizes the *big-bang* and *big-crunch* as singularities, thus ontologically conceiving of the *big-bang* as a sort of absolute beginning of the unique universe, i.e., that in which we live.

The determinism of
this theory requires,
however, a principle
grounding the
existence of such
an object and that
must be necessarily
“outside” the object
itself, given the
impossibility of self-
referential
foundations

However, at the same time, because of its *determinism*, Hawking's model does not really deny, but it rather requires, the presence of a Principle that is capable of justifying why a similar universe exists – as Hawking himself recognized in the words cited above (Cf. p. 461). This question was best metaphysically set by Hawking, without risking confusing such a Principle with the “initial conditions” of the evolution of our universe. The problem of the Foundation of the existence of the universe is metaphysically distinct from that of its hypothetical “absolute beginning”. It is always possible to debate about the need of asking the latter question, just as it is concerning any hypothesis. Whereas, for those who accept with Hawking the use of logic to investigate ultimate questions, without conceding anything to mere sentiment or ideology, the need of formulating the former question is not optional.

The shift that Hawking did not bring about, and did not intend to bring about, is only that of faith, i.e., that characterizing the final phase of the five Thomistic “proofs” of the existence of God. In other words, he did not recognize such a Principle in “God”, or better in the God of the Bible: “if you like, you can believe that God is the answer to this question”. Indeed, Thomistic “proofs”, after having shown the necessity of such a Principle, that explains *why* the universe “does exist”, conclude with the phrase: “and this we call God” (*et hoc dicimus deum* (S. Th., I, 2, 3c)). Therefore, these are not, properly speaking, proofs of the existence of God, but rather of an Absolute Principle of the universe that consti-

tutes its ultimate “why”, i.e., its First Cause. It is up to faith, and to different religious doctrines, to bring about the “shift” and define “God” as the One that is identified – or not – with such a Principle, as long as the “God” of a certain religious doctrine is compatible with such notion of an Absolute and Transcendent Principle. A Buddhist, for instance, could never accept such identification...

The continuity of such a non-theological ontology of modern cosmology (proposed by Hawking) with Thomistic metaphysics is witnessed by the fact that both of them agree on the main assertion, namely they both accept the hypothesis of the eternity of the universe. From a strictly metaphysical viewpoint, according to Aquinas – as we shall shortly see in detail – the necessary existence of the Principle, or of the First Cause, is perfectly compatible both with a cosmological hypothesis of an absolute beginning of the universe – in accordance with the models currently making reference to the hypothesis of the “big-bang”, as the absolute beginning — and with the opposed cosmological hypothesis, denying this.

Accordingly, these considerations, despite their roughness, helped us to delineate a threefold problem:

Three
consequences:
1. even in the case
of an eternal
universe, the
problem of its cause
remains
2. the problem of
the cause of the
universe is different
from that of its
beginning
3. such cause does
not correspond to
any of those
proposed by
Aristotle

- ◆ If we conceive the order of the cosmos as one of its objective properties, a problem immediately arises as to its foundation, i.e., as to “why does the universe bother to exist?”, and “exist in this particular way?”.
- ◆ Such an issue is to be distinguished from that of the presumed temporal beginning of the universe, since the question about a more or less absolute beginning of the universe – i.e., the question about its temporal eternity – cannot be either metaphysically or cosmologically definitively decided. Both of these hypothesis are consistent with the laws of physics, at least with those we know.
- ◆ It is clear that the “why” to which Hawking aims to give an answer does not correspond to any of the four that were identified by Aristotle, through his doctrine of the four causes (Cf. § 6.3.2.1), but somehow it includes and precedes them. In other words, the “why” identified by Hawking concerns a more fundamental notion of cause, i.e., the most fundamental that we can conceive. This is that concerning not only the essence, the secondary substance, but the whole being – primary and secondary substance – of every entity that is in the physical universe. As it is well-known, in the Middle Ages, Aquinas gave to such causality a name and a content, through his doctrine of the participation of ‘being as act’ in Pure Act.

Thomistic theory of causality: physical causes (entities) are contingent *in se*, but necessary (= ordered) for the happening of events *per se*

According to Thomistic doctrine, all the entities in the physical universe – including men – are metaphysically made of *act* and *potency*, being and essence, and, from the perspective of physical ontology, by form and matter. From the viewpoint of general metaphysics, every entity is an *entitas* (thing) that does not exhaust its “intensional” richness in the being as act – as actuality of every form – but, since it has a limited essence, it participates in it in a finite way. This is intrinsically bound to the *contingency of the existence* of such entities, i.e., to the fact that their *existence in act* with the essence, with that set of properties, depends on a concurrence of *necessary* causes that made them *exist*, and made them exist as an *entitas* characterized by certain properties and not by others. As we know, (Cf. § 5.5.2, especially Figure 5-3), such a causal concurrence of causes that are necessary to the existence of an entity is composed of a twofold order:

- ◆ Of entities with limited *being*, which are causes that are *necessary per se* and *contingent in se*, and of their – although not necessarily univocal – *ordering*;
- ◆ Of a Subsistent Being whose *Being* corresponds to the being that is the Cause *transcending* the previous collection, since it is necessary in itself and for itself, so as to give consistency to the whole system.

Absolute metaphysical causality is out of time and *simultaneous* to every moment of time within the universe

For the time being, in order to summarize what we shall say next, we can anticipate that the Subsistent Being participates in the being of all subhuman entities¹⁷² by a *single creative act* “outside time” (see Thomas Aquinas, *S. Th.* I, 45, 3c); which is *simultaneous* to any event happening in the space-time within the universe, but that appears as “diluted” in time to us who live within the space-time. This is due to the fact that the forms determining the essences of those entities come to existence (i.e., are actualized) in matter through the causal concurrence of certain sec-

¹⁷² In terms of physical beings endowed with intelligence and freedom, such as ‘man’ – although this same reasoning can be applied to any other intelligent being from some unknown planet – capable of transcending, with their faculties, the physical determinism of which they are an integral part because of the matter of their bodies, it is necessary to hypothesize (in order to justify that capacity of their minds) a further relationship of transcendental participation in the Subsistent Being. This relationship goes beyond that common to all physical beings and which deals with the matter of which these intelligent are made. The «spirituality» of the mind is therefore essentially a transcendental relationship to the Absolute Being which, from conception onwards, determines the form and the development of the human being from its genetic «matter». The spiritual soul is therefore quite something else than «the ghost in the machine», «a thinking thing» (*res cogitans*) within an «extended thing» (*res extensa*) of Cartesian ilk. Yet we will return to this question in Part Four.

ondary causes that, being contingent, exist, like their effects, only in particular segments of the space-time¹⁷³.

6.3.3.2 Time and creation¹⁷⁴

The absolute being of every entity (essence and existence) depends on an a-temporal first cause

In order to address the issue of the consistency of any evolutionary theory not only of the living, but, more radically, of the entire cosmos of physical entities, we need first to correctly focus the problem, that is metaphysically more fundamental, of the relation between the actual beginning of the existence in time from potency of the matter of the individual entities in the universe, and their shared dependence on the participation in the act of being, *qua* specific *entitates* (things), each of which with its own essence. Put another way, it is necessary to correctly focus the problem of the relation between *time* and *creation* (in terms of biblical, theological language).

As we have just seen, the *absolute being of every entity* (= existence and *entitas* together) depends on the First Cause. If it is true that every entity participates in being according to its essence, if the essence and the existence of a single entity existing in a certain location in space-time *also* depends on the ordered causal concurrence of the whole of second causes that preceded that entity in their actual existence, then it is also true that, far more radically, the *entitas* and the existence of that entity *definitely* depend on the First Cause. Every entity that causally concurs to determine *for itself*, i.e., necessarily, the existence of other entities, precisely because it causally depends in turn, according to a certain ordering, on other entities, it depends in the end on the First Cause. Only a Cause that is placed *outside* the ordered series of the caused-causing

¹⁷³ As far as we identify the Subsistent Being with the God of the Bible, we can say that God is First Cause, since He *creates out of nothing*, outside space-time, the *whole being* of something. In the case of subhuman physical entities, He is First Cause both of their *material substrate*, whose origin in time goes back to the origin of universe, and of the internal organization of their substrate, or *form*, which determines its nature, and thus its essence. Indeed, He is the Cause of the being of those secondary causes, and of their ordering, which, by their action on matter, actualize that form in potency in matter, and thus determine both the *essence* and the *existence* of a given something in a specific space-time location. God is thus First Cause not because He *creates the forms of the matter*, but rather because He creates the entire being of every entity. Somewhere else (*S. Th.*, I, 45, 4c) Aquinas maintains that, properly speaking, the “material”, not spiritual, forms of physical entities are not created by God; they are, rather, “co-created” in the metaphysical unity of every entity that is a substance belonging to the physical world. In brief, “bodily forms are caused not as if they flowed from some immaterial form, but as matter reduced from potentiality to act by some compound (form and matter) agent” (*S. Th.*, I, 65, 4c).

¹⁷⁴ A full convergence with what we seek to reconstruct here regarding Aquinas’ position about this delicate point can be found in Castagnino & Sanguineti 2000, 156ff.

causes, i.e., a causing non-caused Cause, can ultimately justify the existence of a certain ordered set of contingent causes.

The absolute “first metaphysical cause” does not belong to the succession of “secondary” relative physical causes, but it is outside this and justifies its existence and ordering

Accordingly, we should not understand the First Cause as the first in the series of the secondary causes, according to which, for instance, Descartes suggested, claiming that God is the One who, after having created matter and its laws – i.e., essentially, the principle of inertia – and given to the universe the initial impulse to motion, had nothing to do afterwards either with the experience or with the *entitas* of the individual entities in causal sequence. The First Cause is not “first” in a temporal but rather in a *metaphysical* sense: it is *outside time* and, at most, *temporally simultaneous* to all and each moment of time, “measuring” the becoming of the universe. Therefore, as it is meaningless to look for the absolute “time-zero” of the universe (every moment of time presupposes both a *beforehand* and an *afterwards*, as Augustine suggested in the 4th century)¹⁷⁵, it is even meaningless to define such “time-zero” as the moment of creation. Creation is not “the event of all events” because this notion is intrinsically contradictory. Indeed, it implies an infinite regress problem (the event of the event of all events, and so on). The “when” of creation is thus a meaningless issue: creation is not “inside” time, but rather “outside” time. This means that time begun to be with the world and, accordingly, since we are “inside” time and “inside” the becoming of the world, as it is measured by time, we will never be able to establish a time for creation, as Augustine realized from the very beginning of the Christian Age (see Augustine of Hippo, *Confess.*, XI,10-13; *De Civ. Dei*, XI, 6; XII, 16).

The theological notion of creation outside time has nothing to do with the theory of *big-bang*, conceived as initial singularity of the universe

In light of this, we can clearly understand that the *theological* notion of creation has nothing to do with the *physical hypothesis*, connected to relativity theory, of the origin of the universe from the *big-bang*. In the first place, this is because this event is, at least in principle, *datable*, thus it is *inside* time. Secondly, because before the *big-bang* it is not like there was absolutely nothing of the current universe: there must have been, at least, the totality of the chaotically disordered energy (*quantum emptiness*) of which the current (partially) ordered cosmos consists, as by its substrate (*prime matter*). But perfectly consistent with relativity theory is the possibility that different universes preceded ours and will follow it, according to the hypothesis of the *big crunch*, if the cosmological model placing a limit to the current expansion of the universe turns out to be

¹⁷⁵ Also from the perspective of quantum-relativistic cosmology, cosmic time does not have to be necessarily defined exclusively on the straight line of real numbers. In principle, time could also exist on the basis of imaginary numbers, so as to give to cosmological time a *complex* nature. From a metaphysical angle, this is the most interesting aspect of, say, Hawking's cosmological theory.

accurate. In this case, once maximum expansion is reached, the universe should begin to contract, i.e., to “implode”, until it has reached an extremely small size, from which a new explosion and a new universe could begin to exist.

Hypothetical nature of the theories that either affirm or deny the absolute beginning of the universe with the big-bang

Beyond this “cyclical” hypothesis of the sequence of many universes, that is nowadays very popular among cosmologists, relativity theory is consistent with other hypotheses, which, despite their being unknown to a common-sense intuition, have been nonetheless formulated because they have a mathematical validity within certain models of quantum-relativistic cosmology of several (at most, infinite) simultaneous universes (Cf. footnote 171). The possibility of deriving from the same axioms of the theory of relativity cosmological models that seem to reinforce the hypothesis of a temporal beginning of the universe, as well as models that deny such hypotheses, is a sort of “empirical” confirmation of the absolutely *undecidable* nature of the problem of the beginning of the universe, as Scholastic metaphysics has always claimed from Augustine to Aquinas.

Right interpretation of the natural theology notion of “creation out of nothing”

The First Cause is thus “outside” the chain of secondary causes and, actually, underpins it, step by step, entity by entity, since the origin of the universe, just as it is metaphorically symbolized in the myth of Atlas, holding the world on his shoulders (Cf. Figure 5-3). In this sense, in metaphysical theology, we can and must say that God, by a single creative act which is outside time, created all the contingent entities that are in the physical universe, both in their actual and potential components, i.e., *being* and *essence*, *entitas*, *form* and *matter*. But we also have to say that, since the universe is simply the whole of everything that existed, exists and will exist, every entity in the universe, even if it comes to an actual existence long after the origin of the physical universe, is always *created by God*, in that single creative act, which does not suppose anything besides God (e.g., a chaotic matter to order as in theological, non-biblical cosmologies).

“Creation out of nothing”: the entire being of an entity, nothing excluded, derives from the Subsistent Being

By the expression *creatio ex nihilo*, i.e., “creation from nothing” we do not mean in metaphysical – Christian and, in particular, Thomistic – theology that “beforehand” there was nothing, and “then” the being in the totality of entities existing in time. First of all, we mean that *the entire being of an entity, nothing excluded of that entity*, namely matter and form, essence and being, have their origin, i.e., their Principle, their **Archē** in the Subsistent Being, i.e., in the Being without potency of Pure Act (Cf. *S. Th.*, I, 46, 3, ad 2). Scholastic metaphysics has never wanted to argue by the doctrine of *creatio ex nihilo* the existence-of-nothingness of being, or the non-existence-of-the-‘to be’-of-being. These are all *meaningless* statements, besides their being either true or false, since they violate the first

principle of meta-logic and metaphysics: i.e., the p.n.c. In this sense, every entity exists in God, as in its First Cause, *from always* and *forever*, even if it has not either potentially or actually existed, always and forever, in the universe of contingent entities, made up of potency and act, both metaphysically (essence and being) and physically (matter and form). Therefore, the meaning of *creatio ex nihilo* is that of creation *post nihilum* (Cf. *S. Th.*, I, 46, 3, ad 2). This expression is not to be understood in the absurd sense of the primordial existence of nothingness, but rather, as Aquinas well explains, in the sense that, before creation, there was no entity (*nullum ens*) — therefore nothingness ‘is’ not —, except for Subsistent Being, i.e., the Principle on which the entire being of the entity “after” its creation depends. This refutes Severino who, instead, has never understood this point, mistakenly accusing Thomistic metaphysics and, more generally, the Scholastic doctrine of *creatio ex nihilo* of being contradictory, i.e., of implying the existence of *non-being*, or of the non-existence of *being* (Basti & Perrone 1996).

(...) We need to consider not only the emanation of a certain particular being from a certain particular agent, but also the emanation of the entire being from a universal cause, that is God: and this emanation is called *creation*. That which derives from a particular emanation is not presupposed by the emanation: accordingly, if a man is generated, *beforehand there was no man*, since the man comes from what is not man (e.g., from a certain genetic material before conception), and the white comes from non-whiteness. But, if we consider the emanation of the entire being of the universe from a single principle, *it is impossible that any entity is presupposed by this emanation*. This is nothingness, in the sense of *no entity* (*nullum ens*) (*S. Th.*, I, 45, 1c).

Creation is the original, transcendental relation of the entire being of every entity with Subsistent Being

The entire being of that entity, its “absolute being” (*esse absolute*), beyond which there would be nothing of that entity, ultimately depends on God. The being of the entity, i.e., the relative being of the *entitas* — i.e., of the participation in the being as act, which depends on the divine Agent — and, therefore, of the actual existence of an entity in space-time causally depend in their entirety on Subsistent Being: being comes from Being. Creation is not the conversion of nothingness into the being of a given entity, or of the totality of the entities (i.e., the universe), as if, *before* the existence of the universe, *nothingness* existed. Nothingness *is not*, as it should be clear — from Parmenides’ time — to all those who in some way deal with philosophical issues. Accordingly, it cannot exist either beforehand, or afterwards, or ever. Creation is the transcendental causal relation between Subsistent Being and contingent being, independently of any becoming — ‘becoming’ is to be metaphysically grounded, since *it* does not ground anything, as, beginning with Par-

menides, has become clear in Western metaphysics (Cf. § 5.1.4) — and, thus, independently of all temporality. In short, as Parmenides understood, ‘nothingness’ simply *is not*.

Problem of the absolute beginning related to the personal nature of God

As Aquinas underscores correctly, the issue of an absolute beginning of the universe *qua* distinct from the metaphysical affirmation of the *ab aeterno* existence of the world in its First Cause can be addressed only after faith helped us in understanding the personal nature of the First Cause, i.e., that we are faced with a Personal God. Indeed, in that case, the *ab aeterno* dependence of the universe on its First Cause can certainly be compatible with a *non ab aeterno* existence of the universe, given the mediating action of the unquestionable Intelligence and Free Will of a Creator (*S. Th.* I, 46, 2c).

Also according to the Bible at the beginning there was not anything, but only God

According to the Bible, that reveals to us an absolute beginning of the universe, “before” its creation, it is not like there was ‘nothingness’: only God was there and the world *ab aeterno* in His creating will. Similarly, in the case of something done from an intelligent source, “before” being produced by its maker, it is not the case that it does not exist in absolute terms: in fact it exists in the maker’s mind. As the first verse of the Book of Genesis explains, “In the beginning [was] God”, not ‘nothingness’. “In the beginning God created the *heavens* [or the source, immanent to the universe, of its entire order] and the *earth*. And the earth was without form and void” — i.e., the primordial chaos, enclosed and ordered in its immanent ordering principles: “heavens”. The difference lies in this: whereas in the case of the work of human intelligence, this is produced by the maker not only building on the idea that is in his/her mind, but also out of an already existent matter — i.e., every “creation” that is made by human intelligence is *ex nihilo sui*, that is to say “building on nothing of itself”, but not *ex nihilo subiecti*, that is to say “from the nothingness of the subject”, i.e., the matter in which it is implemented and realized — in the case of a divine “creation”, a work is produced also building on the “nothingness of the material subject”, i.e., *ex nihilo sui et subiecti*. In other words, it always depends on the Divine Maker’s creative intention. In this the biblical theology of creation is distinguished from extra-biblical (both Eastern and Western, ancient and modern, religious and philosophical – including Plato’s and Aristotle’s –) myths of creation. These share an anthropomorphism of the notion of creation, i.e., the idea that it consists of the mere ordering of the chaos of primordial matter, of the implementation of a form, i.e., an order, in matter, as in the case of Plato’s Demiurge, or Aristotle’s Motionless Engine or, moving towards the East, in the case of the Taoist doctrines of the *yin* and *yan*, that are currently very popular also in Western countries.

In order to summarise our considerations about this issue, which is fundamental so as to understand correctly the *issue of evolution* – both of the physical and biological universe (to which we shall devote the Third and Fourth parts of the second volume of this work), it is necessary to bear in mind two points regarding what can be demonstrated through reason and what cannot be demonstrated by means of it (and is thus comprehensible only through faith) about the *revealed truth* concerning the beginning of the world through creation by God (“In the beginning, the Lord God created heaven and earth”, *Gen 1,1*):

The necessity of the First Cause can be metaphysically demonstrated

- ◆ We can perfectly demonstrate metaphysically the necessity of a First Cause – placed outside the universe, its becoming and, thus, its time – on which the being of every entity that existed, does exist and will exist depends by participation. In Hawking’s words, this is the metaphysical answer to the *fundamental* and *unavoidable* question, for all those who use their reason, about the reason “why the universe does exist”. However, such a question remains beyond any cosmological hypothesis about the eternity of the universe. Aquinas answered in this way the question “what is creation?” – consistently with what Hawking claimed: “if one believes, s/he can say that God is the answer to this question”, i.e., one can attribute the name “God” to the First Cause:



What is created comes to existence without any becoming or change, because every becoming and every change presuppose that something existed beforehand. Accordingly, when creating, God produces things without any change. [...] If we eliminate the ‘becoming’ from an action, the relation alone will remain in place. [...] Therefore *creation in the creature is simply a certain relation to the creator, as the principle of its being* (*S. Th.*, I, 45, 3c).

Accordingly, in the Christian theological doctrine of creation, the dependence of every entity on the being of a *transcendental Absolute* is something that is both an object of revelation that is accepted by faith – in which case the Absolute is the God revealed by the Bible – and an object of metaphysical demonstration, which we must *necessarily* accept through reasoning – beyond any religious faith – in light of the axioms of a certain metaphysical theory. In the specific case of Thomistic metaphysics, this is the fundamental axiom of *being as act*.

The theological doctrine of an absolute beginning of the universe cannot be metaphysically decided

- ◆ In Christian theological doctrine, what will never be rationally demonstrated is the concept of the absolute beginning of the universe. The temporal existence of the universe is logically compatible both with a positive and a negative answer to this question (*S. Th.*, 46, 1 ad 7), as is also proven by the current cosmological debate concerning the singularity of the *big bang* — in other words, this is an undecidable problem, since it is a classical self-referential, or, more precisely, *impredicative* problem. In Aquinas' words:

We have to say that the fact that the world has not always existed is something that cannot be proven in demonstrative form[...]. The reason for this is that the absolute novelty of the world cannot be demonstrated in a way that proceeds from the existence of the world itself [...]. Everything that is considered in light of its own species, abstracts from space-time modalities (*anstrahit ab hic et nunc*): it is in that sense we say that 'universals' are everywhere and always (*S. Th.*, I, 46, 2).

Impredicativity of the cosmological demonstration of an absolute beginning of the world

In other words, if I wanted to demonstrate, building on the existence of the world, that the world in absolute terms had a beginning, it would be as if I tried to deny at a certain moment the existence of something that, by hypothesis, I have recognized as existing in another subsequent moment, thus having to impose some limits on the variables in order to grant universality to the demonstration, i.e., I have to suppose that such a thing exists.

Impossibility of a theological demonstration of an absolute beginning of the world

Equally, as Aquinas continues to assert in the same article, from the viewpoint of a metaphysical theory, i.e., supposing to have demonstrated the necessity of a First Cause, there are no sufficient rational motives to decide whether such a Cause should create the world *ab aeterno* or not.

The absolute beginning of the universe can only be revealed, not demonstrated, and accepted on faith, not through reasoning

Accordingly, the idea of an absolute beginning of a temporal universe can only be an object of revelation and thus accepted by faith. Put another way, the question about the beginning of the world, as in all self-referential problems, allows *only* decidable answers that are "external" to the system in which it is formulated. In conclusion, from a metaphysical perspective, both building on the world and on the First Cause, we can never exclude the possibility of a co-eternity of the world and the transcendental Absolute on which it is founded, although such a possibility does not exclude in turn the necessity of a dependence in the being of the world on this Absolute.

Indeed, it is clear that the world leads to the knowledge of the creating divine power, both in the case it has not always existed, and in the case it has: everything that has not always existed was

clearly caused, although this cannot be immediately applied to what has always been (*S. Tb.*, 46, 1ad 7).

The revelation of the absolute beginning does not contradict any rational truth because such issue cannot be decided.

The possibility of the co-eternity of the world and God can only be excluded by Revelation, *without ever contradicting any rational, universal and necessary truth*, given the undecidable nature of the issue, both from a cosmological and a theological metaphysical perspective. It can thus be affirmed only by faith, on the basis of an extra-rational revelation, in light of the doctrine according to which the world had an absolute beginning. Such “beginning” is to be conceived of as outside time, in the sense that it includes the entirety of time, in view of what we said about a metaphysically consistent notion of the First Cause, as that presented by Aquinas.

In any case, all this shows both the scientific (in light of the reasons presented above) and metaphysical (in view of what we have just argued) weakness of the confusions, that are now very widespread, between theological and metaphysical doctrines of creation and physical hypotheses about the origin of the universe from the *big-bang*. We need to bear in mind, in this respect, the recommendation put forward by Aquinas at the end of the article of the *Summa* that we have cited above, against those who aspire to make such doctrine of the absolute beginning of the universe an object of rational demonstration, i.e., those who seek to deny the possibility of the eternity of the world through sound reasons:



The idea that the world had an origin is a matter of faith, but it cannot either be demonstrated or known [through reason]. It is important to consider and think about this, so that someone, seeking to demonstrate what is an object of faith, does not ground such an aspiration in motivations that cannot demonstrate anything, thus opening the way to the mockery by those who do not believe, who could then think that we believe certain things not by faith, but in light of such false motivations (*S. Tb.*, I, 46, 2).

Full compatibility of creation and evolution

Having thus made clear the correct metaphysical sense of the relation between time and participation in the act of being, we can immediately understand that the *creation* of entities by the First Cause *outside time*, and their *evolution inside time* by the causal concurrence of secondary causes can perfectly co-exist without contradicting one another. But we shall deal with the relation between the theological theory of creation and the cosmological and biological theory of evolution respectively in the Third and Fourth part of the second volume of this work.

6.4 Summary of Chapter Six



At the beginning of this chapter we briefly dealt with the meta-logic of Thomistic metaphysical language (§ 6.1), as it was expressed in Aquinas' distinction (§ 6.1.1) between *logical analysis of language in material suppositions* (= syntactic analysis) and *in formal suppositions* (= semantic analysis). In particular, we focused on the analysis of the copula "is" of every predicative statement (§ 6.1.2). We saw that in the schematic universal formula of every predicative statement "x is an entity", "to be an entity" does not primarily mean "to be existent", but rather "to be *that which is*", i.e., "to be a singular *entitas*", where the "what is" is expressed by the verbal name at the right hand side of the copula (e.g., "x is a man"). Accordingly, the predicate "exists/does not exist" is a *meta-predicate* that is somehow co-extensive with the other *meta-predicate* "true/false", in the sense that *exists* never refers to the subject of a predicative statement (the "x" of the formula "x is an entity"), but it rather always refers to the *referent* of a *true whole categorical assertive statement* — and, correspondingly, *does not exist* makes reference to the *referent* of a *false whole categorical assertive statement*, and vice-versa. According to modern Aristotelian-Thomistic philosophy of nature, we should understand in this way the typical principle of Galilean modern science, namely the motto according to which nature "is to be questioned" (actively) and not "observed" (passively). Only if we know something (or, at least, we hypothesize something) about the "what is" (i.e., the *entitas* of being) of what we are looking for, can we question nature in order to know whether a certain entity *exists* or not. We also pointed out the strong similarities between Thomistic meta-logic of being and the issues tackled by contemporary *formal ontology*, that, precisely in light of this, devotes much attention to Scholastic (and, in particular, Thomistic) formal ontology.

On the basis of this distinction between "being of the essence", or "proper being", or *entitas*, and "being of existence", or *common being* of an entity, Aquinas reinterprets, in an original way, the classical assertion of all Western thought, from Aristotle and Plato onwards, according to which "being is said in many ways" (§ 6.2). However, until Aquinas, such models essentially represented *modes of existence* (either necessarily or contingently, in potency or in act, materially or physically, etc.). Beyond these modes of existence, Aquinas distinguishes a further way to predicate being, i.e., the *entitas*, or the being of the entity according to the measure of its essence, on which the mode of existence itself depends.

Hence, first of all, we arrive at the distinction of three equivalent – but not identical – terms, namely *essence*, *nature* and *quidditas* of an entity (§ 6.2.1), but above all the fundamental doctrine of Thomistic metaphysics:

i.e., the distinction between *essence* and *act of being* (§ 6.2.2). The *entitas* of every being does not say anything more than the way in which every entity *participates in the fullness of being* in accordance with its *own limited essence*. Similarly, in physics, “matter” imposes a limit on “form” in the sense that “potency” imposes a limit on “act” (e.g., no material concrete individual human being, or a collection of these – class, nation, race, ethnicity, etc. – exhausts the entire richness of the human “form”, or “humanity”). Accordingly, the essence of an entity, or of a specific class of entities, imposes a limit on the ample richness of the being (e.g., “the man” has many actualities (properties), but he has none of those that are specific for a “dog”, and a “dog” has none of the properties of a “cat”, etc.), just like the “potency” of individual matter imposes a limit on the richness of the act of the specific form (= principle of individuation: no man or human group can exhaust the richness of humanity). This is the reason why Aquinas speaks about *being as act* (*esse ut actus*), or *act of being* (*actus essendi*), as “really distinct” from *essence* — i.e., as truly distinct as “act” is from its “potency” — as metaphysical constituents of every (physical, logical, mathematical, material, spiritual, etc.) entity, just as matter and form constitute the essence of every physical entity. The Absolute Being alone has an “intensively rich” essence, like a being with all its actualities, and is thus not a mere “entity”, i.e., “that which participates in being”, but the *Subsistent Being*. This is a Unique, Simple Being, with no distinctions, or Pure Act with no potency, that is distinct, i.e., “separate” and transcending all the other entities in its non-replicable Uniqueness, in which all entities participate. However, since every entity takes part in being in accordance with those properties that define its specific “essence”, or “nature”, some properties of an entity will be either *necessary* (i.e., *essential*), or *contingent* (i.e., *accidental*) (§ 6.3.1).

Moreover, the analysis in the previous chapter clearly illustrated that the novelty of the Aristotelian and Thomistic conception, in comparison with the rest of classical thought, is the *causal foundation* both of *essences* (Aristotle) and of *all being* (Aquinas), in its double component of *being of the essence* (*entitas*) and *being of existence* (common being). Philosophy of nature is thus called to reconsider the Aristotelian doctrine of the four causes in *physics*, in order to see whether it can suggest a model, more adequate than Hume and Kant’s, for the study of complex systems. At the same time, we need to re-evaluate whether the Thomistic model of the *metaphysical causality* of being – participated throughout the universe of physical entities by Subsistent Being – is compatible or not with the principles of current quantum-relativistic cosmology (§ 6.3.1).

Regarding the Aristotelian doctrine of the four causes (§ 6.3.2) — which, in the physical order, are reduced to three (*efficient*, *material* and

formal-final cause), since the *final* cause can hold an explanatory power only at the level of intentional operations — we cannot but notice similarities with current physics. In particular, these are evident in relation to the distinction between stable and unstable systems, and, within these latter, as to stable systems outside equilibrium (dissipative structures) (§ 6.3.2.1). The vast majority of chemical systems — i.e., of inorganic bodies of our ordinary experience — and the totality of biological systems — i.e., living bodies — belong to this specific category of dynamic systems, which are studied by the physics of complex systems. The consistency of this Aristotelian doctrine is revealed also on the basis of its logical analysis, according to the lines of analytical investigation proposed, in the Middle Age, by Aquinas himself (§ 6.3.2.2), and, recently, by Galvan. This is particularly evident if we compare the Aristotelian doctrine of the “final cause” with the doctrine of the absolutely inconsistent finality in Plotinus’ Neo-Platonism (§ 6.3.2.3). The consistency of Aristotelian doctrine emerges from the fact that a) it never confuses efficient and final causality; b) final causality acquires explanatory value only within an intentional order; c) formal-final causality in the physical order is essentially a form of non-reductionism of the causal explanatory factors of a final ordered state of an unstable system in relation to merely initial causes. If we consider the knowing subject, i.e., if we move to an epistemological level, we will be faced with a form of backwards-causation: the ordering of the cause to its effect is not, in principle, *a priori* knowable, but only *a posteriori*, when the process has reached its final state.

Regarding the compatibility between the Thomistic doctrine of participation and quantum-relativistic cosmology, two points are clear (§ 6.3.3):

- ◆ According to Aquinas, what can be metaphysically proved is the necessity of a foundation for the existence and the order of the universe, besides the issue of its possibly being eternal (§ 6.3.3.1).
- ◆ As to the issue of the eternity of the universe, this is a question that cannot be answered by reason alone, but it remains metaphysically undecidable. The metaphysical doctrine of participation is compatible both with theories asserting the eternity of the universe and with those that deny it. A proof of such indecidability can be found in quantum-relativistic modern cosmology, that is compatible both with models that assert and that deny the eternity of the universe (§ 6.3.3.2).

The idea that theology can solve such an undecided issue, opting for an absolute beginning of the universe, is not in contradiction with physics and metaphysics, as long as a twofold condition holds:

- ◆ That such a solution depends on a *surplus* of information, exceeding the normal capabilities of the human mind, i.e., that this comes from God's revelation.
- ◆ That this absolute beginning is conceived "outside" time, since the conception of an absolute beginning inside time is a contradiction in terms (§ 6.3.3.2).

6.5 Bibliography of Chapter Six



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Glossary of significant scientific terms

ALGEBRA. Science of quantities, considered in their most general sense.

According to its most basic version, A. is an extension of arithmetic, through two principles: 1) Numbers are indicated through letters, without specifying their value, so as to define rules of calculation that can directly operate on letters, in order to achieve valid results, whatever the numerical values attributed to them may be; 2) Negative numbers are introduced as an extension of arithmetical numbers, so as to eliminate the operation of subtraction (reduced to that of addition) together with the limits it brings about (necessity that the minuend is larger than the subtrahend). Hence, a more compact set of unified rules, no longer regarding quantities, but rather the logical links between them.

ALGORITHM. General procedure to solve a problem of a given class (e.g., to perform a certain kind of complex calculus) through a finite series of basic directions (that are not further reducible), sequentially defined with no ambiguity. For instance, an A. is the sequence of steps of calculus, together with its directions, to perform a division, or a multiplication with many-place numbers, or to break down numbers into their factors, etc.

ANISOTROPOUS: (SEE ISOTROPOUS).

ATTRACTOR: Geometrical forms that characterize the long run behaviour of dynamical systems described in a *phases space* (see). In other words, an attractor geometrically represents the state, or the set of states, towards which the behaviour of a dynamical system “is attracted”, or made stable.

AXIOMATIC METHOD (SEE FORMALIZATION)

AXIOMATIC SYSTEM (SEE FORMAL LANGUAGE).

CALCULUS OF PREDICATES (SEE PREDICATE LOGIC)

CALCULUS OF PROPOSITIONS (SEE LOGICS OF PROPOSITIONS)

COMPLETENESS (OF FORMAL LANGUAGES OR SYSTEMS): By the C. of a *formal language* (see), or of a *formal system* (see) we mean that property by which a certain system is enough to decide about any correctly built (e.g., consistently deduced) proposition, that is also formulated building on the basic propositions (primitives, axioms, inference rules) of a certain language. In other words, C. of a consistent axiomatic system means that it must be possible to demonstrate within that system either any demonstrable formula, or its negation. Accordingly, in order to actually establish the C. of a formal language, it is essential that we guarantee that all propositions, correctly built in that language, are *decidable* (SEE CONSISTENCY, DECIDABILITY, INCOMPLETENESS, THEOREMS OF).

CONNOTATION (OF AN OBJECT): The whole of logical relations that the denoting term has (see DENOTATION), and through which the (denoted) object itself can be *identified* by the agents of communication, in relation to the linguistic context in which the denoting term is used. The above-mentioned logical relations can be both *intra-linguistic* – both of a *syntactic* and *semantic* kind – between the term and the other parts of the language it belongs to; and *extra-linguistic* – of a *pragmatic* kind – between the agents of communication. This is oriented to the justification of specific functions of C. attributed to the term, within limited specific contexts where a language is used communicatively.

CONSISTENCY (OF FORMAL LANGUAGES): A formal language can be said to be consistent if it does not contain contradictory formulas, i.e., when it is not possible that one of its formulas and its negation are both built (if axioms) or demonstrated (if theorems) within it.

CONSTRUCTIVE (SEE CONSTRUCTIVE METHOD)

CONSTRUCTIVE METHOD (OR RECURSIVE METHOD): Method for the definition and/or demonstration of mathematical items that is not limited simply to affirming their existence, but that rather specifies how these can be defined, or demonstrated, in terms of simpler items, or operations. (See CONSTRUCTIVISM)

CONSTRUCTIVISM: Philosophical theory about the foundations of mathematics that maintains that the existence of a mathematical

entity is not independent of the methods to build it and/or to calculate it (see CONSTRUCTIVE METHOD).

DECIDABILITY: A statement that can be formulated within a given *formal language* (see), or *formal system* (see), is said to be decidable if it can be demonstrated (syntactically, i.e., if it is derivable) to be either true or false within that system, otherwise *undecidable*. (SEE COMPLETENESS; INCOMPLETENESS, THEOREMS OF).

DEDUCIBILITY: Property of a formula to be deduced as the conclusion of a given valid argument, within a given *formal language* (see), or *formal system* (see).

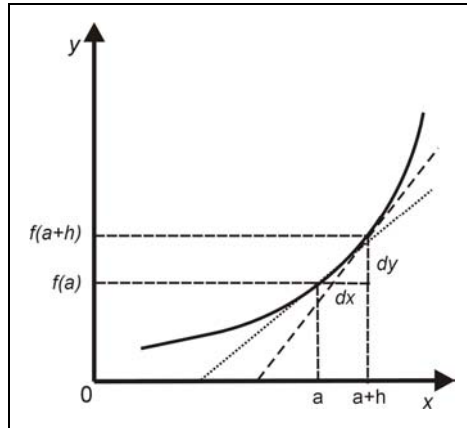
DEDUCTION: Procedure, typical of mathematics and logics, in which a *well-formed* formula (see) of a given *formal language* (see), or *formal system* (see), necessarily follows from the premises. Accordingly, such a conclusion cannot be false when the premises are true.

DEGREE OF FREEDOM: The term denotes one of the one-dimensional mutually independent quantities (variables or parameters) that determine the state of a physical system. For instance, a mass of gas has two degrees of freedom that can be chosen among the thermodynamic variables of volume, temperature and pressure.

DENOTATION (OF A TERM): By D. we mean, in (semantic) logics, the intra- or extra-linguistic object to which a term makes reference. Such an object is contextually identified through the set of logical relations that the term itself has, and that constitute the *connotation* (see) of the considered object, within the limited context of that language, and, possibly, of a certain communicative sphere between the agents engaged in that same communication.

DERIVATIVE: The D. of a function $f(x)$ in a point $x = a$ is defined as the limit of the relation between the increase of the function dy or $df(x)$ and the related increase of the variable dx . This notion has been of capital importance in the development of infinitesimal analysis. Geometrically, the D. corresponds to the notion of a tangent to a curve in a given point. Physically, the D. corresponds to the instantaneous, or local, variation of a quantity (e.g., in mechanics, speed in relation to motion). By D. we also denote the function that is associated to a given function $f(x)$, in accordance with the variation of the point at which it is derived. A D. can be thus expressed:

$$\frac{d}{dx} f(x) \quad f'(x) \quad D_x f(x)$$



Needless to say, there are also derivatives with a *higher order* than the first, i.e., derivatives of derivatives (i.e., *second derivatives*, e.g., in mechanics, the instantaneous variation of speed, or acceleration), derivatives of derivatives of derivatives (i.e., third derivatives), and so on. There are also *partial derivatives* by which we mean the D. of a function with multiple variables, in relation to one of those. A partial D. can be written thus:

$$\frac{\partial}{\partial x} g(x, y) \quad D_x h(x, y, z)$$

DIFFERENTIAL (SEE DIFFERENTIAL EQUATION).

DIFFERENTIAL EQUATION: An equation is said to be differential if the unknown function (or the unknown functions) appears through its value and the value of its *derivatives* (see). An ordinary D. E. — that has as its unknown the value of a function of a real or complex variable (expressed in terms of real or complex numbers) — of order n , has the following general form:

$$F(x, y', y'', \dots, y^{n-1}, y^n) = 0$$

where $y', y'', \dots, y^n$ respectively refer to the first, second, ... derivative of order n of the function $y = f(x)$. Beyond analytical methods for the calculation of the exact solution, there are numerical methods of approximation of the solution (e.g., the method of finite differences) for differential equations (or for systems of differential equations), some of which can be achieved through repetitive procedures that can be executed by a calculator. This makes it possible to simulate on a computer (at least partially) the behaviour of certain systems of differential equations, thus also dealing with those

for which there is no analytical solution (e.g., complex non-linear systems).

DOMAIN (SEE EXTENSION).

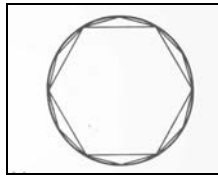
DOUBLE NEGATION (SEE REDUCTIO AD ABSURDUM)

DYNAMICS: (SEE MECHANICS)

EQUALITY: Mathematical relation often associated with different meanings and generally denoted by the symbol “ $=$ ”. As to a relation that is established between two expressions of members of the E., the term suggests that they denote the same numerical value or, at least, the same thing. If in the two expressions we find undefined terms, when the two members are equal, we will speak about an *equation*. If the two members are equal for all the values allowed for the variables (e.g., in the expression: $x^2 - y^2 = (x - y) \cdot (x + y)$), or if they do not contain undefined elements and individuate the same thing (e.g., in the expression: $3^2 + 4^2 = 5^2$), then we shall speak about an unconditioned equality, or *identity* (SEE). Equality is sometimes also used in order to define new notations; in this case the expression of the second member must only contain known elements. In this case, instead of using the symbol “ $=$ ”, we use the symbol “ $:=$ ” (equal by definition) so as to highlight the different role of the two members. For instance, $\tan(x) := \sin(x) / \cos(x)$. An evident abuse of the term E. and of its symbol “ $=$ ” occurs when they are used to denote an *equivalence* (SEE). (SEE IDENTITY; EQUIVALENCE).

EQUIVALENCE: Relation generally expressed by the symbol “ $\equiv$ ”, which satisfies the properties of *reflexivity* ($a \equiv a$), *symmetry* (if $a \equiv b$ then $b \equiv a$) and *transitivity* (if $a \equiv b$ and $b \equiv c$, then $a \equiv c$). In *propositional logic* (see) this corresponds to the relation of *bi-conditionality* (i.e., “if and only if” condition) between statements. In general, in *logic of classes*, when we deal with collections, two classes are said to be equivalent when they are composed by the same collection of elements (SEE IDENTITY; EQUALITY).

EXHAUSTION (PRINCIPLE OF): Principle defined by Eudoxus and that, in Euclid's *Books of Elements* is thus defined: "If from any quantity we subtract a part not smaller than its half [or third, or fourth, or fifth ... part] and if, from the rest, we subtract again no less than the second [or third, or fourth, or fifth... part], and if such process of subtraction is continued, in the end there will be a quantity that is smaller than any other quantity of the same kind previously given".



In this way, for the first time in mathematics, although in a form that is strongly related to a geometrical intuition, the concept of *limit* (see) was introduced, that was fundamental for the modern development of infinitesimal calculus and *mathematical analysis* (see), although in a context that is made independent of geometrical intuition and that is, nowadays, fully formalized and axiomatized.

EXTENSION: Notion of formal logics, taken from *mathematical analysis* (see) and from the theory of *mathematical functions* (see), by which we indicate the collection of terms (not of their possible extra-linguistic referents!) that constitute the *domain* of a given *predicate* (see). Such terms can possibly be considered *denoting* extra-linguistic (natural, ideal, logical, mathematical, imaginary, or other) objects, which, for this reason, are also defined as *referents* of the predicative statements that were built with a given predicate. For instance, given the predicate "to be water", the domain, or E., of such a predicate is constituted by all the names that indicate objects that are water (H₂O molecules, drops, rivers, streams, etc.). (SEE DENOTATION; EXTENSIONALITY, AXIOM OF; SUBSTITUTIVITY, AXIOM OF; EXTENSIONAL, LOGIC).

EXTENSIONAL (SEE EXTENSION).

EXTENSIONALITY (AXIOM OF): Axiom typical of mathematical formal languages, but, in general, of any scientific axiomatic system, according to which if two collections of objects, e.g., *A* and *B*, contain all and only the same elements, i.e., they are *equivalent* (see), they are the same collection. In symbols: $A \subseteq B \cdot B \subseteq A \Rightarrow A = B$, i.e., if *A* is included in *B* and *B* is in *A*, then *A* and *B* are identical (i.e., they are the same thing). This axiom is fundamental for all scientific definitions, in any formalized language, and not only in physical and

mathematical ones. Indeed, when we define an object, or a collection of objects (e.g., the fluid coming out of a faucet), in a given “technical” language, we do this by a certain predicative statement (e.g., in chemistry, by the statement “ x is H_2O ”) that, we presume, can replace that of another language (e.g., in ordinary language, “ x is water”), without changing the logical truth of the matter, simply because both of the predicates, and/or predicative statements, have the same *extension* (see), or *domain*. In other words, the axiom of E. immediately derives from the *axiom of substitutivity* (SEE). All languages that include among their own rules the axiom of E. and the axiom of substitutivity are considered *extensional*, or built in accordance with *extensional logic* (SEE). (SEE EQUIVALENCE, EXTENSION; EXTENSIONAL, LOGIC; SUBSTITUTIVITY, AXIOM OF)

FORMAL LANGUAGE (OR FORMAL THEORY). By formal language (or theory) we mean, in modern logic, a language that is built in a non-ambiguous way, i.e., a *formal system* (see) for which an *interpretation* (see) is offered. In other words, this is a language in which the terms and/or propositions belonging to it are rigorously *declared*, or *defined*, or *demonstrated* according to a non-contradictory form as soon as they are added to the language itself.

In particular, in such a language it is necessary to *declare*, in the first place, the *primitives* of that language, i.e., the basic terms and propositions (subject-predicate) that are not rigorously defined within that language, but that we presume to be known, since they will be used to build any following definition. What characterizes a formalized language are its *basic propositions*:

1) In the first place, we can find the *axioms*, i.e., propositions that are not demonstrated and that cannot be demonstrated within that language, from which further propositions are formed by demonstration. It is essential for the rigorous construction of a formalized language that its axioms are in a finite number, that they can be demonstrated not to be in contradiction, and that they are actually as such, i.e., not-deducible from other axioms of that language.

2) Another kind of basic proposition is represented by the *definitions* of the terms and operations used for those demonstrations.

3) There are, in the end, *inference rules* through which other propositions will be subsequently non-ambiguously demonstrated building on axioms and definitions.

All propositions built on *basic propositions* will be the *theorems* of that language.

A particular kind of formal language is represented by *operational languages* (see), that are typical of modern mathematical-experimental

sciences, and that should in no way be identified with formal languages *tout - court*. (See FORMALISATION, OPERATIONAL LANGUAGE, OPERATIONISM).

FORMAL SYSTEM (OR FORMAL CALCULUS). Symbolic system without *interpretation* (see), whose *syntax* (see) is defined in a rigorous way, and on the basis of which a relation of *deducibility* (see) is defined in purely syntactical terms. Like *formal languages* (see), F.S. consist of *primitive terms, definitions, axioms, inference rules* and *theorems*. In particular, the only “meanings” allowed for the primitive terms of a F.S. are those determined by their use within the axioms of the system. In this sense, we can say that the “primitives” “satisfy” their corresponding axioms.

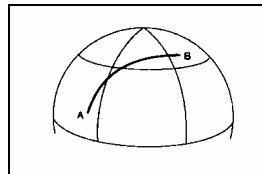
FORMALIZATION: Systematic effort to translate a theory, expressed up to that moment in a non-rigorous form, generally through ordinary language, in the terms of *formal language* (see). Thus, the term F. has practically become synonymous with axiomatization, or of the use of axiomatic method in the construction and exposition of scientific, mathematical, philosophical theories. (SEE FORMAL LANGUAGE, OPERATIONAL LANGUAGE, OPERATIONALIZATION).

FORMALISM (SEE FORMALIST, RESEARCH PROGRAM)

FORMALIST (RESEARCH PROGRAM): Research program in logical science, started by David Hilbert at the beginning of the 20th century, that identifies formal language with a pure manipulation of symbols according to syntactic rules, so as to set aside, in logical analysis, the semantic content of linguistic symbols themselves.

FUNCTOR: (SEE PREDICATE)

GEODETIC: The shortest curve, entirely situated on the surface, connecting two points on a curved surface. For instance, the bold-face curve in the figure is a G. connecting the two points A and B on the hemisphere. G. is also an arc of *maximum circle* (see) of the sphere.



GEOMETRY. Part of mathematics that studies the properties of figures, their location in space and their, even very abstract, generalisations. From the Greeks until the last century, it was the paradigm of every apodictic-deductive science. Nowadays, after its rigorous axiomatiza-

tion that changed it into a hypothetical-deductive science, several different branches can be distinguished, such as analytical, differential, Euclidean, non-Euclidean, Riemannian, projective... G.

HISTORICAL-DIALECTICAL METHOD: Method of the Hegelian philosophy that aspired to reduce philosophy to history of philosophy, applying to history itself the dialectical method of Hegelian logic and metaphysics, that were ultimately considered *identical*, according to the guiding principle: *the ideal is real* and the other way around (= transcendental idealism). This method intended to ground differences, i.e., determinations, in being through the logical procedure of double negation (the *reductio ad absurdum* (see) of classical logics). Put another way, supposing that being is the most abstract and undetermined concept (the most general of *genres*) and, accordingly, the negation of every determination and specification, it was argued that by progressively denying every negativity (indetermination) it was possible to found, step by step, through such a dialectical process, the whole set of missing determinations that separate one entity from the others. The reference to history was thus a guarantee of the necessary condition for such a process, namely that it was never to pass where it had already passed (history never repeats itself identically). The systematic mistake of this method is that it forgets the sufficient condition for its existence, namely that the *reductio ad absurdum* can never be used as a constructive method (= constitutive of the existence) of objects (as, for instance, the differences between the entities), but its function can only be that of highlighting what in another way (e.g., through appropriate axioms of existence, like in mathematics as to the notion of *continuum*) was already taken as existing. In this case, there will be two possibilities available:

- 1) *either being already has in itself all determinations*, all differences among entities that the dialectical method will simply highlight, without ever aspiring to constitute them. This was, for instance, Plato's position, that is also re-proposed by Emanuele Severino, within a modern immanentistic approach, by his idea of *whole*, or *totality of being* already defined and differentiated in full and where, accordingly, there is neither time nor becoming (except as mere appearance internal to the dialectical process of manifestation/hiding of the differences themselves). However, such an approach, precisely in light of its immanentism, becomes contradictory like all notions of *infinite universal totalities* (e.g., total classes, or universal sets) that aspire to have in themselves the graph (fully defined) of the relations between the elements differentiated within them.
- 2) Or, *it is necessary to hypothesize a causal foundation of being and of its determinations in different entities*, "from the outside" of the universe of entities, and,

accordingly, “from the outside” of the common being (existence) of all contingent beings. This is the scheme of the participation in *being as act*, as the principle of all determinations, that was developed in the Middle Ages by Thomas Aquinas, and re-proposed in the 20th century by Cornelio Fabro, in order to solve the systematic inconsistencies of modern immanentistic metaphysics, starting with Hegel’s.

IDENTITY: Relation associating identical elements. Generally and roughly speaking, one or more things are identical if these are the same thing. Logically (semantically), two expressions (terms or propositions) having the same form are identical if and only if they denote the same object. Such an object is what the Ancients defined as *essence*. Therefore, metaphysically, we can say that two objects are identical if *their essence is one* (Aristotle), where “is” is to be understood not in the sense of “to exist”, but in the sense of the “being of the essence”, or *entitas* (Aquinas). For instance, a man who is a child before and an adult afterwards is the same entity, or the same thing (*entitas*), because their essence is one. Mind that it is incorrect to say that they “share the ‘same’ essence”, because the essence is an incomparable *unicum*, that makes reference only to itself. Indeed, if we were to use such an incorrect expression about the essence, we would open the way to an infinite regression problem, since it would make sense to ask: “same” in relation to what? In this sense, every individual is identical only to his/herself, since s/he is, before any other attribution, his/her own *individual, unique* essence that makes her/him different from any other individual. In the terms of the Thomistic doctrine of transcendentals, we can say that every entity is a unique, incomparable and unrepeatable *entitas*, besides its being an *esse aliquid*, i.e., a “being something” in relation to something else. Moreover, two separate individuals can be said to be identical if and only if they respond to the same definition, i.e., if the same *quidditas*, or *specified* essence, can be correctly attributed to them. For instance, Jack, John and Mary, if considered in themselves as individuals, are non-identical. However, since they are human, i.e., they are identically “human beings”, they satisfy the definition of “being human”. Anyway, it is a serious mistake, typical of scientific reductionism, to mix up I. with *equivalence* (SEE) and *equality* (SEE).

IDEOLOGY: Collection of ideas, more or less systematized in a consistent set, that purports to encompass all of reality, but which in fact works for the pursuit and maintenance of the interests and/or ends of a particular social group.

INCOMPLETENESS (THEOREMS OF): Demonstrations originally achieved by Kurt Gödel concerning axiomatic arithmetic, but then

extended by A. M. Turing and A. Tarski to all formalized languages or axiomatic systems, having a minimum degree of consistency so as to include formal arithmetic — generally, the arithmetic formalized in Peano's axiomatic paradigm (see footnote 20). According to such theorems, every formal language that satisfies the above-mentioned requirements *necessarily* contain *undecidable* formulas, i.e., that do not have the property of *decidability* (see). Therefore, none of those languages has the property of *completeness* (see) [First theorem of incompleteness]. Among such undecidable assertions a particular meaning is attached to *self-referential* assertions, i.e., those assertions that are constructed in that language through which such semantic properties as *consistency* (see) [Second Gödel theorem] or the *truth* [Tarski Theorem] of that language are affirmed. (SEE COMPLETENESS).

INTENSION: According to a common understanding, I. indicates “what is meant” by a given term or proposition, *qua* distinct from the object to which the same term or proposition makes reference. For instance, what is meant by (i.e., on the intension of) the term “salt”, *qua* distinct from the extra-mental physical object is the white crystal matter contained in a saltshaker. In logic and in linguistic analysis, the I. (which is not to be confused with the epistemological and psychological notion of “intention”) is the term by which we define the connoting *property* — as separate from the *extension* (see) or *domain* — of a given predicate. In other words, by I. we define the *reference* — as separate from the *referent* (see) — of predicative statements that are built by that predicate. Generally speaking, the I. indicates that quality, or property, expressed by the predicate and attributed by it to all those objects denoted by the terms that constitute the *domain*, or *extension* (see), of that predicate. A characteristic of the I. of a predicate is that two different predicates, having different I., may have the same referent and, formally, the same extension. Put another way, two predicates can make reference to the same object, or collection of objects, having different I., or different meanings (e.g., the predicate “to be water” and the predicate “to be H₂O”). In short, the *axiom of extensionality* (see) does not apply to intensional objects and meanings. From the viewpoint of the referential object, this implies that the same object, in different contexts, can be characterised by different qualities, or properties.

INTENSIONAL (V. INTENSIONAL LOGIC).

INTENSIONAL LOGIC . Part of the *logic of propositions* (see) in which the truth-value of composite propositions does not only depend on the truth-value of the propositions, but also on considerations regarding the *meaning*, or *intension* (see), of *propositions* and/or, within basic

propositions, the *meaning*, or *intension* of the composing *terms*. For instance, the truth-value of the composite proposition «Caesar wrote the *De Bello Gallico*, while he was fighting in Gaul» cannot be decided only on the basis of the (indubitable) truth alone of the two composing propositions, but it depends also on considerations regarding their meaning. In other words, in order to decide the truth of the proposition «It is necessary that all men die» it is not enough to know that it is true that all men die, but we need to know that mortality is an *essential* property of theirs. There are several L.: *temporal*, *modal*, *deontic*, *epistemic*, etc. all sharing the negation of the *axiom of extensionality* (see).

INTERPRETATION. Attribution of meaning to the terms of a *formal system* (see). This is an attribution of *denotation* (see) to a term, or to the *extension* (see) of a predicate in a formal system, so that the *truth* (see) of the system have a value of truth in the interpretation.

ISOTROPOUS. Feature of abstract mathematical and physical space according to which all directions around a point are equivalent. Thus, for instance, in an I. physical context (e.g., a gas), all light waves are propagated at the same speed in all directions. Under the same conditions, the same property applies to thermal conductivity, electrical resistance, etc. that have the same values in all directions. Contrary: *anisotropic* (see).

KINEMATICS (SEE MECHANICS).

LIMIT. Value l that is associated with a *mathematical function* $f(x)$ (see) and with a point of accumulation a belonging to the set $\{x\}$ that represents its *domain* (see). This value l exists if, having set an arbitrarily small number ϵ , it is possible to individuate a surrounding of a so that for every value of x belonging to it and different from a , $f(x)$ differs from l for a quantity δ , smaller than ϵ . In this case, we say that $f(x)$ tends to l for x tending to a . The concept of L. is fundamental for a rigorous, axiomatic presentation of all infinitesimal calculus (see MATHEMATICAL FUNCTION, DERIVATIVE).

LOGIC OF CLASSES (SEE PREDICATE LOGIC)

LOGIC OF PROPOSITIONS: Part of logic that studies propositions and their connectives in the constitution of a language. The L. is sometimes also defined as *calculus of propositions*, or *propositional calculus* as far as, following the *formalist program* (see) identifying formal language with a pure manipulation of symbols in accordance with syntactic rules, we set aside the semantic content of the symbols themselves. The L.P. conceived in these terms is also referred to as extensional,

since – following Frege – the extension of a statement is identified with its truth-value. In ordinary propositional calculus, the truth-value of composite propositions is only determined by the truth-value of the single propositions. Accordingly, logical connectives, or propositional predicates, are defined as *truth-functional*. However, it is clear that not all L.P. can be identified with extensional logic. Most propositions of ordinary language and, in general, of non-scientific languages (i.e., philosophical, ethical, juridical, religious, etc.) follow an *intensional logic* (see).

LOGIC OF TERMS: Part of logic that has as its object of study the terms constituting a proposition, and not the propositions themselves (SEE LOGIC OF PROPOSITIONS; PREDICATE LOGIC).

MATHEMATICAL ANALYSIS. Part of mathematics that can be defined in general as having as its object the study of *mathematical functions* (see), i.e., functions with either real or complex variables, and their manipulations, as the calculus of *derivatives* (see), of integrals, of serial developments, etc., and of the possibility of having them as solutions of *differential equations* (see), integrals, and so on.

MATHEMATICAL FUNCTION: (or “application”, “map”, “univocal correspondence”). Relation between two sets of mathematical quantities, respectively named *domain* (see) and *co-domain* (*range*) of the function. It associates to one or more elements of the domain (independent variable(s)) one and only one element of the range (dependent variable, or image). In short, a mathematical function can be written thus $y = f(x)$, where y represents the dependent variable, or “co-domain”, and x is the independent variable, or “domain” (see DERIVATIVE, LIMIT).

MAXIMUM CIRCLE. A circle drawn on the surface of a sphere that has its very same radius. The minor arc of a maximum circle, passing by two points placed on the surface of a sphere, is the shortest of all the arcs connecting them, or *geodesic* (see), entirely situated on the surface.

MECHANICS: Part of physics that studies the motion of bodies and the laws that govern it. M. can be divided into three branches: 1) *kinematics*, that studies the geometrical formalization of the motions of bodies, i.e., the motion of bodies independently of the forces that determine it; 2) *statics*, that studies the forces that operate on a body at rest; 3) *dynamics*, that studies the laws of motion in relation to the forces that determine the motion itself.

META-LANGUAGE: Language used to describe and define whole categories of languages. As to Tarski, Carnap and Gödel’s meta-

linguistical theory (= *formal semantic theory*) the M. must be necessarily of a superior logical order in comparison with the object-language that it seeks to define together with its fundamental semantic notions (i.e., consistency, truth, etc.).

META-LOGIC: According to our very generic conception, we mean by M. every theory concerned with the foundations of logic (SEE META-LANGUAGE).

META-MATHEMATICS: According to our very generic conception, we mean by M. every theory concerned with the foundations of mathematics.

MODEL. An *interpretation* (see) of a *formal system* (see) in relation to which the theorems that can be derived within that system are *true*, i.e., a part of a given *formal language* (see), or *formal theory*, reflecting some aspects of a phenomenon, or of a physical, social, or technological process, and allowing predictions in relation to that. In this sense every *scientific theory applied* to the study of some objects of the physical or human world, *qua* formal language or theory, is a M. of an underpinning formal system. For instance, *arithmetic*, *qua* scientific theory of natural numbers, is a M. of the formal system based on Peano's five axioms.

OPERATIONAL LANGUAGE: Particular form of formal language to express scientific theories of an experimental kind, both in natural and human sciences. Such a language adds to the usual axiomatization, typical of every formal language, the further condition of the transformation of every event or process expressed, described or explained by that theory in terms of a mathematical operation on quantities that can be measured and experimentally controlled. Such control can be performed either by means of an appropriate system of measurement, or through a simulation on a system of numeral calculus (PC). The definition of the operation of measurement and/or the method of numeral simulation, as methods of experimental control of the assertions and predictions of the theory, is a part of its *operationalization* (see), i.e., its translation in terms of an operational language.

OPERATIONAL METHOD (SEE OPERATIONALIZATION).

OPERATIONALIZATION: Concise term meaning the use of the operational method in modern empirical and mathematical sciences. By O. we mean the systematic effort to translate into the terms of an *operational language* (see) a theory about certain facts, that are objects of experience, that was expressed up to that moment in a non-rigorous

way, by means of ordinary language (e.g., in a certain philosophical theory). In general, *formalization* (see) can represent the first step towards O., but it should not be confused with it, otherwise *operationism* (see) can be jeopardized. Indeed, certain theories (e.g., metaphysical theories) can be formalized, but not operationalized, since many of the objects that metaphysics deals with are not objects of experience, let alone of measurement. (SEE OPERATIONAL LANGUAGE, FORMAL LANGUAGE, FORMALIZATION, OPERATIONISM).

OPERATIONISM: (SEE OPERATIONIST, RESEARCH PROGRAM)

OPERATIONIST (RESEARCH PROGRAM): Philosophical theory typical of Neo-Positivist science and of its scientific reductionism, according to which only *operational languages* – and the theories that are couched, or that can be couched, in that form – make sense (see OPERATIONAL LANGUAGE, FORMAL LANGUAGE).

PHASES SPACE: Rational mechanics and statistic mechanics take into consideration the phases space, i.e., the space having $2n$ dimensions given by the n coordinates and by the n moments of a dynamic system with n degrees of freedom. The state of this system is described from a point in phases space, and its dynamic behaviour (that is a sequence of states) is described by a trajectory. In phases space Liouville's theorem applies: let us imagine the points of a region of the phases space on a surface: if the states described by the points of this region evolve dynamically, the points will move along a well-defined trajectory, compatible with the law of motion, and the surface that formerly contained it will change accordingly. Liouville's theorem maintains that in such change the volume contained in the surface remains constant. For instance, if we represent in a bi-dimensional P., placing on the x-coordinate the quantity of motion and on the y-coordinate the positions, the dynamic evolution of a pendulum that is slowing down its oscillation, such evolution will be represented by a trajectory having the form of a spiral in the bi-dimensional P., with increasingly narrow spirals, until its ending in a *fixed point* that represents the condition of final *static* stability of the pendulum, towards which such a system tends (= fixed point attractor). Conversely, an opportunely accelerated pendulum that is stabilized in a defined oscillation (e.g., a charged pendulum clock) will describe in the same bi-dimensional P., after a while, a trajectory having the shape of a closed curve, representing the condition of *dynamic* final stability of the pendulum, towards which such a system tends (= limit cycle attractor).

PRAGMATICS (SEE SEMIOTICS)

PREDICATE: In general, P. are defined as those parts of speech that, applied to terms and/or propositions, *constitute*, or *produce*, propositions. In modern formal logic they are also defined as *functors* determining a particular *argument* that can be constituted:

1) either by a *term*, whether it be a noun or a verb. In this case, P. are defined as *terminal predicates*. For instance, “to be red” has as its argument such nouns as “fire”, “blood”, etc., hence “fire is red”, “blood is red”;

2) or by one or more *propositions*. In this case, P. are defined as *propositional predicates*. Typical examples of these P. are *n* – argumental predicates, typical of the *logic of propositions* (see) (negation, implication, alternation, exclusion, etc.: *see below*), or, more generally, such semantic meta-predicates as “is true”, “is false”, “is consistent”, “is contradictory”, “means”, etc., like in the proposition “it is true that men are rational animals”.

This definition of P. as functors is connected to the origin of modern symbolic logic, and in particular it is connected to Frege and to his definition of the expressions that are object of analysis of formal logic as *propositional functions* (see). Therefore, in general, in Russell’s symbolism, for example, terminal predicates are written by such expressions as “ $A(x)$ ” or, more synthetically without using parenthesis by means of Greek letters, “ ϕx ” – where x denotes the argument and the first letter the P. – in accordance with mathematical symbolism of functions $f(x)$.

Below is a table illustrating the main propositional *mono* and *bi* – argumental predicates of the *logic of propositions* (see), according to Lukasiewicz’s, Russell’s and Hilbert’s symbolism:

	SYMBOLISM OF			EXAMPLES
	ŁUKASIEWICZ	RUSSELL	HILBERT	
PROPOSIT. VARIABLES	$p, q, r, s, \dots$	$p, q, r, s, \dots$	$A, B, C, D, \dots$	
NEGATION	Np	$\sim p$	$\overline{A}$	"It is <i>not</i> raining"
IMPLICATION	Cpq	$p \supset q$	$A \rightarrow B$	"If it rains, <i>then</i> it is cold"
ALTERNATIVE	Apq	$p \vee q$	$A \vee B$	"It rains, or it snows, or <i>both</i> of them"
EXCLUSIVE	Dpq	$p \mid q$	$A \mid B$	" <i>Either</i> it rains, or it is sunny"
EQUIVA- LENCE	Epq	$p \equiv q$	$A \sim B$	"It rains <i>if and only if</i> drops of water fall from the sky"
COPULATIVE	Kpq	$p \cdot q$	$A \& B$	"It rains <i>and</i> it is windy"

Let me explain what this means in practice. For instance, take the structure of the so-called *first figure syllogism* (see) studied by Aristotle, according to Hilbert's symbolism:

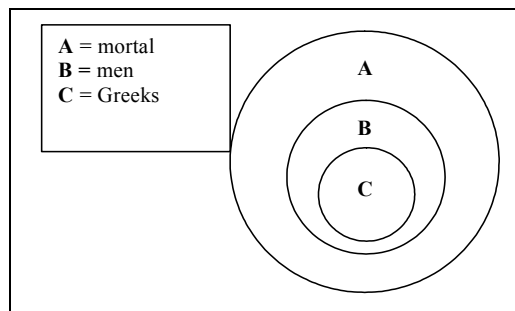
$$\begin{array}{c}
 A \& B \\
 B \& C \\
 \therefore A \& C
 \end{array}$$

"All (the entities that are) B are also A , all C are also B , then all C are also A ". It is clear that no expression that may be used instead of the variables represented by the alphabet letters A, B, C , changes the validity of the reasoning (syllogism), as long as each variable is consistently replaced by the same expression throughout. For instance, "All (the entities that are) men are (also) mortal, all (the entities that are) Greeks are (also) men, thus all (the entities that are) Greeks are (also) mortal". Or, "All quadrilaterals are polygons, all rectangles are quadrilaterals, thus all rectangles are polygons". According to Russell's interpretation of syllogistic calculus, each of the three formulas within the above-mentioned

syllogism should be interpreted through the notion of *formal implication* (= implication with apodictically true premises) thus justified in a purely *extensional* sense:

$$\forall x (Bx \supset Ax) \cdot \forall x (Cx \supset Bx) \supset \forall x (Cx \supset Ax)$$

i.e., “for all x : if x is B, then x is A and if x is C, then x is B; thus, if x is C, then x is A”. Its justification is purely extensional since, from Leibniz onwards, this fundamental kind of syllogism has been interpreted as the inclusion of three classes of objects. More precisely, given the *axiom of comprehension** of Russell’s theory of classes, each of the three predicates, Ax , Bx , Cx defines a class of objects, respectively **A**, **B**, **C**. A correspondence is thus established between the *implication* “ $\supset$ ” between the propositions constituted by their predicates and the inclusion “ $\subseteq$ ” of the corresponding classes. Thus, for instance, there is a correspondence between the implication “ $\forall x (Bx \supset Ax)$ ” and the inclusion of the corresponding classes **B** $\subseteq$ **A** (“**B** is included in **A**”). Indeed, if it is true that for all entities x of a given collection (e.g., that of animals) to be a man implies to be mortal, this is because the class of men is included in the class of mortals. According to Euler’s set theory with concentric circles, this extensional interpretation of Aristotelian first figure syllogism can be thus summarized:



PREDICATE LOGIC: Part of the *logic of terms* (see) that studies particular terms of propositions, namely predicates. P. is sometimes also called *predicate calculus* as far as, following the *formalist* (see) program identifying formal language with a pure manipulation of symbols in accordance with syntactic rules, we set aside the semantic content of the linguistic symbols themselves. In this sense, the domain of objects on which predicates are defined is exclusively identified with *names*, taking no notice at all of the semantic problem of the *designation* of

objects by means of those names. P. is also deeply connected to the logics of classes, where by class we mean that collection of objects to which the predicate designating the class can be univocally attributed.

PROPOSITIONAL FUNCTION: Expression in which the formal structure of the proposition is treated in an analogous way with mathematical expressions and, in particular, with *mathematical functions*. Indeed, as Aristotle taught us, what generally characterizes the validity of demonstrative procedures is the *form* of the demonstration, regardless its *content* (= terms or propositions having various meanings), which can thus vary without exercising any influence on the validity of the demonstration. In this way, every expression containing *variables*, regardless of the fact that these are terms or propositions, can be defined as a *propositional function*, modelled on *mathematical functions* (see) of the kind $f(x)$, which are similarly expressions containing *variables*. On the basis of such analogy, the *predicate*, or *functor*, corresponds to the “*f*” considering that both of them determine their *variable argument* “*x*”. However, it is necessary to operate a distinction between propositional functions and mathematical functions, in accordance with two series of reasons:

- 1) because, whereas the variables in the former functions can be replaced by terms or propositions, in the latter they can be replaced only by numerals, i.e., by terms indicating numbers;
- 2) because, whereas the former functions can be said to be true (or false) if and only if the variables are *explicitly bound* by the use of quantifiers, the latter do not require that *quantifiers* are made explicit, even if we can say that they always make reference to the “universal quantifier” (“for all” and “for each”).

RATIONAL MECHANICS: Axiomatic formulation of classical mechanics built following an analogy with geometry as a hypothetical-deductive *formal system* (see). Its axioms are the fundamental laws of mechanics and, its primitives are such notions as material point, rigid body, frictionless joint, perfect fluid, etc. that are idealizations of concrete physical objects.

REDUCTIO AD ABSURDUM: or “method of double negation”. Dialectical procedure of demonstration through which a given proposition is necessarily concluded from certain premises since its negation is demonstrated to be contradictory. The demonstrative formula is as follows: “it *cannot not* be”. For instance, in the famous method of “exhaustions”, invented by Eudoxus (IV century B.C.), given a circumference, an inscribed and a circumscribed polygon, if we infi-

nately multiply the sides of the two polygons, their perimeter *cannot not* to coincide with the circumference. This kind of demonstration is said to be *weak*, because it has always to suppose as already “elsewhere” *logically existent*, i.e., as fully *determined* and *defined*, both the *term* to demonstrate (= notion of limit of two successions) and the *compact* and *homogeneous infinity* of the terms that are necessary to achieve it (= notion of continuous). In other words, the R.A. is not a constructive method of demonstration: for this reason (among others) the Hegelian *historical-dialectical method* (see) is logically inconsistent. In our intuitive example, the circumference has to be already described if we want the demonstration to be valid. If we wanted to construct the circumference by infinitely multiplying the sides of polygons, at what number of sides should we stop to complete our construction? Infinity can never be approximated “piece by piece”: an infinite in act, i.e., progressively actualized building on an infinite in potency (= privative infinite), is a contradictory notion, unlike the idea of actual infinity (= negative infinity). Archimedes tried to exploit this limit of the R.A., proposing a “heuristic”, or “mechanical”, method for the demonstration of geometrical theorems. This consisted of a first empirical calculation, through experimental simulations, of the solution to a geometrical problem (e.g., the center of gravity of a fulcrum), then he tried to demonstrate the logical necessity of such a solution through the R.A. By this he provided us, for the first time in Western history, with a physical-mathematical example, although we have to wait until the modern creation of infinitesimal calculus and of *mathematical analysis* (see) by Newton, and their axiomatic justification in the works of D’Alembert, Cauchy and Weierstrass, in order to see Archimedes’ “mechanical method” acquire the dignity of a “formal system”.

REFERENT. Linguistic object denoted by a given part of language, whether a term or a proposition.

REFERENCE (SEE INTENSION).

SELF-REFERENTIAL: Property of a language to have the capacity to speak of itself and, in particular, of the rules on which the meaning of its formulas is based (e.g., in natural languages, it is possible to write grammars of a given language in that same language, i.e., to write in Italian grammars of Italian). In formal languages, self-referential propositions are the best candidates for undecidability (see *completeness*). Thus, we generally maintain that natural languages are self-referential only because they conceal inconsistencies (i.e., contradictions) and/or ambiguities.

SEMANTICS (SEE SEMIOTICS). (IN FORMAL LANGUAGES (SEE)). Study of the *interpretations* (see) and of the *models* (see) of a formal system, and, accordingly, of the relation between the *structure* (see) of a formal system and all of its objects. Accordingly, S. has as its object the principles determining either the truth or falsity of formulas, and the references of terms within a given *formal system* (see), or *formal language* (see).

SEMIOTICS: By S. we mean the general theory of signs, regardless of their being linguistic or of some other kind. From this perspective, logic can be considered as a part of S., namely that part dealing with the theory of linguistic signs. By *syntax* (see) we mean, in general, the theory of relations between signs; by *semantics* (see) we mean the theory of signs in relation to their meanings; by *pragmatics* we mean the theory of signs in relation to those who use them. In the domain of semantics, we can operate a distinction between *denotation* – or “meaning” of a sign, following Frege –, conceived as the reference of a sign to an object, and *connotation* – or “sense” of a sign, following Frege – as that through which a sign denotes an object and thus somehow expresses (e.g., in a linguistic form) its meaning (e.g., as a set of properties characterizing the object). For this reason, connotation is also defined as the semantic “content” of a symbol. The object denoted, or indicated, by the linguistic sign is also defined, in S., as the *referent* (see) of the predicative statement, whereas the connotative content expressed by the sign represents the *reference* (see) or *intension* (see) of the statement.

STATICS (SEE MECHANICS)

STOCHASTIC (SEE STOCHASTIC SYSTEM)

STOCHASTIC SYSTEM. This refers to any physical or mathematical process in which there is at least one casual or uncertain variable. A classical example of a S. is that following Langevin's equation which, in the definition of the variation of the state of the system, clearly separates the deterministic term from the non-deterministic, or uncertain, one.

STRUCTURE. Attribution to a language of a non-empty set of elements (or *universe*) constituted by *individuals*, and, accordingly, attribution of predicates (see) to the symbols of that language. A S. for a formal system in which some axioms are true is defined as *model* (see) of that system.

SUBSTITUTIVITY (AXIOM OF): Typical axiom of mathematical formal languages, but also, in general, of any axiomatic scientific system, ac-

according to which if two statements, f and f' , are *equivalent* then one can be substituted by the other without influencing the truth-value (either truth or falsity) of the statement itself. A rather simple case of *equivalence* (see) between statements is when the predicates that constitute them have the same extension, or domain. In this sense we say that the axiom of S. immediately derives from the *axiom of extensionality* (see). However, the notion of *equivalence* (see) is larger than that of *identity* (see), since two statements can be equivalent, i.e., they can satisfy the *bi-conditionality* relation, even if their meanings, or *intentions* (see), do not satisfy the *identity* relation, even though it is always the case the other way around. The problem is that the notion of identity always implies also that of the identity of the classes of belonging, as it is spelled out by the *axiom of extensionality* (see). Conversely, the notion of equivalence is not expected to satisfy such a condition. In light of this, it is necessary to distinguish between substitutivity of statements in view of their equivalence, and substitutivity of statements in view of the identity of their corresponding referents.

For instance, if we admit, *ex hypothesi*, that “Frank met John if and only if Frank was in Rome”, the truth of the expression “either Frank met John, or John could not sell Frank the car” does not change if we replaced the first statement of the alternative sentence by its supposed equivalent, so as to obtain “either Frank was in Rome, or John could not sell Frank the car”. This does not absolutely imply the identity of the class of objects defined by the predicate “being in Rome” and that of the objects defined by the predicate “to be met by John”, or that the object “Frank” can only be defined as belonging to these two classes (since “to be Frank” is not the same as the sum of “being in Rome” and “being met by John”) although in this example “Frank” actually was an element belonging to both classes.

These paradoxes of substitutivity are at the basis of the Aristotelian distinction between “essential definition” (*per se*) and “accidental definitions” (*per accidens*) of an object. According to Aristotle, two objects are indeed *identical*, only if their essence is *one* (mind: not if they “have” the same essence, since every entity *is*, does not “have” its essence, see § 5.4.2; see *identity*); they are, instead, *similar* if they have the same quality, or are *equal* if they have the same quantity. Since modern science has chosen, in general, to define the identity of its objects simply by means of the fulfilment of the equivalence condition (bi-conditionality) of the corresponding quantitative and/or qualitative statements denoting them, it is precluded from the possibility of accruing a distinction between what is essential, or necessary

(*per se*), and what is merely accidental, or occasional (*per accidens*), in the definition of an object (SEE IDENTITY; EQUIVALENCE; INTENSION; EXTENSIONALITY, AXIOM OF)

SYLLOGISM (= literally: technique for combining words). In general, deductive inference composed by three propositions (namely, major premise, minor premise and conclusion), constructed in order to demonstrate the conclusion from the major premise, through the mediation of the minor premise. According to Aristotle — who first systematically studied the S., making it the main object of study of formal logic — this is the perfect kind of deductive reasoning, and has, as its essential characteristic, that of proving the necessary connection between the subject and the predicate of some propositions (or conclusions), building on the necessary connection between subject and predicate of other propositions that were appropriately chosen (premises). This chain of connections allows us to infer some assertions from others (see PREDICATE). Besides the different forms of deductive syllogisms (categorical or apodictical, with true premises; and hypothetical, with hypothetical premises), Aristotle also studied inductive syllogisms, in which the conclusion, opportunely generalized, can be taken as a universal premise for a hypothetical deductive syllogism (causal syllogism), typical of physical sciences.

SYNTAX (see SEMIOTICS). (in FORMAL LANGUAGES (see) and in FORMAL SYSTEMS (see)). A system of rules (generally expressed in terms of an *algorithm* (see)) determining all *well-formed formulas* (see) within a given *formal system* (see). Such rules are fully established in the terms of the structure to which they are meant to be applied, but they are completely independent of the meaning, or of the truth of the formulas to which they are applied.

UNDECIDABILITY (SEE DECIDABILITY, COMPLETENESS, INCOMPLETENESS).

WELL-FORMED (sentence, formula, expression, etc.). Built in order to be *grammatically correct*. In formal systems and languages, a *well-formed formula** (wff) is that built in accordance with the rules of formation (construction and transformation), or *syntactic* rules of a given *formal language* (see), or *formal system* (see).

WFF. *Well-formed* formula (see WELL-FORMED).

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